
MATHCOUNTS®

2013

■ National Competition ■

Answer Key

The appropriate units (or their abbreviations) are provided in the answer blanks.

Note to coordinators: Answers to the Tiebreaker Round problems appear in the Tiebreaker Round Booklet.

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04-N13ANS

Sprint Round Answers

- | | | |
|--------------------|---|-----------------------|
| 1. 60 cents | 11. 7 | 21. $5\sqrt{6}$ units |
| 2. 5 ordered pairs | 12. $3 - 2\sqrt{2}$ or $-2\sqrt{2} + 3$ | 22. -24 |
| 3. $\frac{1}{9}$ | 13. 16 subsets | 23. 338 people |
| 4. -2013 | 14. 81 units ² | 24. $\frac{5}{2}$ |
| 5. 37 | 15. 3 weeks | 25. 44 times |
| 6. 24 ways | 16. $\frac{1}{3}$ | 26. $\frac{2}{3}$ |
| 7. 36 students | 17. 84 teachers | 27. 3 |
| 8. 6 ft | 18. $\frac{34}{5}$ | 28. $\frac{25}{72}$ |
| 9. 13 | 19. 8775 | 29. $41\frac{3}{5}$ |
| 10. 27 | 20. 21 ordered triples | 30. $\frac{85}{8}$ |

Target Round Answers

- | | | | |
|---------------------------|---|-------------------|---------------------------------------|
| 1. \$80 or \$80.00 | 3. 28 | 5. $\frac{10}{3}$ | 7. 3 |
| 2. 320 units ² | 4. $\frac{13\sqrt{3}}{16}$ units ² | 6. $\frac{5}{2}$ | 8. $\frac{27}{32}$ units ² |

Team Round Answers

- | | |
|-------------------------|----------------|
| 1. 2 | 6. 846 |
| 2. 33.5 | 7. 400 gallons |
| 3. 413,280 permutations | 8. 2 games |
| 4. 27 | 9. -1.44 |
| 5. 64 pairs | 10. 6 |

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Sprint Round

Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete[®]. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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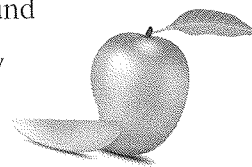
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1. _____ cents Eighty percent of a mango is edible. If mangos sell for \$2 a pound and Mary bought a mango that weighed $1\frac{1}{2}$ pounds, how many cents did she pay for the part of the mango that was not edible?



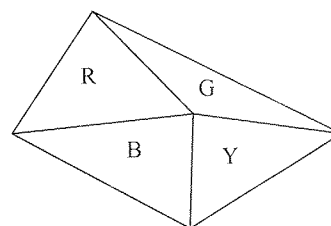
2. _____ ordered pairs How many ordered pairs of primes (a, b) satisfy $a + b = 22$?

3. _____ Among all three-digit positive integers ending in 9, what is the probability of randomly selecting one that is divisible by 9? Express your answer as a common fraction.

4. _____ What is the product of all of the values of x that satisfy $x^2 + 2012x = 2013$?

5. _____ The sum of seven distinct positive integers is 91. What is the greatest possible value of the second largest of these integers?

6. _____ ways A large quadrilateral is divided into four smaller non-congruent regions, and each region is painted with a different color. The possible colors are red, blue, green and yellow. One possible way to paint the quadrilateral is shown below. Including the way shown, how many total ways could the quadrilateral be painted?

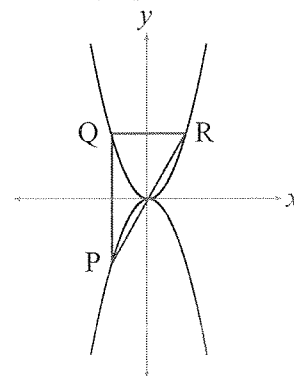


7. _____ students There were ten more girls than boys in a basket weaving class. Today six new boys enrolled, and the ratio of girls to boys is now five to four. How many students are now in the class?

8. _____ ft Three concentric circles have radii of lengths 2, 4 and 8 feet. What is the length of the shortest line segment that has at least one point in common with each of the three circles?

9. _____ If $w^a w^5 = w^{15}$ and $(w^4)^b = w^{12}$, what is the value of $a + b$?

