
MATHCOUNTS®

2004

■ State Competition ■
Sprint Round
Problems 1–30

Name _____

School _____

Chapter _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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9. For the set of numbers $\{a, b, c, d, e\}$, the following five inequalities are true: $c < d$, $a < b$, $d < a$, and $e < d$. Which number in the set is the median?

9. _____

10. Using the map to the right, John is creating a new map with a scale of $1 \text{ cm} = 5 \text{ km}$. The representation of the portion of road between the two points on the map shown here is 3.75 cm . In centimeters, how long will this same stretch of road be on John's new map?



10. _____

11. In an arithmetic sequence, the 7th term is 30, and the 11th term is 60. What is the 21st term of this sequence?

11. _____

12. What number, when added to the numerator and to the denominator of $\frac{5}{8}$, results in a fraction whose value is 0.4?

12. _____

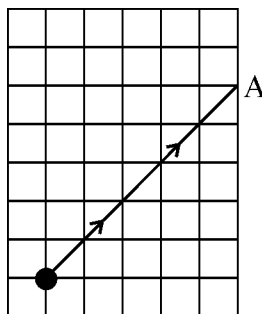
13. Let m and n denote the greatest and least positive three-digit multiples of 7, respectively. What is the value of $m + n$?

13. _____

14. Of the final five contestants in a television show, three are female and two are male. If two are chosen randomly to be the final contestants, what is the probability that both are female?
Express your answer as a common fraction.

14. _____

15. A ball is shot from the lower left part of the table along a path of 45 degrees, as shown. After contact with a side, it continues along a path that is a reflection of the path prior to contact. The line of reflection is the line perpendicular to the side of the table the ball hit, at the point of contact. The first point of contact is labeled A. After initially shot, how many times will the ball touch a side of the table before it reaches a corner of the table?



15. _____

16. A positive multiple of 45 less than 1000 is randomly selected. What is the probability that it is a two-digit integer? Express your answer as a common fraction.

16. _____

17. What is the sum of the tens digit and the units digit in the decimal representation of 9^{2004} ?

17. _____

18. Forty-three percent of Americans have Type A molecules in their blood, 15% have type B molecules, and 46% have neither Type A nor Type B molecules. What percent of Americans have both Type A and Type B molecules in their blood?

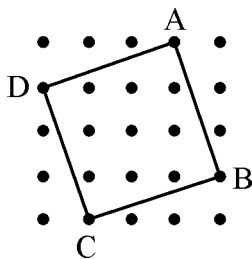
18. _____

19. The points (x, y) represented in this table lie on a straight line. The point $(13, q)$ lies on the same line. What is the value of $p + q$? Express your answer as a decimal to the nearest tenth.

x	y
2	-5
p	-14
$p + 2$	-17

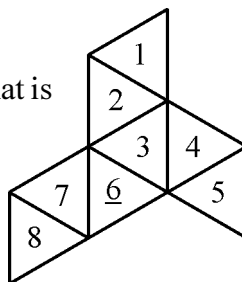
19. _____

20. On the 5 by 5 square grid below, each dot is 1 cm from its nearest horizontal and vertical neighbors. What is the product of the value of the area of square ABCD (in cm^2) and the value of the perimeter of square ABCD (in cm)? Express your answer in simplest radical form.



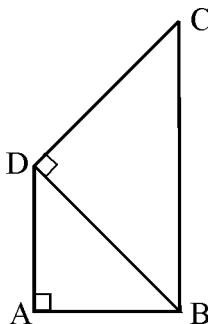
20. _____

21. This net is folded into a regular octahedron. What is the sum of the numbers on the triangular faces sharing an edge with the face with a "1" on it?



21. _____

22. Each triangle in this figure is an isosceles right triangle. The length of \overline{BC} is 2 units. What is the number of units in the perimeter of quadrilateral ABCD? Express your answer in simplest radical form.



22. _____

23. Of the five points $(3, 10)$, $(6, 20)$, $(12, 35)$, $(18, 40)$ and $(20, 50)$, what is the sum of the x -coordinates of the points that lie in the region above the line $y = 2x + 7$ in the coordinate plane?

23. _____

24. The terms $x, x + 2, x + 4, \dots, x + 2n$ form an arithmetic sequence, with x an integer. If each term of the sequence is cubed, the sum of the cubes is -1197 . What is the value of n if $n > 3$?

24. _____

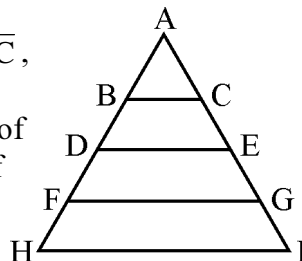
25. The set $\{5, 8, 10, 18, 19, 28, 30, x\}$ has eight members. The arithmetic mean of the set's members is 4.5 less than x . What is the value of x ?

25. _____

26. If all angles are measured in degrees, the ratio of three times the measure of $\angle A$ to four times the measure of the complement of $\angle A$ to half the measure of the supplement of $\angle A$ is 3:14:4. What is the number of degrees in the measure of the complement of $\angle A$?

26. _____

27. Triangle AHI is equilateral. We know \overline{BC} , \overline{DE} and \overline{FG} are all parallel to \overline{HI} and $AB = BD = DF = FH$. What is the ratio of the area of trapezoid $FGIH$ to the area of triangle AHI ? Express your answer as a common fraction.



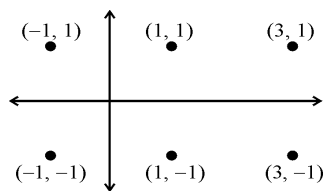
27. _____

28. The dartboard below has a radius of 6 inches. Each of the concentric circles has a radius two inches less than the next larger circle. If nine darts land randomly on the target, how many darts would we expect to land in a non-shaded region?



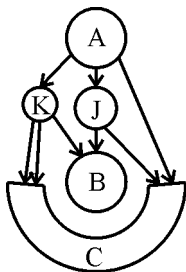
28. _____

29. The graph shows six labeled points. How many distinct circles of radius 2 units are in the coordinate plane and pass through exactly two of the labeled points on this graph?



29. _____

30. Regions A, B, C, J and K represent ponds. Logs leave pond A and float down flumes (represented by arrows) to eventually end up in pond B or pond C. On leaving a pond, the logs are equally likely to use any available exit flume.



Logs can only float in the direction the arrow is pointing. What is the probability that a log in pond A will end up in pond B? Express your answer as a common fraction.

30. _____