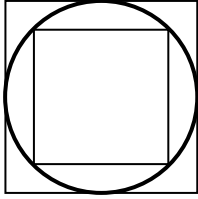
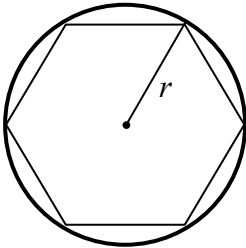


1. Joan estimates the area of a circle by averaging the areas of inscribed and circumscribed squares.



If the circle has a radius of 2 centimeters, what would be Joan's approximation for its area?

- (A)  $4\pi \text{ cm}^2$
  - (B)  $10 \text{ cm}^2$
  - (C)  $12 \text{ cm}^2$
  - (D)  $8\sqrt{2} \text{ cm}^2$
2. Use the diagram.



A regular hexagon is inscribed inside a circle of radius  $r$ . What is the difference between the circumference of the circle and the perimeter of the hexagon?

- (A)  $r(\pi - 6)$
- (B)  $r(3\sqrt{3} - \pi)$
- (C)  $2r(\pi - 3)$
- (D)  $2r(3\sqrt{3} - \pi)$

3. Complete the description of Cavalieri's principle.

If two solids have the same  $\frac{\text{---}}{\text{---}}$  <sup>(i)</sup> and the same cross-sectional area at every level, then the two solids have the same  $\frac{\text{---}}{\text{---}}$  <sup>(ii)</sup> .

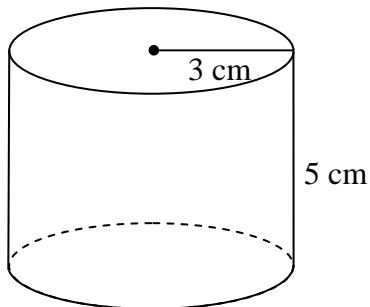
- (A) *i.* height    *ii.* volume  
(B) *i.* radius    *ii.* volume  
(C) *i.* height    *ii.* surface area  
(D) *i.* radius    *ii.* surface area
4. A right circular cylinder has a height of 7 centimeters. The radius of the base is 5 centimeters. What is its volume?

- (A)  $58\frac{1}{3}\pi \text{ cm}^3$   
(B)  $70\pi \text{ cm}^3$   
(C)  $175\pi \text{ cm}^3$   
(D)  $245\pi \text{ cm}^3$

5. A sphere has volume  $36\pi$  cubic inches. What is its surface area?

- (A) 18 square inches  
(B)  $18\pi$  square inches  
(C) 36 square inches  
(D)  $36\pi$  square inches

6. Use the diagram.



What is the volume of the cylinder?

- (A)  $15\pi \text{ cm}^3$
  - (B)  $45\pi \text{ cm}^3$
  - (C)  $75\pi \text{ cm}^3$
  - (D)  $225\pi \text{ cm}^3$
7. The diameter of a golf ball is approximately 40 millimeters.  
The diameter of a billiard ball is approximately 60 millimeters.

The volume of a billiard ball is approximately how many times the volume of a golf ball?

- (A)  $\frac{8}{27}$
- (B)  $1\frac{1}{2}$
- (C)  $2\frac{1}{4}$
- (D)  $3\frac{3}{8}$

8. The great pyramid at Giza is a square pyramid. Its height is approximately 139 meters and it has a total volume of about 2.5 million cubic meters.

Which expression shows the approximate length of its base in meters?

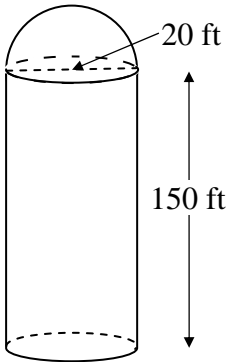
(A)  $\frac{(3)(2.5 \times 10^6)}{139}$

(B)  $\frac{2.5 \times 10^6}{139}$

(C)  $\sqrt{\frac{(3)(2.5 \times 10^6)}{139}}$

(D)  $\sqrt{\frac{2.5 \times 10^6}{139}}$

9. A grain storage silo consists of a cylinder and a hemisphere. The diameter of the cylinder and the hemisphere is 20 feet. The cylinder is 150 feet tall.



What is the volume of the silo?

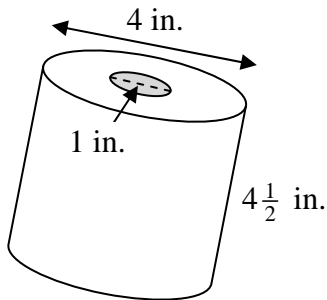
(A)  $\frac{17000\pi}{3} \text{ ft}^3$

(B)  $\frac{47000\pi}{3} \text{ ft}^3$

(C)  $\frac{49000\pi}{3} \text{ ft}^3$

(D)  $\frac{182000\pi}{3} \text{ ft}^3$

10. A regular roll of bathroom tissue (toilet paper) is  $4\frac{1}{2}$  inches high with a one inch inner diameter. The outside diameter is 4 inches.