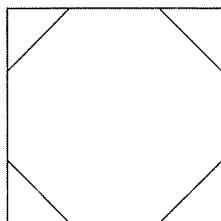
10. _____ For $\triangle PQR$, shown here, points $Q\left(-\frac{2}{3}, q\right)$ and $R\left(\frac{2}{3}, r\right)$ lie on the graph of $y = ax^2$, and point $P\left(-\frac{2}{3}, p\right)$ lies on the graph of $y = -ax^2$, where $a > 0$. If the area of $\triangle PQR$ is 16 units², what is the value of a ?



11. _____ What is the tens digit of the sum of the first 40 terms of this sequence?

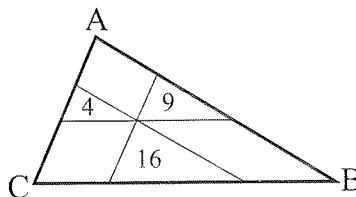
9, 99, 999, 9999, 99999, . . .

12. _____ A square circumscribes a regular octagon, as shown. If a point is randomly selected inside the square region, what is the probability that it will not be inside the octagonal region? Express your answer in simplest radical form.



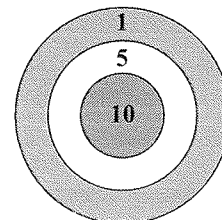
13. _____ subsets How many non-empty subsets of $\{1, 2, 3, 4, 5\}$ contain an element whose value is the number of elements in the subset?

14. _____ units² In $\triangle ABC$, line segments are drawn parallel to each of the sides dividing the triangle into six regions. The areas of three regions are shown in the figure. What is the total area of $\triangle ABC$?



15. _____ weeks A baseball league has four teams, and each team plays a game against one other team each week. Assuming there have been no ties, what is the minimum number of weeks after which it is possible to have one team that is undefeated but no team that has lost every game?

16. _____ To play a game of darts Michael throws three darts at the dart board shown. The number of points (1, 5 or 10) for each of the three regions is indicated. His score is the sum of the points for the three darts. If the radii of the three concentric circles are 1, 2 and 3 units, and each dart Michael throws hits this dart board at random, what is the probability that his score is evenly divisible by 3? Express your answer as a common fraction.

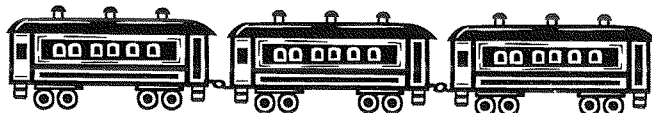


17. _____ teachers In a certain school district, the student-teacher ratio is 25:1. Currently, the school district has 3150 students. If the student enrollment increases by 20% and none of the current teachers leave, how many additional teachers must be hired to improve the student-teacher ratio to 18:1?

18. _____ In the coordinate plane, \overline{LM} passes through the origin. If $\overline{LM} \perp \overline{RS}$, and they intersect at $(5, 3)$, what is the x -coordinate of the point where \overline{RS} intersects the x -axis? Express your answer as a common fraction.

19. _____ The sequence 3, 4, 13, 15, 30, 33, 54, 58, 85, 90, ..., contains, in increasing order, all the positive integers that yield a triangular number when multiplied by 7. What is the 100th term of this sequence?

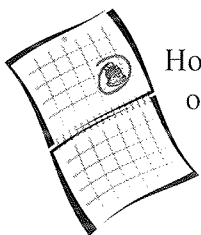
20. _____ ordered
triples Five people board a train containing three cars. The conductor records the ordered triple (a, b, c) indicating how many of the five passengers take a seat in the first, second and third car, respectively. How many such ordered triples are possible?



21. _____ units Two circles share a common chord. The chord cuts off a 90° arc of circle A and a 60° arc of circle B. The radius of circle A is $5\sqrt{3}$ units. What is the radius of circle B? Express your answer in simplest radical form.

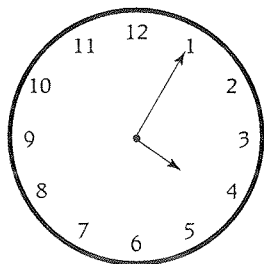
22. _____ If $f(x) = x + 3$ and $g(x) = [f(x)]^2$, what is the product of all possible values of $f(x - 1)$ for which $g(x) = 25$?

23. _____ people How many people must be in a room to guarantee that at least two of them have the same birthday or at least one of them was born in February?



