Name_________________________ Period ______

Primes

Number Theory

A **Prime Number** is a whole number whose only factors are 1 and itself. To find all of the prime numbers between 1 and 100, complete the following exercise:

1. Cross out 1 by **Shading** in the box completely.
   1 is neither prime nor composite. It has only 1 factor - itself.
2. Use a forward **Slash ** to cross out all multiples of 2, starting with 4.
   2 is the first prime number.
3. Use a backward **Slash /** to cross out all multiples of 3 starting with 6.
4. Multiples of 4 have been crossed out already when we did #2.
5. Draw a **Square** on all multiples of 5 starting with 10. 5 is prime.
6. Multiples of 6 should be X'd already from #2 and #3.
7. **Circle** all multiples of 7 starting with 14. 7 is prime.
8. Multiples of 8 were crossed out already when we did #2.
9. Multiples of 9 were crossed out already when we did #3.
10. Multiples of 10 were crossed out when we did #2 and #5.

All of the remaining numbers are prime.

How many prime numbers are left between 1 and 100? ______

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Answer: Use your chart for help.

Is 51 prime? If not, what are its factors? _____________

Is 59 prime? If not, what are its factors? _____________

Is 87 prime? If not, what are its factors? _____________

Is 91 prime? If not, what are its factors? _____________

Name________________________ Period _____

Divisibility Rules

Number Theory

There are some easy tricks you can use to determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9 and 10.

A number is divisible by:
2 - if it is even.
3 - if the sum of its digits is divisible by 3.
4 - if the number formed by the last 2 digits is divisible by 4. (ask me why this works)
5 - if the ones digit is 5 or 0.
6 - if it is divisible by 2 AND 3. (All even multiples of 3.)
7 - there is no good trick for 7.
8 - if the number formed by the last 3 digits is divisible by 8. (ask me why this works)
9 - if the sum of the digits is divisible by 9.
10 - if the last digit is a 0.
11: We will learn this trick next. It is more complicated.

Practice: Write **y**es or **n**o in each blank.

Determine whether 21,408 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Determine whether 1,345,866 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Determine whether 222,222,225 is divisible by:

2 - ____ 6 - ____
3 - ____ 8 - ____
4 - ____ 9 - ____
5 - ____ 10 - ____

Trickier Divisibility Problems

Examples:

1. What is the smallest 4-digit number that is divisible by 2, 3, 4, 5, 6, 8, 9, and 10?

Reasoning: The number must end in zero. Lets assume that to be the smallest it should start with a 1. Since the digits must add up to 9, the last three digits must add up to 8. **1,080** is the smallest four-digit integer divisible by 8 and 9.

A second method will be explained in the first practice problem below.

2. Using only 1s and 2s, what is the smallest integer you can create which is divisible by both 3 and 8?

Reasoning: The last digit has to be a 2. Since 12 and 22 don't work, we need a 3-digit number that is divisible by 8. 112/8 = 14, but it is not divisible by 3. Unfortunately, 122, 212, and 222 are not divisible by 8 so we must go to a 4-digit number that ends in 112 and whose digits are a multiple of 3. The only 4-digit number that works is 2,112 (we want the sum of the digits to be 6) so it is the smallest.

Practice:

1. What is the smallest positive integer that is divisible by 2, 3, 4, 5, 6, 8, 9, and 10? (There is a good way to do this without guess-and-check).

2. The digits of a number are all 8s, and it is divisible by 9. What is the smallest positive integer that fits this description?

3. What is the smallest positive integer that is divisible by 2 and 3 that consists entirely of 2s and 3s, and has at least one of each?

4. What is the smallest 5-digit integer divisible by both 8 and 9?

1. 360 2. 888,888,888 3. 2,232 4. 10,008

Divisibility Rule: Eleven

The divisibility rule for 11 is seldom taught in regular classes.

Practice:
First, take a moment to multiply several numbers by 11:

504	1723	abcd
x11	x11	x11

You should see some patterns with the digits.
In the final example, the digits become: **a a+b b+c c+d d**

If you add the alternating digits you get the same result.

a **(a+b)** **b+c** **(c+d)** **d**

To find out if a number is divisible by eleven:
Sum the alternating digits. Subtract these two numbers. If the result is zero **or** is divisible by 11, the number is divisible by 11.

Examples: Determine if each number is divisible by 11 without a calculator:

1. 495 **2.** 9,835 **3.** 14,806 **4.** 918,291

Practice: Determine if each number is divisible by 11 without a calculator:

1. 3,951 **2.** 987,654 **3.** 14,256 **4.** 65,768

Harder Practice: Solve each without a calculator.

1. What digit could fill-in the blank if 89_43 is divisible by 11?

2. What five-digit multiple of 11 consists entirely of 2s and 3s?

3. What is the largest five-digit multiple of 11?

4. What is the remainder when you divide 1,234,567 by 11?

Name________________________ Period _____

Divisibility Practice

Number Theory

Practice:
Solve each using what you have learned about divisibility.

1. What digit could be used to fill in the blank and make the following number divisible by both 3 and 8?

45,2_8

_____

2. What is the smallest three-digit prime?

_____

3. How many multiples of 3 less than 1,000 use only the digits 2 and/or 4.

_____

4. 360 is divisible by both 8 and 9. How many integers less than 360 are also divisible by both 8 and 9? (hint: First find the smallest integer that is divisible by both 8 and 9.)

_____

5. A three-digit integer is divisible by 9. If I subtract the tens digit from the hundreds digit, I get the ones digit.
What is the largest number that meets these conditions?

_____

6. There are two ways that the digits 1, 2, 3, and 4 be arranged to create a four-digit multiple of 8. Find them both.

_____ _____

7. Consecutive integers are placed in order to form a three-digit integer. The integer will ALWAYS be divisible by what prime number?

_____

8. For the number ABC, each distinct letter represents a **different** digit. If ABC, CAB, and BCA are all divisible by 6 and 9, find the value of ABC + CAB + BCA.

_____

9. What is the largest seven-digit number that contains each of the digits 1 through 7 and has the property that the sum of any two consecutive digits is a prime number?
(source: MATHCOUNTS 2000 National Team Round)

_____

Name________________________ Period _____

Quiz: Divisibility

Number Theory

Solve:

1. How many of the following integers are factors of 12,345?
2, 3, 4, 5, 6, 8, 9, 10.

1. _______

2. What number could fill-in the blank to make 56,74_ divisible by both 3 and 4?

2. _______

3. How many prime numbers are greater than 20 but less than 40?

3. _______

4. Using only the digits 1, 2, and 3 with at least one of each, what is the smallest integer that can be created which is divisible by 8 and 9?