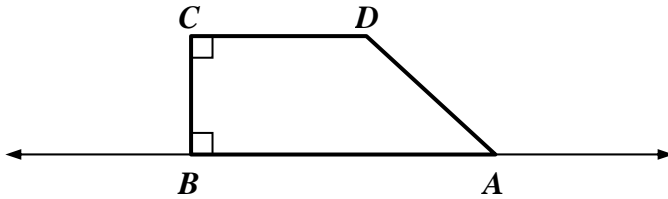
What is the volume of the bathroom tissue?

- (A)  $\frac{15\pi}{4}$  in.<sup>3</sup>  
(B)  $15\pi$  in.<sup>3</sup>  
(C)  $\frac{135\pi}{2}$  in.<sup>3</sup>  
(D)  $\frac{135\pi}{8}$  in.<sup>3</sup>
11. A cone-shaped paper cup (see picture) with radius 1.5 inches and height of 4 inches has a capacity of 154 milliliters. If the cup currently holds 77 milliliters of water, what is the height of the water?



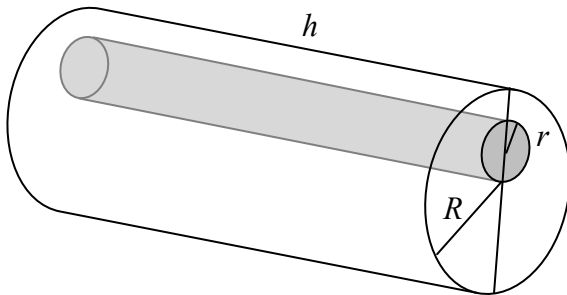
- (A)  $\sqrt[3]{32}$  inches  
(B)  $\sqrt[3]{16}$  inches  
(C) 2 inches  
(D) 3 inches
12. A cube is intersected by a plane. Which shape could NOT be the resulting cross-section?
- (A) triangle  
(B) pentagon  
(C) hexagon  
(D) octagon

13. In the diagram,  $ABCD$  is a trapezoid where  $\overline{AB} \parallel \overline{CD}$ , angles  $B$  and  $C$  are right angles, and  $m\angle A = 60^\circ$ .



The trapezoid is rotated  $360^\circ$  about  $\overleftrightarrow{AB}$ . Which describes resulting three-dimensional figure?

- (A) The union of a cylinder and a cone.
  - (B) The union of two cones.
  - (C) The union of a prism and a pyramid.
  - (D) The union of two pyramids.
14. An object is consists of a larger cylinder with a smaller cylinder drilled out of it as shown.



What is the volume of the object?

- (A)  $\pi(R^2 - r^2)h$
- (B)  $(\pi R^2 - r^2)h$
- (C)  $\pi(R - r)^2 h$

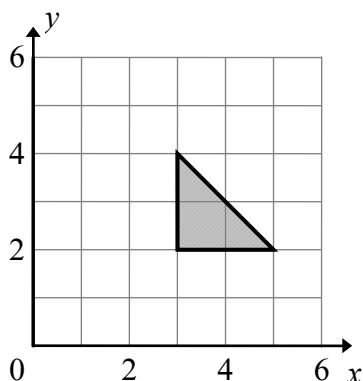
For questions 14–15, use the following scenario.

A swimming pool is in the shape of a rectangular prism with a horizontal cross-section 10 feet by 20 feet. The pool is 5 feet deep and filled to capacity.

Water has a density of approximately 60 pounds per cubic foot, or 8 pounds per gallon.

15. What is the approximate weight of water in the pool?
- (A) 8,000 lb  
(B) 16,700 lb  
(C) 60,000 lb
16. About how many gallons equal one cubic foot of water?
- (A) 0.13 gal  
(B) 4.8 gal  
(C) 7.5 gal

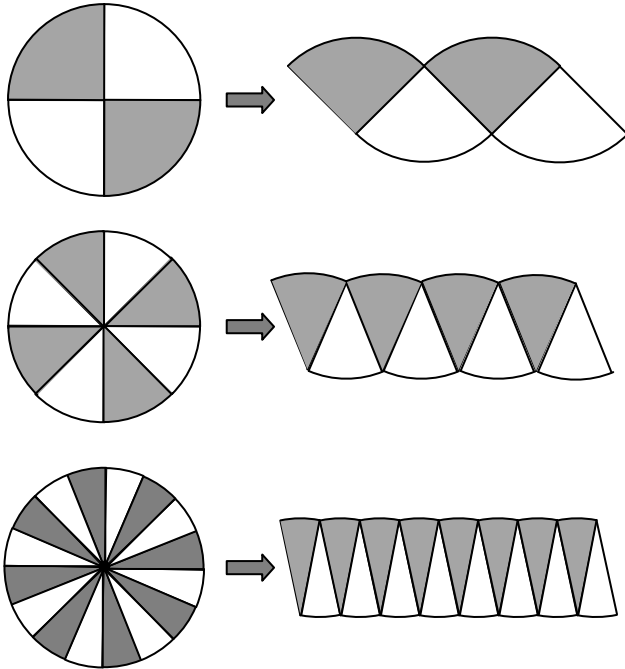
17. An isosceles right triangle is located in the coordinate plane as shown.



