Warm-Up 12 Solutions

1. $ 33 — 35% = 7/20. 7/20 · 1/4 · 880 = 33.

2. 212 or 212_3 — The largest 3-digit prime in base-3 is 222_3 = 2(9) + 2(3) + 2 = 26. The greatest prime less than 26 is 23, which is 2(9) + 1(3) + 2 = 212_3.

3. 2.56 oz — The new cube will be 4cm tall, and the ratio of the volume of the new cube to the original is $4^3/5^3$ = 64/125. Multiply this by 5oz to get 64/25 = 2.56oz.

4. 56 — The least possible median for a set of distinct positive integers is 4 (the smallest integers would be 1, 2, 3, and 4). This makes the mean 8 and the sum of the seven integers 7(8) = 56.

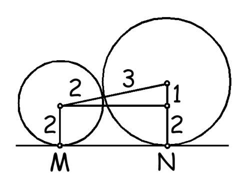
5. ¾ — There are ${}_4C_3$ = 4 ways to select three poles from a set of 4. Of these sets, only the combination of 3m, 5m, and 9m poles will not form a triangle (because 3 + 5 < 9). This gives us a probability of (4-1)/4 = ¾.

6. 6 — The prime factorization of 990 is $2 \cdot 3^2 \cdot 5 \cdot 11$. We are looking for factors which end in 5. These are all odd multiples of 5, which means that the factors cannot include a 2 but must include a 5. We can use 0, 1, or two 3's and 0 or one 11 along with the 5 to create each factor for a total of 2 · 3 = 6 factors (5, 15, 45, 55, 165, and 495).

7. 9 — There are a number of ways to find coordinates B(9, 3) and D(6, -4). Easiest involves sketching a quick graph. It is also possible to find the equation of the line which passes through B and D, which is the perpendicular bisector of the segment passing through A and C. Plug-in the given values to solve for x and y. This is much more difficult. Either way, we get 3 + 6 = 9.

8. 64 — There are ${}_7C_1$ = 7-unit paths (choose and of the 7 vertical lines to use to move down a unit). We can trace a 9-unit path by going down, then up, then down again while moving from left to right. This requires that we select 3 of the seven vertical segments. There are ${}_7C_3$ = 35 ways to do this. Continuing, there are ${}_7C_5$ = 21 ways to trace an 11-unit path and ${}_7C_7$ = 1 path that is 13-units long and uses all 7 vertical segments. 7 + 35 + 21 + 1 = 64 paths.

9. $2\sqrt{6}$ cm — Given the diagram as shown, we are looking for the long leg of a right triangle of hypotenuse 5 and short leg 1. Apply the Pythagorean theorem to get $\sqrt{24}$ or $2\sqrt{6}$cm.



10. 1/3 — There are 4! = 24 ways to arrange the digits. The divisibility rule for 11 requires that the difference of alternating digit sums be equal or divisible by 11. Because the digit sum for the whole number is 10, alternating sums must be 5: 1+4 and 2+3. This gives us 4 · 2 = 8 possible multiples of 11 (the first digit can be any of the four digits and the next can be one of the two from the other pair: 1243, 1342, 2134, 2431, 3124, and 3421 4213, and 4312). 8/24 = 1/3.

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