MATHCOUNTS

2008-09
■ 42nd Mock Mathcounts ■
Sprint Round
Problems 1 - 30

Name _____

State _____

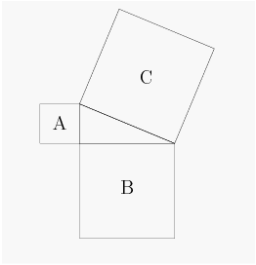
DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books, or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1) In the figure below, the letters represent the area of the figure they lie in. Find the value of $\frac{A+B}{C}$. Assume that the triangle given is right and that the "squares" are actually squares. 1)_____



2) Find the sum 2)_____

$$1 + 2 + 5 + 13 + 34 + 89 + 233 + 610$$

3) Find the sum of all A such that the six-digit number $63A5A7$ is divisible by 9. 3)_____

4) Eleven times a number leaves a remainder of 7 when divided by 13. What is the remainder when 9 times the number is divided by 13? 4)_____

5) Eight people, all of which have different IQs, are lined up in a row. The probability that they are lined up either by increasing IQ or decreasing IQ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find the remainder when $m + n$ is divided by 1000. 5)_____

6) Let x and y be real numbers such that $x \geq 4$ and $y \geq 5$. What is the minimum possible value of $(x-3)^2 + (y-6)^2$? 6)_____

7) In a class of 20 people, 13 have eaten pie and 17 have eaten muffins. What is the minimum possible number of people who have eaten both pies and muffins? 7)_____

8) How many positive integers under 100 can be written as the sum of two, not necessarily distinct, positive perfect cubes? 8)_____

9) In triangle ABC, $m\angle BAC = 45°$, $m\angle BCA = 30°$, and $AB = 4\sqrt{2}$. To the nearest whole number, find the area of ABC. 9)_____

10) Define $f(a,b) = (a-b)(a^2+ab+b^2)$. Find the value of $\dfrac{f(12,3)+f(10,7)}{3}$ 10)_____

11) There are two spheres and a cube oriented in space such that every vertex of the cube is on the larger sphere and each face of the cube is tangent to the smaller sphere. Let R be the radius of the larger sphere and let r be the radius of the smaller sphere. If the volume of the cube is 64, find the value of $R^2 + 2r$. 11)_____

12) The vertex of an upward-opening parabola is $(3,0)$. Let $f(x)$ be the function whose graph is this parabola. Find $f(5)$. Assume when x is 0 the function is equal to 9. 12)_____

13) Let a be an integer and let p be a non-even prime. Find the remainder when $a + 2a + 3a + \cdots + (p-1)a$ is divided by p. 13)_____

14) If $x + \frac{1}{x} = 7$, what is $x^3 + \frac{1}{x^3}$? 14)_____

15) Billy has 64 unit squares. What is the most amount of distinct squares he can create with these 64 unit squares, if each of the distinct squares must have a positive integer side length and all unit squares must be used? Squares are considered distinct if they have different side lengths, and an $n \times n$ obviously must use n^2 unit squares. 15)_____

16) Define $z(S)$ to be the sum of the elements of the set S. Let A be any non-empty subset of the set $\{1,2,3,\ldots,42\}$. How many possible values are there for $z(A)$? 16)_____

17) Let AB be a chord of circle O and let C be a point on the circle. Let D be the intersection of OC and AB. If $AD = 3$, $BD = 6$, and $CD = 2$, find the square of the distance from O to AB. 17)_____

18) What is the largest value of a for which $a^2 - 11a + 24 \leq 0$? 18)_____

19) How many integers x are there such that 0x100 and 361 is a perfect square when written in base x? 19)_____

20) A rectangle with integer side lengths has a perimeter of 424. Let A be the positive difference between the maximum possible area and the minimum possible area. Find $A \div 25$. 20)_____

21) How many of the first 1337 positive integers are either perfect squares, perfect cubes, or perfect fifth powers? 21)_____

22) For some value of n, it is true that 22)_____

$$-1 + 2 - 4 + 8 - 16 + \cdots + 2048 - 4096 + 8192 = 1 + 4^1 + 4^2 + \cdots + 4^n$$

Find n.

23) Find the last two digits of 2008^{2009}. 23)_____

24) $ABCD$ is a tetrahedron with A as the apex, $BC = 6$, $BD = 8$, and $CD = 10$. Suppose that a cone is such that A is the tip of the cone and B, C, and D lie on the circular base. The percent of the cone's volume that is inside the tetrahedron is $x\%$. Find x to the nearest whole number. 24)_____

25) What is the sum of the prime divisors of $2^{26} - 1$ less than 1000? 25)_____

26) What is the smallest positive integer that leaves a remainder of 3 when divided by 5, a remainder of 1 when divided by 8, and a remainder of 2 when divided by 9? 26)_____

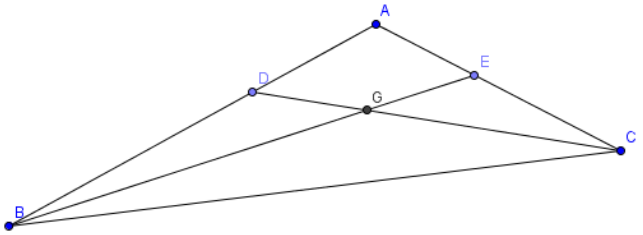
27) Let $g(a)$ be the sum of the distinct roots of $x^2 - ax + 16$. Find the value of 27)_____

$$\left\lfloor \left| \frac{g(1) + g(2) + g(3) + \cdots + g(50)}{5} \right| \right\rfloor$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

28) The sum of the reciprocals of the proper factors of 496 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m - n$. 28)_____

29) In the figure below, ABC is a triangle and D and E are points on AB and AC, respectively, such that the ratio of AD to DB is 3 to 5 and the ratio of AE to EC is 4 to 8. If the area of EC is 12, find the area of ABC. 29)_____



30) Phil has 4 indistinguishable oranges and 5 indistinguishable apples that he wishes to give to 3 kids. In how many ways can he do this if some kids can get no fruit and every fruit must go to some kid? 30)_____