The triangle is rotated  $360^\circ$  about the  $y$ -axis.

- (a) Sketch the resulting solid.  
(b) What is the volume of the solid?

18. A circle is cut into increasingly larger numbers of sectors and rearranged as shown.



Explain how this process can be used to develop the formula for the area of a circle.

19. Stephanie has an aquarium that is in the shape of a right rectangular prism, 50 centimeters long, 25 centimeters wide and 30 centimeters tall.

For decoration, Stephanie wants a layer of marbles in the bottom of the tank about 5 centimeters deep. The marbles have a diameter of 1 centimeter and come in bags of 500.

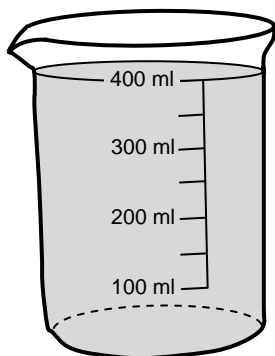
(a) How many bags of marbles will Stephanie need?

Stephanie pours all of the marbles into the tank. She now adds water until its level is 3 centimeters below the top of the tank.

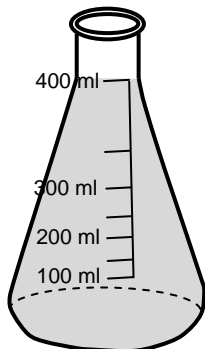
(b) How much water is in the tank? Express your answer in liters (1 liter = 1000 cubic centimeters).



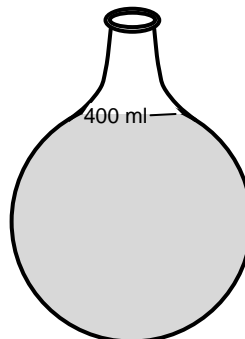
20. The diagram (not to scale) shows three types of glassware used in chemistry (from left to right): a beaker, an Erlenmeyer flask, and a Florence flask. All have 400 milliliters of liquid in them.



beaker



Erlenmeyer Flask



Florence Flask

When measuring liquid, one milliliter is equivalent to one cubic centimeter.

The beaker and Erlenmeyer flasks both have diameters of 8.0 cm.

- What is the approximate height of the liquid in the beaker?
  - What is the approximate height of the liquid in the Erlenmeyer flask?
  - The Florence flask is essentially a sphere with a small neck on it. Would the Florence flask fit inside the beaker? Explain.
  - The graduated markings on the beaker are equally spaced. Explain why they are not equally spaced on the Erlenmeyer flask.
  - Add 100 ml, 200 ml, and 300 ml graduated markings to the Florence flask. Explain why you drew them where you did.
21. A silicon wafer is a circular disc 80 millimeters in diameter. One side of the wafer is coated with 0.06 milligrams of a substance, called photoresist, to a uniform thickness. Photoresist has a density of 1.2 milligrams per cubic millimeter.
- What is the volume of photoresist used on the wafer?
  - What is the thickness of photoresist on the wafer?

22. A parabola has focus at  $(0, 3)$  and vertex at the origin. Which could be the equation of the directrix?

- (A)  $y = -12$
- (B)  $y = -3$
- (C)  $y = 0$
- (D)  $y = 3$

23. Jillian and Tammy are considering a quadrilateral  $ABCD$ . Their task is to prove  $ABCD$  is a square.

Jillian says, “We just need to show that the slope of  $\overline{AB}$  equals the slope of  $\overline{CD}$ , and the slope of  $\overline{BC}$  equals the slope of  $\overline{AD}$ .”

Tammy says, “We should show that  $AC = BD$  and that  $(\text{slope of } \overline{AC}) \times (\text{slope of } \overline{BD}) = -1$ .”

Whose method of proof is valid?

- (A) Only Jillian’s is valid.
- (B) Only Tammy’s is valid.
- (C) Both are valid.
- (D) Neither is valid.