4. _______

5. Using only the digit 2, **how many** 2s must be used to create an integer that is divisible by both 9 and 11?
(Example: Using four 2s: 2,222 is divisible by 11 but not 9.)

5. _______

6. What is the remainder when 456,654,465,645 is divided by 6?

6. _______

7. The digits 0,3,6, and 9 are used to create the smallest integer that is divisible by 2, 3, 4, 5, 6, 8, 9, 10, and 11. What is this integer?

7. _______

Name________________________ Period _____

Quiz: Divisibility

Number Theory

Solve:

8. The digits 5, 6, and 7 are arranged to create a three-digit number. Which of the following cannot be a factor of the number formed? (There may be more than one answer, list all that apply.)

2 3 4 5 6 8 9

8. _______

9. Using two 5s and two 6s, it is possible to create four 4-digit numbers which are divisible by 11. What is the sum of these four numbers?

9. _______

10. Each of the digits 0-9 is used exactly once to create a ten-digit integer. Find one of the many ten-digit numbers which uses each digit once and is divisible by 8, 9, 10, and 11.

10. ___________________

Divisibility Combo

Rules:
Each player gets 6 cards, each with a single digit.
Each hand, players' goal is to find a number using *at least 3* of their 6 digits which meets the divisibility requirements.

Example:
You hold 1,4,4,5,7 and 8.
Goal: Divisibility by 3 and 5.
Possible answer: 8,415 4,875 etc.

Example:
You hold 1,2,3,3,8 and 9.
Goal: Divisibility by 3 and 8.
Can you find a solution?

Sometimes it will be impossible using the cards you have. After a player gets three combos, (s)he wins the round and everyone gets a new hand.

Create a 3+ Digit Number that is Divisible by:

6	2 and 3
3 and 4	5 and 6
2 and 11	3 and 11
8 and 9	4 and 6
11	5 and 8
3 and 8	4 and 9
9	4 and 5
2 and 11	5 and 9
3 and 10	2 and 3 and 5

More Work With Ones

Use a calculator to determine each of the following perfect squares:
(you should know the first two)

$1^2 =$

$11^2 =$

$111^2 =$

$1{,}111^2 =$

$11{,}111^2 =$

... after this your calculator will probably start giving you weird answers.

Think!

1. Explain why this pattern does what it does.
(hint: try writing the multiplication out).

2. Predict the value of $111{,}111{,}111^2$

3. Why does the pattern start to break up at 1,111,111,111?

A similar problem:
Pick any three-digit number.

a. Multiply your number by 7.
b. Multiply your answer by 11.
c. Multiply that answer by 13.

Notice anything?

Think!

1. Can you explain why this happens every time?

2. What number would you multiply a four-digit number by to create the same effect?

3. Pick any four-digit number and multiply it by 73, then multiply that answer by 137. Without checking on your calculator, do you know what 73 times 137 is?

4. The following trick works on the same principle:

Take the first three digits of your phone number. Multiply by 80. Add 1. Multiply by 250. Add the last four digits of your phone number. Add the last four digits again. Subtract 250. Divide by 2. What do you get?
The reasoning behind this one requires some Algebra.

Testing for Primes

Testing to see if a number is prime:
If we want to know if 401 is prime, do we need to test to see if the following numbers are factors: **6? 7? 10? 13? 20? 23?**

Is 401 prime? How can you tell?

To determine whether a number *n* is prime:
Check for divisibility by primes < $\sqrt{n}$ starting from least to greatest.

Think:
You do not need to check composites like 6 and 14 because if 6 were a factor, 2 and 3 would be factors. If 14 were a factor, 7 would also be a factor.

You do not need to check primes greater than the square root because they would be multiplied by a number less than the square root (which we would have already checked).

Example:
Is 181 prime?
Check in your head: 2, 3, 5, and 11. Check 7 and 13 on paper if you need to. Make sure you understand why you do not need to check numbers like 4 and 17 as factors.

Practice: Answer each (you wil need a calculator).
If a number is composite, write two numbers whose product is the number.

1. Is 391 prime? **2.** Is 287 prime? **3.** Is 503 prime?

4. The number 13 is prime, and when its digits are reversed, 31 is also prime. In addition to 13 and 31, five other 2-digit primes satisfy this condition. What are they?

Challenge: What is the smallest 4-digit prime?

1. 17x23 2. 7x41 3. Prime 4. 11-11, 17-71, 37-73.
C. 1001=7x11x13 1003=17x59 1007=19x53 1009 is :prime Notice that you do not need to check 7, 11, 13, 17, or 19 (for 1009) since they are factors of 1001, 1003, and 1007.

A Brief Intro to Exponents

One of the things that will come up frequently in this class which you all must be familiar with is the use of exponents.

Base:

The repeated factor in a power.
In the expression **n^3**, n is the <u>base.</u>

Exponent:

Represents the number of times a factor is being multiplied.
In the expression **n^3**, the ^{3} is the <u>exponent.</u>

The expression 5^3 means that you multiply $5 \cdot 5 \cdot 5$

The expression x^5 means that you multiply $x \cdot x \cdot x \cdot x \cdot x$

The expression $(ab)^4$ means that you multiply $(ab)(ab)(ab)(ab)$

Practice: Write-out without using exponents: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

1. 3^5 **2.** n^6 **3.** $(3n)^5$ **4.** $2^3 \cdot a^4$

Practice: Write using exponents. **Ex.** $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

1. $a \cdot a \cdot a \cdot a$ **2.** $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$ **3.** $(n \cdot n \cdot n)^5$

Practice: Evaluate. (solve) **Ex.** $2^5 = 32$

1. 7^2 **2.** 3^4 **3.** $2^3 \cdot 3^2$ **4.** $5^2 \cdot 2^4$

Fundamental Theorem of Arithmetic

Today we will learn something so crucial to number theory and mathematics that it is actually called the **FUNDAMENTAL THEOREM OF ARITHMETIC!**

The theorem states that
Every Positive Integer has a Unique Prime Factorization.

Example:
$5{,}544 = 2^3 \times 3^2 \times 7 \times 11$ and there is no other way to factor 5,544 into a product of primes.

To find the prime factorization of a number, it is often easiest to create a factor tree. If you remember your divisibility rules this should be easy:

Examples: Factor each (try without a calculator).

120 448 1,518

Practice: Write the prime factorization of each number below using exponents and placing the prime factors in order from least to greatest.

440 432 209

Practice: Write the prime factorization of each number below. These three numbers all have something in common. Can you tell what it is?

441 256 576

The numbers above are all perfect squares.
If a number is a perfect square, each prime factor will have an even exponent.
Example:

$$3^4 \cdot 5^2 = (3^2 \cdot 5)^2 = 45^2 = 2025$$

What perfect squares are represented below?

