forthegreatergood

MATHCOUNTS 2014 Mock Sprint

Username on AoPS: ______________________________

Problems: 30

Time: 40 minutes

Calculators: Not permitted

Type: Individual

Score: ______________________________

Important Information: The answer to every question on this test is an integer ranging from 000-999 inclusive. When submitting answers, please submit them on ONE line. If your answers to questions 1,2,3,4,5 are: 7,23,143,333,770 respectively, submit: 007023143333770, and continue the string in a similar fashion for the next 25 answers. Your string should contain $3 * 30 = 90$ digits. **Include leading zeros**. If your answer is one or two digits, add the appropriate number of zeros before that number so it is now three digits (e.g.: 90 becomes 090 and 3 becomes 003). If your answer is zero, submit 000.

Private message **forthegreatergood** on AoPS with the subject being **MOCK2014** in order to have your test graded.
Your test will be copy and pasted into a computer program in order to be graded. Please adhere to these rules in order for grading to be done quickly and effienctly.

Test on Next Page

1. Find the sum of all integral values of x such that:

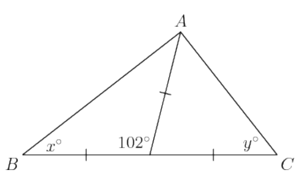
$$2014^{(-1)^x} = 2014$$

1. ____________

2. Triangle ABC has side lengths $5, 12$ and 13. It laying flat on the x axis of the xy plane such that one side is completely on the x axis. Let M be the point with the maximum positive y value which lies on the perimeter of $\triangle ABC$. The minimum value of M can be expressed as $\frac{a}{b}$ where a and b are relatively prime. Find $a+b$.

2. ____________

3. Find the value of $x+y$ in the following triangle:



3. ____________

4. Find the value of the following fraction:

$$\frac{1^3+2^3+3^3+4^3+5^3+\ldots+100^3}{(1+2+3+4+5\ldots+100)^2}$$

4. ____________

5. If $a + \dfrac{1}{b + \frac{1}{c}} = \dfrac{13}{9}$ where a, b, c are positive integers, find the value of $100a + 10b + c$.

5. ____________

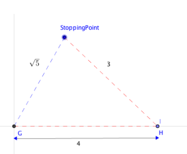
6. Four knights and King Arthur sit at a round table. After dinner is served, two of the knights switch seats. Dessert is then served, starting from King Arthur and then moving counterclockwise. Find the number of orders in which the knights can receive their dessert.

6. ____________

7. Find the maximum possible value of x^2y if $2x + y = 12$.

7. ____________

8. An ant starts at the coordinate the origin and travels along the positive x axis 4 meters. Next, he travels 3 meters in some direction on the xy plane. He then stops. The distance the ant would need to travel from the place he stopped to the origin in a straight line is $\sqrt{5}$. The shortest distance from the place he has stopped to the y axis can be expressed as $\frac{p}{q}$ where p, q are relatively prime. Find $p + q + 1$.



8. ____________

9. For every term after the first, the next term in a sequence is created by doubling the sum of all of the previous terms. If the first term of the sequence is 1, what is the largest number in the sequence less than 1000?

9. ____________

10. A cube and sphere have the same surface area. The ratio of the volume of the sphere to that of the cube can be expressed as $\frac{a\sqrt{b}}{\sqrt{\pi}}$ where a and b are integers. Find $10a + b$

10. ____________

11. Two programmers begin programming. The first programmer can program the whole program alone in 2 hours, and the second can program the whole program alone in 4 hours. How many minutes will it take both of them to program the program working together?

11. ____________

12. Let $p(x)$ be a polynomial of degree 2014 such that $p(1) = p(-1) = 99$ and $p(0) = 26$. Call the coefficents of the even powers of x good (excluding the constant term). Find the sum of all good coefficients.

12. ____________

13. Let x and y be integers with x greater than y. If:

$$\frac{2xy}{19} + \frac{2x}{19} = 118$$

What is the value of the sum $x + y$?

13. ______________

14. What is the smallest positive integer with exactly 12 divisors, including itself?

14. ______________

15. How many zeros does the number 100! end in when expressed in base 12?

15. ______________

16. Which unique integer is not in the range of: $y = \dfrac{3x+2}{x}$?

16. ______________

17. During a group test, a class of 8 students is split into groups of 2, with each student belonging to one of these four groups. How many different ways is it possible to split the students?

17. ____________

18. The approval rating of the newest president of the school district increased from 50 % to 70 %. By what percent did the approval rating increase?

18. ____________

19. In $\triangle ABC$, BM is drawn such that $AM = MB = MC$. Moreover, $AB = 7$ and $MC = 12.5$. Find the value of $4 \cdot BC$.

19. ____________

20. A square is drawn with side length 2. Another square is drawn by connecting the midpoints of the original square. A third square is drawn by connecting the midpoints of this second square. This process continues an infinite amount of times. What is the sum of the areas of all of these squares?

20. ____________

21. A pool is in the shape of a rectangular prism. The outside rim of the pool is 24 meters and the depth of the water is currently 10 meters. A diver jumps into the pool, and the water level rises by 2 meters. What is the maximum possible value of the volume of the diver?

21. ____________

22. What is the sum of all of the digits in the following sequence:

$$2, 3, 4, 5 \ldots 96, 97, 98, 99, 100$$

22. ____________

23. An point in an equalateral triangle is chosen. The distances from that point to each of the verticies is 5, 6 and 7 units. The area of the triangle can be expressed as $n\frac{\sqrt{3}}{4}$ where n is an integer. Find the value of n.

23. ____________

24. An ordered n-touple of n real numbers is called $n-$cyclic if each number is equal to the product of the other two numbers. How many ordered 3-tuples: (x_1, x_2, x_3) are there?

24. ____________

25. The sum of all four digit palindromes can be expressed as $p_1^{q_1} p_2^{e_2} e_3^{e_3} \dots p_k^{e_k}$ where p_i are prime numbers, and e_i are integers. Find the sum: $e_1 + e_2 + e_3 + \dots e_k$.

25. ____________

26. Find the value of $|a+b+c|$ if:

$(a+b)(a+c) = 6$
$(a+b)(b+c) = 30$
$(b+c)(a+c) = 45$

26. ____________

27. A mathlete is taking a five question test. The probability of getting each question correct decreases linearly, starting with 100% (so the probability that he gets the last question correct is 20%). The probability that he gets at least three out of the five questions correct can be expressed as $\frac{p}{q}$ where p and q are relatively prime. Compute $p + q - 500$.

27. ____________

28. A regular tetrahedron has side length 12. The midpoints of two opposite edges are connected. The length of this segment can be expressed as $a+b\sqrt{3}$ where a, b are integers. Find the value of $a+b$.

28. ____________

29. After the MATHCOUNTS sprint round in Plano, Oregon, the test transporter, Bill, needs to send 3,000 tests 1,000 miles from the test site to his house. Bill's car can carry only 1,000 tests at a time. Every time Bill travels a mile towards his house, he must throw one test out the window as a payment of tax. However, if he goes from his house to the test site, he does not need to pay anything. What is the greatest number of tests that Bill can get to his house?

29. ____________

30. Call a positive number *distod* if it has distinct digits, and is odd. Find the number of *distod* numbers less than 1000.

30. ____________