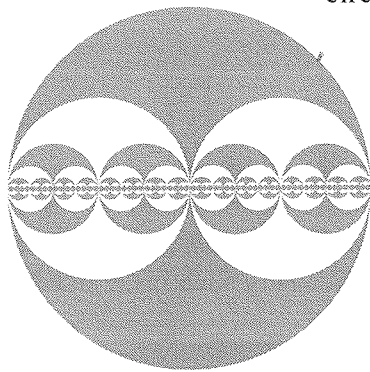
24. _____ If a and b are real numbers such that $a + b = 1$ and $a^2 + b^2 = 2$, what is the value of $a^3 + b^3$? Express your answer as a common fraction.

25. _____ times



How many times in a 24-hour day will the hour and minute hands of a 12-hour analog clock form a 90-degree angle?

26. _____

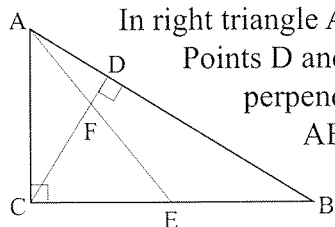


To create the figure shown, a circle of radius 1 was drawn at Stage 1. Two new circles of radius $\frac{1}{2}$ were drawn at Stage 2. Then 2^{n-1} new circles of radius $\frac{1}{2^{n-1}}$ were drawn at Stage n . Each circle is tangent to one other circle of the same size as well as to one other circle of the next larger size. This process is repeated infinitely. The interior of each circle drawn at an odd stage is shaded. The interior of each circle drawn at an even stage is unshaded. What fraction of the largest circle is shaded? Express your answer as a common fraction.

27. _____

What is the sum of the values of x that satisfy $9(x^3 - 3x^2 - 28x + 60) = 1$?

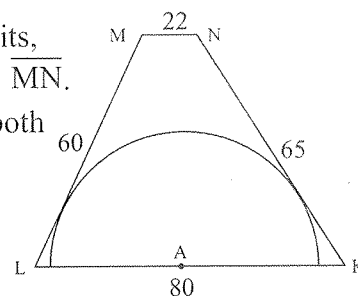
28. _____



In right triangle ABC, shown here, $AC = 5$ units and $BC = 12$ units. Points D and E lie on \overline{AB} and \overline{BC} , respectively, so that \overline{CD} is perpendicular to \overline{AB} and E is the midpoint of \overline{BC} . Segments \overline{AE} and \overline{CD} intersect at point F. What is the ratio of AF to FE? Express your answer as a common fraction.

29. _____ units

Trapezoid KLMN has sides $KL = 80$ units, $LM = 60$ units, $MN = 22$ units, and $KN = 65$ units, with \overline{KL} parallel to \overline{MN} . A semicircle with center A on \overline{KL} is drawn tangent to both sides \overline{KN} and \overline{ML} . What is the length of segment KA? Express your answer as a mixed number.



30. _____ units

A triangle has sides of length 10, 17 and 21 units. What is the radius of its circumscribed circle? Express your answer as a common fraction.

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Target Round

Name _____

State _____

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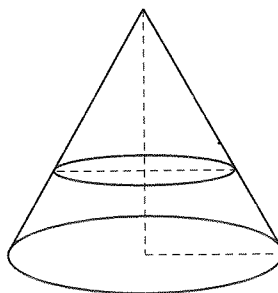
04-N13TAR7

1. \$ _____

Bobby saved \$32 when he purchased a jacket at a clearance sale. If the clearance price was 40% less than the regular price, what was the regular price of the jacket?

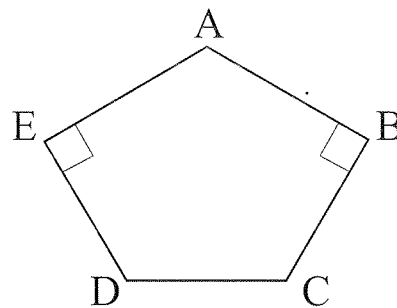
2. _____ units²

The two cones in the diagram share a common apex and have parallel bases. Millie found the volumes of the smaller and larger cones to be 125π units³ and 512π units³, respectively. She found the lateral surface area of the smaller cone to be 125 units². What is the lateral surface area of the larger cone?



3. _____ A set contains four integers. The sums of the integers in each of its three-element subsets are 82, 88, 79 and 84. What is the median of the original set of four distinct integers?

4. _____ units² Pentagon ABCDE has $AE = AB = 1$ unit. Angles B and E each have measure 90° , and $m\angle A = 120^\circ$. The two remaining angles are congruent to each other, and the three remaining sides are congruent to each other. What is the area of pentagon ABCDE? Express your answer as a common fraction in simplest radical form.