1. 5^4 **2.** $2^4 \cdot 3^2$ **3.** $2^2 \cdot 3^6 \cdot 5^2$

Fundamental Counting Principle

Another important Fundamental in mathematics is **The Fundamental Counting Principle:**

See Probability Unit for worksheet

Counting Factors

One way to count the number of **factors (divisors)** that a number has is to list them.

Examples:

<u>40:</u> <u>96:</u> <u>196:</u>

When you list factors, list them in pairs and go from least to greatest.
There is a nice relationship between the prime factorization of a number and the number of factors (divisors) that it has.

Example: Look at the prime factorization of 40:

$$200 = 2^3 \cdot 5^2$$

Every factor of 40 is a combination of 2s and 5s.

$1 = 2^0 \cdot 5^0$	$2 = 2^1 \cdot 5^0$	$4 = 2^2 \cdot 5^0$	$8 = 2^3 \cdot 5^0$
$5 = 2^0 \cdot 5^1$	$10 = 2^1 \cdot 5^1$	$20 = 2^2 \cdot 5^1$	$40 = 2^3 \cdot 5^1$
$25 = 2^0 \cdot 5^2$	$50 = 2^1 \cdot 5^2$	$100 = 2^2 \cdot 5^2$	$200 = 2^3 \cdot 5^2$

If you are only asked HOW MANY factors a number has, there is an easy shortcut that involves the prime factorization (maybe you can recognize what this shortcut is by looking at the example above):

Example:
How many factors does 56 have? First find the prime factorization.

$$56 = 2^3 \cdot 7^1$$

Each factor can have either 0, 1, 2, or 3 twos and 0 or 1 seven.
This means that 56 has 4 choices for the number of twos and two choices for the number of sevens in each of its factors, for a total of 4x2=8 factors.
They are 1, 2, 4, 7, 8, 14, 28, and 56

This is a great trick and we will practice more with it.

Name________________________ Period _____

Counting Factors

Number Theory

Practice: How many factors (divisors) does each number have? (do not list them)

1. $72 = 2^3 \cdot 3^2$

2. $180 = 2^2 \cdot 3^2 \cdot 5^1$

3. $210 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 7^1$

4. $112 = 2^4 \cdot 7^1$

5. How many factors does 600 have?

Apply what you know:

6. What is the smallest positive integer that has exactly 6 factors?
Think: What does the prime factorization of a number with 6 factors look like?

7. What is the smallest positive integer that has exactly 10 factors?

8. What is a characteristic of all numbers with exactly two positive factors?

9. What is a characteristic of all positive integers that have an odd number of factors?

10. What is the prime factorization of the smallest positive integer that has exactly 31 factors?

Challenge. Five 2-digit positive integers have exactly 12 factors. List them.

Counting Factors

Trickier factor counting:

Examples:

1. How many *odd* factors does 240 have?

Reasoning: First look at the prime factorization of 240:

$$240 = 2^4 \cdot 3^1 \cdot 5^1$$

Any number times 2 is even, so the odd factors are the ones that have no twos. I can only use the 3s and 5s for a total of 2x2=4 factors: 1, 3, 5, and 15.

2. How many perfect squares are factors of 360?

Reasoning: Look again at the number's prime factorization:

$$360 = 2^3 \cdot 3^2 \cdot 5^1$$

We need to find factors that have two twos, two threes, or both. 5^1 cannot be a factor of a perfect square.
Our perfect squares are $2^0=1$, $2^2=2x2=4$, $3^2=3x3=9$, and $2^2x3^2=2x2x3x3=36$.

Finding the prime factorization of a number quickly is the key to MANY number theory problems.

Practice:

1. How many perfect squares are factors of 400?

2. How many *even* factors does 210 have?

3. If a number *n* has 7 factors, how many factors does n^2 have?

4. The number *n* is a multiple of 7 and has five factors.
How many factors does 3n have?

Challenge: The number *p* is a multiple of 6 and has 9 factors. How many factors does 10*p* have?

1. 6: 1, 4, 16, 25, 100, 400 2. 8: 2, 6, 10, 14, 30, 42, 70, 210 3. 13 (consider n^6 and n^{12})
4. The original number must be 7^4 so the new number $3x7^4$ has 10 factors C. 24 (p=2x2x3x3)

Name________________________ Period _____

Counting Factors

Number Theory

Basics: How many factors (divisors) does each number have?
(do not list them)

1. 480

2. 400

Basics: How many **ODD** factors (divisors) does each number have?
(do not list them)

3. 900

4. 6,600

Basics: How many **EVEN** factors (divisors) does each number have?
(do not list them)

5. 450

6. 3,200

Basics: How many **Perfect Square** factors (divisors) does each number have? **List them**.

7. 210

8. 1,296

Apply what you know:

9. How many two-digit numbers have EXACTLY three factors?

10. What is the smallest positive integer that has 18 factors?

Review:

Divisibility

100. Find the smallest positive integer greater than 90,000 that is divisible by 11.

200. What positive integer could fill-in the blank to make the following divisible by six? 8,76_

300. What positive *integer* could fill-in both blanks to make the following divisible by both 4 and 9? 8_,8_8

400. What is the smallest 5-digit integer that is divisible by 2, 3, and 5?

500. What is the least positive integer that is divisible by 9 using only the digits 3 and 4, and having at least one of each?

600. What is the least positive integer that is divisible by 8, 9, 10, and 11?

Primes and Prime Factorization (calculators o.k.)

100. What is the prime factorization of 432?

200. What is the prime factorization of 352?

300. List all of the primes less than 1000 that use only one digit. (Start with 11.)

400. What is the prime factorization of 323?

500. 911 is prime. What is the next prime integer after 911?

600. What is the prime factorization of 1,681?

How many Factors does each have?

100. $160 = 2^5 \cdot 5^1$

200. 126

300. $5,665$

400. 80^2

500. $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$

600. $24,750$

Name________________________ Period _____

Practice Quiz: Factors & Divis.

Number Theory

Solve:

1. *How many* of the following integers are factors of 888?
2, 3, 4, 5, 6, 8, 9, 10.

1. _______

2. What number could fill-in the blank to make 8_,045 divisible by 11?

2. _______

3. What is the smallest prime number that is greater than 150?

3. _______

4. What is the Prime Factorization of 264 (written with exponents)?

4. ______________________

5. How many factors does 600 have?

5. _______

6. The digits of a 5-digit positive integer are 1s, 2s, and 3s with at least one of each. What is the smallest such integer that is divisible by both 8 and 9?

