MATHCOUNTS

2008-09
■ 42nd Mock Mathcounts ■
Target Round
Problems 1 and 2

Name ____

State ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record only final answers in the designated blanks on the problem sheet. All answers must be complete, legible, and simplified to lowest terms. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. If you complete the problems before time is called, use the time remaining to check your answers.

Total Correct	Scorer's Initials

1) How many arithmetic sequences have the property that the first term is 1337, the common difference is -7, and the sum of the terms in the sequence is 3990?

1)_______________

2) John hits either 2, 3, or 4 on his calculator, then either $+$ or $\times$, then either 2, 3, or 4, then either $+$ or $\times$, then either 2, 3, or 4, and then presses “enter.” If the calculator follows the order of operations, how many distinct values can the end result be? For example, $2 + 3 \times 4$ gives 14.

2)_______________

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Problems 3 and 4

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3) How many positive integers less than 1000 can be written as the sum of at least two distinct perfect numbers? 3)_______________

4) Let $a_0, a_1, a_2, \ldots$ be a sequence such that $a_0 = 1$ and $a_n = 2a_{n-1} + 1$. Find the remainder when $a_0 + a_1 + a_2 + \cdots + a_{12}$ is divided by 1000. 4)_______________

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Problems 5 and 6

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5) Consider the sequence $k_1, k_2, k_3, \ldots$ defined such that 5)_______________

$k_1 = 2$

$k_n = k_1 k_2 k_3 \ldots k_{n-1} + 1$

The first term that is not prime ends up factoring into the product of two distinct primes. Find their sum.

6) Joe buys his ice cream in a right cylindrical cup. The cup has a radius of 5 cm and a height of 6 cm. Joe's cup was full when Jimmy came by and decided he wanted some ice cream too. Jimmy buys his ice cream in a right circular cone. All cones at the place where he buys his ice cream have a radius of 4 cm, but he can choose the height. The ice cream fills up the whole cone and then there's a hemispherical section at the top with the same radius as the cone. If Jimmy wants the same amount of ice cream as Joe, what height of the cone should he choose? Express your answer as a decimal rounded to the nearest thousandth. 6)_______________

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Target Round

Problems 7 and 8

Name ______________________________________

State ______________________________________

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7) Find the remainder when the sum of the divisors of 360 is divided by 100. 7)________________

8) Let ABC be a triangle and let D, E, and F be points inside the triangle such that D lies on CF, E lies on AD, and F lies on BE. Furthermore, $AE{:}ED = 7{:}3$, $BF{:}FE = 5{:}2$, and $CD{:}DF = 9{:}4$. If the area of the shaded region is 391, find the area of ABC. 8)________________