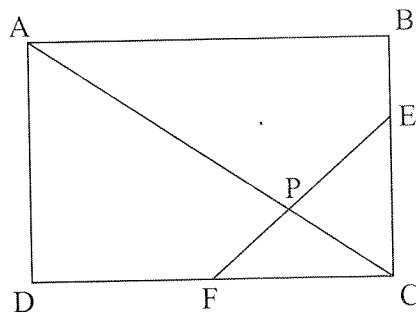


5. _____ We know the following five facts about the numbers a , b , c and d . What is the value of $a + b$?

- 1) $ab = 1$
- 2) $b + c = 0$
- 3) $b + c + d = 0$
- 4) $bc = -9$
- 5) $c + a < d$

Express your answer as a common fraction.

6. _____ In rectangle ABCD, shown here, points E and F lie on sides BC and CD, respectively. Point F is the midpoint of \overline{CD} and $BE = \frac{1}{3}BC$. Segments AC and FE intersect at point P. What is the ratio of AP to PC? Express your answer as a common fraction.



7. _____ What is the smallest possible value of the sum $|x - 2| + |x - 4| + |x - 5|$?

8. _____ units² What is the maximum possible area of a triangle with vertices $(a, 0)$, $(b, 0)$ and (c, d) , where $1 + a - 2a^2 = 1 + b - 2b^2 = 0$, $1 + c - 2c^2 = d$ and $d > 0$? Express your answer as a common fraction.

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Team Round

Problems 1–10

State _____

Team
Members _____, Captain

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This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers in the official team booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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04-N13TEA

1. _____ What is the sum of all real numbers a that satisfy the equation $a + a = a \times a$?

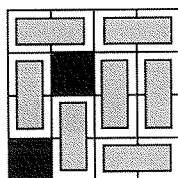
2. _____ The average of ten numbers is 41. The average of six of these numbers is 46. What is the average of the other four numbers? Express your answer as a decimal to the nearest tenth.

3. _____ permu-
tations Eight-letter permutations can be made by using four letters from the word MAINE and four letters from IDAHO. How many unique eight-letter permutations are there?



4. _____ The first term of a geometric sequence is 12, and the geometric mean of the first three terms is 18. What is the third term in the sequence?

5. _____ pairs



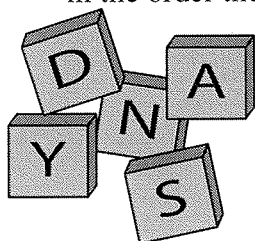
Two unit squares are removed from a 4×4 grid. Seven dominoes are available to cover each of the remaining 14 squares such that each domino covers two adjacent unit squares. One such example is shown. Including the pair shown in this example, how many pairs of unit squares can be removed so that the seven dominoes can cover the remaining 14 unit squares?

6. _____ The letters A through I denote distinct nonzero digits in the addition problem shown. What is the largest possible value of the three-digit number GHI?

$$\begin{array}{r} ABC \\ DE \\ + \quad F \\ \hline GHI \end{array}$$

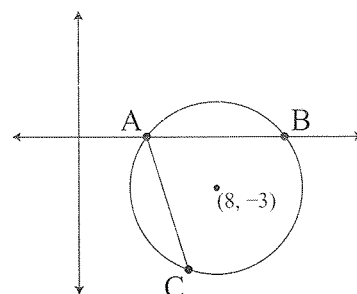
7. _____ gallons A tank containing 600 gallons of a water and sand mixture is 98% water. After some of the water evaporates, the remaining mixture is now found to be 97% water. How many gallons of mixture are in the tank now?

8. _____ games In a three-person Scrabble® tournament, Mary, Sherry and Terry each play three games against each of the other two opponents, with Sherry playing Mary first. A player faces the same opponent for three games before playing against the other opponent. On the score sheet, wins and losses are entered from left to right in the order they were played. A win is recorded as a 1, while a loss is recorded



as a 0. There were no ties. When the tournament is over, each player's score sheet contains a six-digit string of 1's and 0's that can be interpreted as a base 2 numeral. When these base 2 numbers were converted to decimal numbers, Sherry's number was 4 more than Mary's. How many more games did Mary win than Sherry?

9. _____ A circle with radius 5 units has its center at $(8, -3)$. Chord \overline{AB} has its endpoints on the x -axis with $A(4, 0)$. Point C is on the circle with $AC = AB$. What is the sum of the coordinates of point C ? Express your answer as a decimal to the nearest hundredth.



10. _____ If the sum of the first n positive integers equals the sum of the next k positive integers, where $n < 20$, what is the largest possible value of k ?