6. _______

7. What is the smallest positive integer that has exactly 10 factors?

7. _______

Name________________________ Period _____

Practice Quiz: Factors & Divis.

Number Theory

Solve:

8. Distinct positive integers *a* and *b* have 5 and 6 factors respectively. What is the smallest possible product *ab* if a and b do not have any factors in common greater than 1?

8. _______

9. How many even factors does 990 have?

9. _______

10. How many 3-digit integers have exactly 3 factors?

10. _______

Name________________________ Period _____

Quiz: Factors & Divisibility

Number Theory

Solve:

1. *How many* of the following integers are factors of 9,216?
2, 3, 4, 5, 6, 8, 9, 10.

1. _______

2. What number could fill-in the blank to make 13,_45 divisible by 11?

2. _______

3. What is largest 2-digit prime number?

3. _______

4. What is the Prime Factorization of 378 (written with exponents)?

4. ______________________

5. How many factors does 350 have?

5. _______

6. A positive 6-digit integer has two different digits.
What is the smallest such integer that is divisible by both 5 and 6?

6. _______

7. What is the smallest counting number (positive integer) that has exactly 8 factors?

7. _______

Name_________________________ Period _____

Quiz: Factors & Divisibility

Number Theory

Solve:

8. How many of the *factors* of 900 have exactly 18 factors?

8. _______

9. What is the smallest *odd* counting number that has exactly 12 factors?

9. _______

10. How many primes are there between 100 and 144?

10. _______

Name________________________ Period _____

Practice: Challenge Questions

Number Theory

Solve:

1. What is the smallest odd integer that has 8 factors?

1. _______

2. How many perfect squares are factors of 12,321?

2. _______

3. Two four-digit primes use only the digits 3 and 5. Find them both.

3. _______

4. What is the smallest prime factor of 1,517?

4. _______

5. What is the maximum number of factors that any three-digit number has?

5. _______

6. Which two three-digit multiples of 11 have exactly 10 factors?

6. _______ _______

7. How many different rectangles of integer length and width have an area of 1,200cm^2?

7. _______

8. The year 1,849 was the last year with 3 factors. What is the next calendar year with 3 factors?

8. _______

Product of the Factors (Divisors)

Now that you can count the number of factors of any natural number (positive integer) quickly, lets find a quick way to find the product of those factors.

Begin again by making a list of all factors of 120 and 144:

120: 144:

Using the method we used before, it is easy to see that the factors come in pairs (except for perfect squares, we will discuss this in a moment).

Fill in the blanks to complete the sentences below:

120 has _______ factors, which can be divided into _______ pairs, and the product of each pair is _______. Therefore, the product of all of the *pairs* is _______ to the ______ power.

144 has ____ factors. Since each pair of factors has a product of ______, we can replace every factor with the number _____ without changing the product. Therefore, the product of the factors of 144 is ______ to the ______ power.

If you know a little about fractional exponents, the products of the factors can be written as:

$$120^{\frac{16}{2}} = 120^{8} \text{ and } 144^{\frac{15}{2}} = \left(144^{\frac{1}{2}}\right)^{15} = \sqrt{144}^{15} = 12^{15}$$

Using the logical statements above may be more helpful to most of you.

Examples: Find the product of the factors of each.

1. 40 **2.** 225

Practice: Find the product of the factors of each.

1. 45 **2.** 81

Name________________________ Period _____

Product of the Factors Practice

Number Theory

Solve:

1. The product of the factors of 250 is 250^x. What is x?

1. _______

2. Place in order from least to greatest:

a. The product of the factors of 15.
b. The product of the factors of 25.
c. The product of the factors of 12.

2. _______

3. 199 is a prime number. What is the product of the factors of 199?

3. _______

4. A number x has 2n factors. What is the product of the factors of x in terms of x and n?

4. _______

5. Write the prime factorization for the product of the factors of 35.

5. ____________

6. The product of the factors of 30 is equal to $2^x \cdot 3^x \cdot 5^x$. What is x?

6. _______

7. The factors of the number one can be expressed as 1^1.

The product of what number's factors can be expressed as 3^3?

7. _______

Challenge: The product of the factors of 36 is divided by the product of the factors of 54. Write the result as a fraction in simplest form.

C. _______

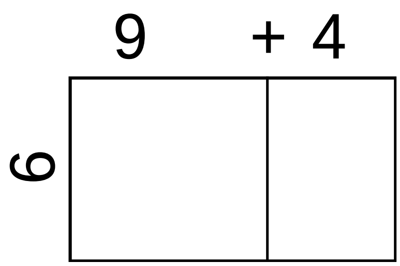
Sum of the Factors

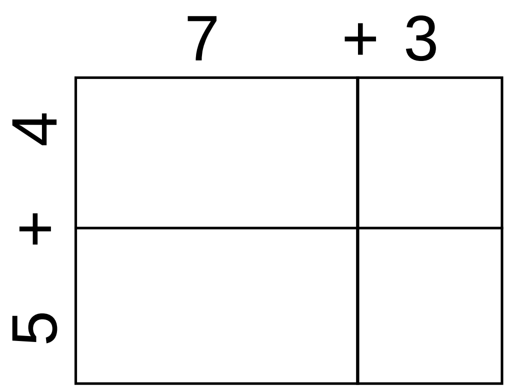
Example:
Consider the following problem:
Find the sum of the factors of 72.

This appears to be a very tedious task. We *could* list all of the factors (you should know now that there are 12 of them) and then add them up.

Like most of what we have learned in Number Theory, there is a shortcut. The shortcut requires that you know some Algebra, specifically how to distribute.

We will learn to distribute with integers.
This will be review if you have studied Algebra.
Begin by finding the area of each rectangle below.





There are two ways to find the area:

1. Find the length of both sides, and multiply to get the entire area.
2. Find the area of the pieces and add them together.

Distribution is the same as finding the pieces and adding them.

$$6(9+4)=6(9)+6(4)=54+24=78$$

$$(5+4)(7+3)=5(7)+5(3)+4(7)+4(3)=35+15+28+12=90$$

More Practice: Do these the long way!
Check your work using the short way.

$(5+2)(7+3)=$ $\qquad\qquad$ $(8+3)(5+4+2)=$

Sum of the Factors

Now, apply this to finding the sum of the factors of a positive integer:

Example:
What is the sum of the factors of 20? (this one is easy to check)

$$20 = 2^2 \cdot 5^1$$

We can get all of the factors using the trick we just learned with distribution. Multiply the sum of the powers of 2 by the sum of the powers of 5:

$(2^0 + 2^1 + 2^2)(5^0 + 5^1) =$ **First, lets make it easier to read.**

$(1 + 2 + 4)(1 + 5) =$ **Distribute. This gives us every factor!**

$1+5+2+10+4+20=42$ **42 is the sum of all the factors.**

Of course, we don't need to use the long way!

$$(1+2+4)(1+5) = (7)(6) = 42$$

This is more important with bigger numbers!
Examples: Find the sum of the factors of each.

$108 = 2^2 \cdot 3^3$ $\quad$ $378 = 2 \cdot 3^3 \cdot 7$ $\quad$ $408 = 2^3 \cdot 3 \cdot 17$

Practice: What is the sum of the factors of each?

1. $18 = 2^1 \cdot 3^2$ **Check this one by writing all the factors out.**

2. $150 = 2^1 \cdot 3^1 \cdot 5^2$

3. 441

1. 39 2. 372 3. 741

Name________________________ Period _____

Sum of the Factors

Number Theory

Solve:

1. What is the sum of the factors of:

a. 50 **b.** 405 **c.** 210

2. Which single-digit integer has a greater factor sum: 6, 8, or 9?

3. Find the sum of the factors of each number below and look for a pattern to determine the sum of the factors of 2^{30}.

a. 64 **b.** 128 **c.** 256

4. The prime factorization of a number is n^3. The sum its factors is 400. What is n?

5. What is the average of all the factors of 144?

6. 6^n has a factor sum that is greater than 1,000. What is the smallest integer value of n?

7. The sum of the factors of 2 x 3 x 5 x 7 x 11 is divided by the number of factors in $2^{11} \times 3^7 \times 5^5 \times 7^3 \times 11^2$. What is the result?

Name________________________ Period _____

Perfect/Abundant/Deficient

Number Theory

Solve:

1. What is the sum of the factors of:

a. 54 **b.** 300 **c.** 1568

2. The numbers 6 and 28 are the two smallest 'perfect' numbers. Find the property that makes these numbers 'perfect'. Is 496 perfect?

A perfect number is a number in which the sum of its *proper divisors* (all of its factors, not including itself) is equal to the number itself (this is the same as saying the sum of the factors is twice the number)

3. There are very few 'perfect' numbers. Most numbers are either 'abundant' or 'deficient'. If 30 is an abundant number, and 35 is considered deficient, do you think that 100 is abundant or deficient? Explain.

4. Are all prime numbers abundant or deficient?
Explain why.

5. Are all multiples of 6 (greater than 6) abundant or deficient?
Explain why.

6. Are all powers of 2 abundant or deficient? Try to explain why.

GCF (also called GCD) and LCM

GCF stands for Greatest Common Factor
The Greatest Common Factor (also called Greatest Common Divisor) for a pair or set of integers is the largest number that is a factor of each.

LCM stands for Least Common Multiple
The Least Common Multiple for a pair or set of integers is the smallest integer for which each number is a factor.

GCF and LCM should be taught extensively in your regular math class, so we will only have a short lesson on GCF and LCM (we will learn a technique or two that is different than what is taught in most classrooms).

Sometimes The GCF and LCM are obvious:
Practice: What is the GCF for the following pairs of numbers?

1. 15 and 35   2. 40 and 50   3. 36 and 54 **(careful!)**

Practice: What is the LCM for the following pairs of numbers?

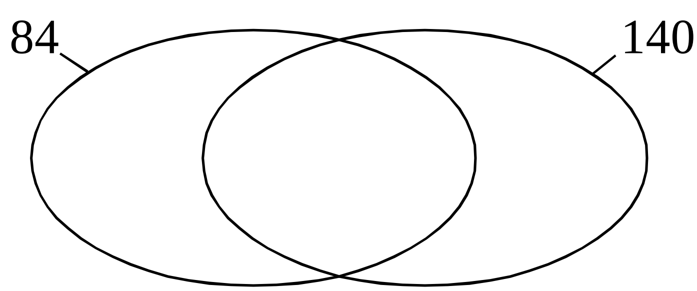
1. 4 and 6   2. 10 and 12   3. 24 and 40 **(careful!)**

It is not always so easy. What if you were asked to find the GCF and LCM of 84 and 140... you may not 'see' the answer in your head.

Venn Diagrams are a great way to solve GCF and LCM problems.

Example: Use a Venn diagram to find the GCF and LCM between 84 and 140.

$$84 = 2 \cdot 2 \cdot 3 \cdot 7$$
$$140 = 2 \cdot 2 \cdot 5 \cdot 7$$



Example: Use a Venn diagram to find the GCF and LCM for 75 and 105:

Practice: Use a Venn diagram to find the GCF and LCM for each.

1. 45 and 60   **2.** 80 and 112   **3.** 28, 42, and 105

GCF and LCM

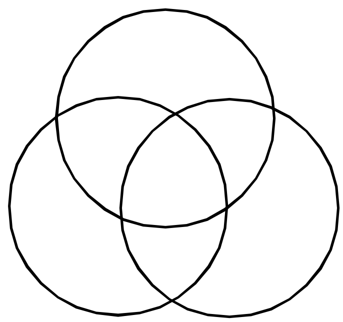
Review: Find the GCF and LCM for 45 and 105.

Now, multiply the GCF by the LCM.
Finally, multiply the original numbers.
Can you explain why this works? (Try looking at the Venn diagram.)

The product of a pair of numbers is equal to the product of their GCF and LCM.

Before we move on, lets try using our technique with three numbers:

Example: Find the GCF and LCM of 30, 45, and 50 using a Venn Diagram.



Practice: Find the GCF and LCM of 56, 126, and 210 using a Venn Diagram.

Note: *There are slightly quicker methods that you may know already, or will learn which can be used for finding the GCF and LCM. I like this one because it illustrates very clearly the concepts of shared/common factors and helps bright students understand the other methods.*

Trickier LCM problems:

1. You are buying cups, plates, and napkins for a picnic. Cups come in packs of 24, plates come in packs of 30, and napkins come in packs of 100. How many packs of napkins will you need to buy if you want to buy the same number of cups, plates, and napkins?

2. Janice and Kiera begin jogging around a track, starting at the finish line and going the same direction. Janice completes a lap every 78 seconds, while Kiera takes 90 seconds. At the end of their workout, they cross the finish line together in a whole number of minutes.
a. What is the minimum number of minutes that they could have run for?
b. How many more laps did Janice run than Kiera?

3. Ken gets his hair cut every 20 days. Larry gets his cut every 26 days. Ken and Larry get their hair cut on the same Tuesday. What day of the week is it the next time they get their hair cut on the same day?

4! = 24

Factorials
You may have seen an exclamation point in a math problem at some point and wondered, "What is so exciting about that number?"

No, 4! does not mean **"FOUR!!!"**

That exclamation point is actually not there to represent excitement or volume, it is **factorial notation.**

n! is equal to the product of all integers less than or equal to n.

Examples make this simple:

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$$

$$15! = 15 \cdot 14 \cdot ... \cdot 3 \cdot 2 \cdot 1 = 1{,}307{,}674{,}368{,}000$$

You can see that factorials can get big fast! You will not be asked to simply compute large factorials, but there are many interesting problems involving factorials that we will review.

First, some simple arithmetic and Algebra:
Examples:

1. $\dfrac{5!}{4!} =$ **2.** $\dfrac{5!}{10} =$ **3.** $\dfrac{5 \cdot 4!}{5!} =$

Harder Practice Problems:

1. $\dfrac{21!}{19!} =$ **2.** $\dfrac{6!}{60} =$ **3.** $\dfrac{5!+4!}{3!} =$

Factorials

We will solve many factorial problems using the same techniques we have learned in this unit involving prime factorization.

Examples:

1. Write the prime factorization of 5! using exponents.

2. How many factors does 5! have?

3. Find the sum of the factors of 5!.

4. Find the product of the factors of 5!.

More Examples:

1. What is the GCF of 5! and 11!?

2. Find the largest prime factor of 15!.

3. What is the LCM of 5! and $(3!)^2$?

4. What is the GCF of 6! and 12^3 ?

More Problems Involving Factorials:

1. Find the largest power of 3 that divides (is a factor of) 9!.

2. Find the smallest possible value of n if n! is divisible by 2^{15}.

3. When 100! is written as an integer, how many zeroes does it end with?

4. In the prime factorization of 1000!, what is the power of 11?

Name_________________________ Period _____

Factorials Practice

Number Theory

Practice:

1. Write the prime factorization of 7! using exponents.

2. *How many* factors does 6! have?

3. What is the LCM of 6! and 600?

4. Find the *largest prime factor* of 100!.

5. Find the *sum of the factors* of 7! (hint: use #1)

6. Find the largest power of 6 that divides 18!.

7. Find the smallest possible value of n if n! is divisible by 3^{15}.

8. When 125! is written as an integer, how many zeroes does it end with?

Quiz Review

Skills to know:

Using 924 and 792:

100. Find each prime factorization.

200. Find the GCF of 924 and 792.

300. Find the LCM of 924 and 792.

400. Find the number of factors of 924.

500. Find the product of the factors of 924 (written as 924 raised to a power).

600. Find the sum of the factors of 924.

Factorials:

100. Find the prime factorization of 8!.

200. Solve: 8! divided by 640.

300. In the prime factorization of 56!, what is the power of 7?

400. Find the product of the factors of 6!.

500. Find the largest prime factor of 50!

600. When $(25!)^5$ is written as an integer, how many zeroes does it end with?

Name________________________ Period _____

Practice Quiz: Factor Tricks

Number Theory

Solve:

1. What is the product of all factors of 100?

1. _______

2. What is the sum of all factors of 84?

2. _______

3. Is 42 perfect, abundant, or deficient?

3. _______

4. What is the GCF of 126 and 162?

4. _______

5. What is the LCM of 126 and 162?

5. _______

6. Write-out the prime factorization of 8! with exponents.

6. ______________________________

7. How many factors does 8! have?

7. _______

Name________________________ Period _____

Practice Quiz: Factor Tricks

Number Theory

Solve:

8. Find the sum of the factors of 6!.

8. _______

9. If n! ends in exactly 10 zeroes, what is the smallest possible value of n?

9. _______

10. What is the LCM of 6!, 216, and 300?

10. _______

Name_________________________ Period _____

Quiz: Factor Tricks

Number Theory

Solve:

1. What is the product of all factors of 60?

1. ________

2. What is the sum of all factors of 56?

2. ________

3. Is 32 perfect, abundant, or deficient?

3. ________

4. What is the GCF of 96 and 168?

4. ________

5. What is the LCM of 96 and 168?

5. ________

6. Write-out the prime factorization of 7! with exponents.

6. ________________________________

7. How many factors does 7! have?

7. ________

Name________________________ Period _____

Quiz: Factor Tricks

Number Theory

Solve:

8. Simplify: $\frac{12!}{11!}+\frac{10!}{9!}+\frac{8!}{7!}+\frac{6!}{5!}+\frac{4!}{3!}+\frac{2!}{1!}$

8. _______

9. In the prime factorization of 343!, what is the power of 7?

9. _______

10. If n! is NOT divisible by 1024, what is the largest possible value of n?

10. _______

Different Number Bases

We have ten fingers and ten toes, which is probably why we have a number system which uses ten digits.

With a ten-digit numbering system:
The first digit is the <u>ones</u> place (10^0).
The second digit is the <u>tens</u> place (10^1).
The third digit is the <u>hundreds</u> place (10^2).
etc.

In our base ten system, **$352 = 3(10^2) + 5(10^1) + 2(10^0)$ or $300 + 50 + 2$**

Suppose we were born with 6 fingers and toes, and we based our numbering system on six digits. What would our numbers look like then?

1. How many digits would we use? What would the largest digit be?
2. What would the first three digits in a 3-digit number represent?
3. Try to write the number 12 in base 6. Try 37.
4. What 'normal' (base 10) number is represented by 352 (base 6)?

The easiest way to think about the conversion is to convert "ones, tens, hundreds, thousands..." into "ones, sixes, thirty-sixes, etc." (for base 6).

Examples:
In base 8, what place values do the first four digits represent?
In base 2, what is the place value of the 1 in 10,000?

A note about notation.
To represent a number in a different base, a subscript is used.

Ex. 123_4 means 123 in base 4, or $1(4^2)+2(4^1)+3(4^0) = 16+8+3 = 27_{10}$
If there is no subscript, it is assumed that you are using base 10 notation.

Practice: Find the base 10 value of each:

1. 215_6 **2.** 101_2

3. 777_8 **4.** 20202_3

Name________________________ Period _____

Number Bases Practice

Number Theory

Solve: Convert each number to base 10.

1. $12{,}321_4$

1. ____________

2. $5{,}515_6$

2. ____________

3. $11{,}011_2$

3. ____________

Solve each.

4. What is the largest base 10 number that can be represented as a 3 digit number in base 6?

4. ____________

5. How many digits would it take to represent 242_{10} in base 3?

5. ____________

6. Base 2 is called binary and is used by computers. How many binary digits are needed to represent 999 (the largest 3-digit base 10 numeral)?

6. ____________

7. How would you represent $5x8^6 + 2x8^2$ as a base 8 numeral?

7. ____________

8. Convert 222_8 to base 4.

8. ____________

Challenge. How would you represent 9^6 as a base 3 numeral?

C. ____________

Things that Work the Same

Part of the reason why it is good to study other number bases is that it helps us to understand some of the simple things we have taken for granted in our own number system. Lets look for some patterns that translate across all number bases:

1. **Find an easy way to multiply a base 5 numeral by 5.**
   - **a.** Convert 14 and 70 to base 5.
   - **b.** Convert 21 and 105 to base 5.
   - **c.** Find an easy way to multiply by 5 in base 5.
   - **d.** Solve in base 5: $10_5 \times 43_5$
   - **e.** Will the same method work in any base? Try multiplying any base 8 number by 8. ($10_8 \times 43_8 =$)

2. **Find an easy way to add numerals in different bases.**
   - **a.** Convert 112_6 and 23_6 to base 10.
   - **b.** Find the sum of the converted numerals in base 10.
   - **c.** Convert the sum back to base 6.
   - **d.** Add 112_{10} $+ \ 23_{10}$
   - **e.** Add 112_6 $+ \ 23_6$
   - **f.** Now, try to add $245_6 + 453_6$. What makes this harder?
   - **g.** Add $376_8 + 274_8$

3. **Find an easy way to subtract numerals in different bases.**
   - **a.** Convert 154_6 and 23_6 to base 10.
   - **b.** Subtract the converted numerals in base 10.
   - **c.** Convert the difference back to base 6.
   - **d.** Subtract 154_{10} $- \ 23_{10}$
   - **d.** Subtract 154_6 $- \ 23_6$
   - **f.** Now, try to subtract $212_6 - 34_6$. What makes this harder?
   - **g.** Subtract $371_8 - 274_8$

Name________________________ Period _____

Base Number Arithmetic

Number Theory

Solve: Complete each operation in the given number base. Try to complete each problem without converting back and forth into base 10.

Multiplication:

1. $25_8 \times 10_8$

1. ______________$_8$

2. $436_8 \times 1000_8$

2. ______________$_8$

Addition:

3. $333_8 + 333_8$

3. ______________$_8$

4. $333_6 + 333_6$

4. ______________$_6$

5. $333_4 + 333_4$

5. ______________$_4$

Subtraction:

6. $567_8 - 456_8$

6. ______________$_8$

7. $213_6 - 144_6$

7. ______________$_6$

8. $10000_2 - 1011_2$

8. ______________$_2$

Challenge: $444_9 - 11111_3$

C. ______________$_9$

The Units Digit

There is an easy way to answer each of the multiple choice questions below without ever touching a calculator. See if you can figure it out (hint: Look at the title of today's lesson.)

Solve: What is 6437 x 7654?

A. 49,268,796 **B.** 49,268,797 **C.** 49,268,798

Solve: What is $65{,}656^2$?

A. 4,310,710,334 **B.** 4,310,710,336 **C.** 4,310,710,338

Solve: Which of the following integers is a perfect square?

A. 7,921 **B.** 7,922 **C.** 7,923 **D.** 7,927

There are several properties of the units digit which we will explore:

Addition:

Obviously, the units digit of the sum of two numbers is easy to figure out.

Ex. What is the units digit of the sum when 1,023 is added to:

1. 62 **2.** 744 **3.** 986 **4.** 1,098,569

Multiplication:

The units digit of a product is also easy.

Ex. What is the units digit of the product when 1,023 is multiplied by:

1. 62 **2.** 744 **3.** 986 **4.** 1,098,569

Squares:

The units digit of a perfect square is simple as well.

Ex. What is the units digit each perfect square below?

1. 62^2 **2.** 744^2 **3.** 986^2 **4.** $1{,}098{,}569^2$

Here is a trickier example:

Ex. How you can tell that 595,378,263,068,723,132 is NOT a perfect square.

Now answer the following:

Which of the ten digits can be the units digit of a perfect square?

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Powers and The Units Digit

At first, it seems impossible to know the units digits of a number like 1079^{87}, but look for a pattern and the problem becomes trivial:

$1{,}079^1$ ends in a 9.
$1{,}079^2$ ends in a 1.
$1{,}079^3$ ends in a ___.
$1{,}079^4$ ends in a ___.
$1{,}079^{87}$ ends in a ___.

Since all we care about is the units digit, we can quickly see that all odd powers of 9 end in a 9.

There are similar patterns for every units digit.
Find the units digit of each:

1st set: These are the easiest ones:

1. $1{,}355^{94}$ **2.** $81{,}001^{31}$ **3.** $465{,}376^{308}$

2nd set: These are a little harder (but still easy):

1. $987{,}654^{35}$ **2.** $81{,}069^{51}$

3rd set: These actually require a little thought:

1. 652^{39} **2.** $45{,}983^{61}$

3. $61{,}777^{102}$ **4.** $888{,}888^{777}$

Here is a trickier example:
Ex. How you can tell that 595,378,263,068,723,132 is NOT a perfect square.

Now answer the following:
Which of the ten digits can be the units digit of a perfect square?
0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Name________________________ Period _____

Units Digit Practice

Number Theory

Solve:

1. What is the units digit of the sum of $89^2 + 98^2$?

1. ____________

2. What is the units digit of $(893+928)^{120}$?

2. ____________

3. What is the units digit of the product of $329^{21} \times 956^{25}$?

3. ____________

4. What is the units digit of $557(892+675)^{57}$?

4. ____________

5. What is the units digit of $10{,}452^{134}$?

5. ____________

6. *How many* of the ten digits can be the units digit of a perfect cube (n^3)?
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

6. ____________

7. *How many* of the ten digits can be the units digit of a number raised to the 4th power (n^4)?
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

7. ____________

Challenge. Carolyn randomly selects three-digit whole numbers a and b. List all the possible units digits of the sum of a^4 and b^4.

C. ____________

Decimals

There is an easy way to determine whether a fraction represents a **repeating** or a **terminating** decimal. First, let's use our calculators to determine the decimal expansion of some common fractions:
(Use bar notation for decimals that repeat).

$\frac{1}{2} =$ $\frac{1}{3} =$ $\frac{1}{4} =$ $\frac{1}{5} =$ $\frac{1}{6} =$ $\frac{1}{8} =$

$\frac{1}{11} =$ $\frac{1}{13} =$ $\frac{1}{40} =$ $\frac{5}{9} =$ $\frac{24}{99} =$ $\frac{817}{999} =$

Converting by hand:
The most common method used to convert a fraction into a decimal by hand is to divide the denominator into the numerator:

$$\frac{1}{2} = 2\overline{)1.0}\ \ (.5) \qquad \frac{2}{3} = 3\overline{)2.0}\ \ (.\overline{6}) \qquad \frac{4}{13} = 13\overline{)4.000000}\ \ (.\overline{307692})$$

Terminating/Repeating Decimals:
The decimal expansion of a fraction in simplest form will terminate if the denominator contains no prime factors other than 2 or 5. Otherwise, the decimal will repeat.

Practice: Which of the following can be represented by terminating decimals?

$\frac{4}{9}$ $\frac{7}{8}$ $\frac{3}{125}$ $\frac{5}{24}$ $\frac{7}{2^{10}}$ $\frac{3}{48}$

Ninths, Ninety-Ninths, etc.
The repeating block of a fraction containing 9, 99, 999, etc. in the denominator is just the numerator (use leading zeroes if necessary)

$\frac{5}{9} = 0.\overline{5}$ $\frac{24}{99} = 0.\overline{24}$ $\frac{817}{999} = 0.\overline{817}$ $\frac{1}{9{,}999} = 0.\overline{0001}$

Practice: Express as a decimal (without using a calculator):

$\frac{7}{9} =$ $\frac{41}{99} =$ $\frac{2}{99} =$ $\frac{71}{999} =$ $\frac{2}{33} =$ $\frac{5}{111} =$

Repeating Decimals

Converting a repeating decimal into a fraction is more difficult and requires some Algebra.
Here is an easy conversion of a decimal that you should already recognize:

$$x = 0.\overline{5}$$
$$10x = 5.\overline{5}$$
$$10x - x = 5.\overline{5} - 0.\overline{5}$$
$$9x = 5$$
$$x = \frac{5}{9}$$

Examples: Convert the following to fractions using a similar method:

$x = 0.\overline{25}$ $\quad x = 0.1\overline{25}$ $\quad x = 0.01\overline{3}$

There is a nice shortcut for problems like the third example.

If $x = 0.01\overline{3}$, then $100x = 1.\overline{3}$, which means that $100x = 1\frac{1}{3}$.

Since $100x = \frac{4}{3}$, dividing by 100 gives us $x = \frac{4}{3} \cdot \frac{1}{100} = \frac{4}{300} = \frac{1}{75}$.

You can use this method quickly for just about any repeating decimal.

Practice: Convert each to a fraction in simplest terms.

1. $0.\overline{08}$ **2.** $0.1\overline{8}$ **3.** $0.10\overline{1}$

Intro to Modular Arithmetic

Try this:
In the decimal expansion of two-sevenths, what is the hundredth digit to the right of the decimal point?

We have learned to solve similar problems involving the units digit. The trick is to recognize that the decimal repeats in blocks of six. The 6th, 12th, 18th, 24th, 30th, … 90th, and 96th digits are all the same (4).

Since two-sevenths is just $0.\overline{285714}$, we can see that if the 96th digit is a 4, the hundredth digit must be a 7 (four decimal places later).

Modular Arithmetic
What we have been doing in problems like these is called **modular arithmetic**, and revolves around remainders. 100 divided by 6 leaves a remainder of 4, so asking for the 100th digit is equivalent to asking for the 4th digit.

Try this example:
It is now 5 o'clock. What time will it be 1,000 hours from now?

The pattern repeats every 12 hours. You know that in 12 hours it will be 5 again. In 24, 36, 48, 60, … 984, and 996 hours it will be 5 o'clock again. Therefore, in 1,000 hours it will be 4 hours later or 9 o'clock. All we needed was the remainder of 1,000 divided by 12. Asking for the time 1,000 hours from now is equivalent to asking for the time 4 hours from now.

The **modulus** is the length of the repeating block. Two numbers are considered equal if they leave the same remainder when divided by the modulus.

For example: 13 is congruent to 1 in modulus 12, or $13 \equiv 1 \pmod{12}$
In modulo 12, only the numbers 0 through 11 are used.

Practice: Find the value of each in mod 12.

1. 30 **2.** 100 **3.** 361 **4.** 4,800,005

Name________________________ Period _____

Modular Arithmetic

Number Theory

Solve each:

1. What is the 1,896,253rd digit in the decimal expansion of $\frac{1}{41} = .\overline{02439}$?

1. ____________

2. During her history class, Priyanka writes her name over and over again on a sheet of paper. She completes 955 letters before the paper is taken away by her teacher and she is reminded to pay attention in class. What is the last letter she writes?

2. ____________

3. The 350 sixth graders at Ligon Middle School stand in a big circle. They count off to form groups, starting with Katy and working to the left. They count off from 1 to 8 and then repeat until everyone has a number, and students who share the same number form a group. If Meera wants to be in the same group as Katy, what is the fewest number of places to Katy's right that she should stand?

3. ____________

4. A soccer team has 11 players, each wearing a jersey numbered from 1 through 11. They stand in a circle, so that their numbers are in order, with numbers increasing to the players' left. Marty begins with the ball, wearing #3. He passes it to the player four spaces to his left (#7), who in turn passes the ball to the fourth player to his left (#11) and they continue in this manner for 100 passes. What number is on the jersey of the player who receives the 100th pass?

4. ____________

5. Your digital clock is broken. To set the minutes, when you push the **>** button the minute value jumps ahead by 7 minutes, and when you push **<** , the minutes value goes back by 7 minutes. The time says 6:56, and when you push **>** , the time says 6:03. From 6:03, what is the fewest number of times you can push either button to get the clock to read 6:04?

5. ____________

Name_________________________ Period _____

Practice Quiz: Bases, Decimals, Modular Arith.

Number Theory

Solve:

1. What is the base-10 value of 222_3?

1. _______

2. Convert $0.\overline{8}$ to a fraction in simplest form.

2. _______

3. Use bar notation to represent the decimal expansion of $\frac{2}{11}$.

3. _______

4. What is the *base-10 value* of the smallest five-digit number in base-4?

4. _______

5. Five-hundred people stand in a circle. Starting with Roy and working to his left, each person counts off a number from 1 through 6 and then starting over again (1, 2, 3, 4, 5, 6, 1, 2, 3, ...) until everyone has counted a number. What number is counted off by the person standing to Roy's right?

5. _______

6. The number 222 is raised to the 222^{nd} power and then multiplied by 9. What is the units digit of the result?

6. _______

7. Convert $0.1\overline{4}$ to a fraction in simplest form.

7. _______

Name_________________________ Period _____

Practice Quiz: Bases, Decimals, Modular Arith.

Number Theory

Solve:

8. Add $0.\overline{2}+0.0\overline{2}+0.00\overline{2}$. Express the result as a fraction in simplest form.

8. _______

9. The minute hand of a clock points directly at the 11. What number will the minute hand point at after 5,555 minutes?

9. _______

10. For how many values of n where $n \le 100$ is $\frac{1}{n}$ represented by a terminating decimal?

10. _______