24. Which equation describes a line passing through  $(-3, 1)$  and is parallel to  $y = 4x + 1$ ?

- (A)  $y = 4x + 13$
- (B)  $y = 4x - 11$
- (C)  $y = -\frac{1}{4}x + \frac{1}{4}$
- (D)  $y = -\frac{1}{4}x + \frac{7}{4}$

For questions 24–25, line  $\ell$  has equation  $y = \frac{2}{5}x$ .

25. The slope of any line parallel to  $\ell$  is  $\frac{2}{5}$ .

- (A) True
- (B) False

26. The slope of any line perpendicular to  $\ell$  is  $\frac{5}{2}$ .

- (A) True
- (B) False

27. The slope of  $\overline{PQ}$  is  $\frac{2}{3}$ . Point  $M$  lies  $\frac{1}{3}$  of the way from  $P$  to  $Q$  on  $\overline{PQ}$ . What is the slope of  $\overline{PM}$ ?

- (A)  $\frac{2}{9}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{4}{9}$
- (D)  $\frac{2}{3}$

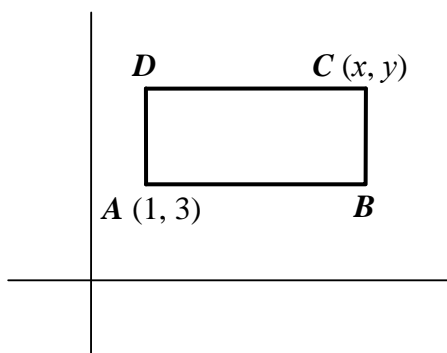
28. The point  $P$  divides  $\overline{AB}$  in a ratio of 4:1, where  $AP > BP$ . If  $A(-9, -5)$  and  $B(11, -2)$ , where is  $P$ ?

- (A)  $\left(7, -2\frac{3}{5}\right)$
- (B)  $\left(6, -\frac{1}{4}\right)$
- (C)  $\left(-4, -3\frac{1}{4}\right)$
- (D)  $\left(-5, -3\frac{3}{5}\right)$

29. The segment  $\overline{AB}$  has endpoints at  $(17, 1)$  and  $(-9, 3)$ . Where is  $M$ , the midpoint of  $\overline{AB}$ ?

- (A)  $(4, 2)$
- (B)  $(8, 4)$
- (C)  $(13, -1)$
- (D)  $(26, -2)$

30. Use the diagram.

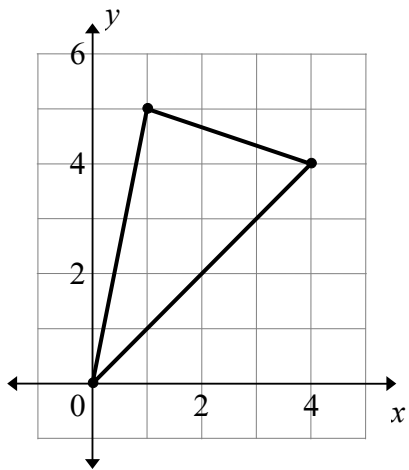


$ABCD$  is a rectangle where the slope of  $\overline{AB}$  is 0.

What is the area of the rectangle?

- (A)  $xy$
- (B)  $xy - 3$
- (C)  $(x - 1)(y - 3)$
- (D)  $2(x - 1) + 2(y - 3)$

31. Use the diagram.



What is the perimeter of the triangle, to the nearest whole unit?

- (A) 12
- (B) 14
- (C) 16
- (D) 18

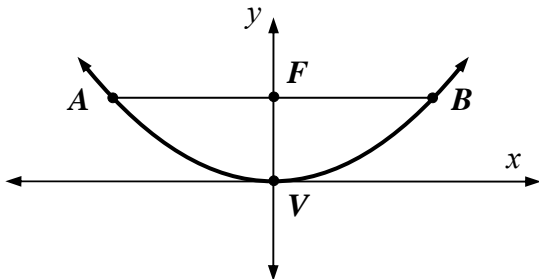
32. Find the area of the triangle with vertices at  $(-3, 2)$ ,  $(1, -2)$ , and  $(1, 3)$ .

- 8 units<sup>2</sup>
- 10 units<sup>2</sup>
- 12 units<sup>2</sup>
- 20 units<sup>2</sup>

33. Given  $M(-4, -1)$  and  $N(0, 3)$ .

- (a) Find the point  $P$  on  $\overline{MN}$  such that  $MP = 3 \cdot PN$ .
- (b) Translate the figure by  $(x, y) \rightarrow (x + 4, y)$ .
- (c) Prove  $MM'N'N$  is a parallelogram.
- (d) Prove the area of  $MM'P'P$  is 3 times the area of  $PP'N'N$ .

34. Three collinear points on the coordinate plane are  $A(x, y)$ ,  $B(x+4h, y+4k)$ , and  $P(x+3h, y+3k)$ . What is  $\frac{AP}{BP}$ ?
35. On a sheet of graph paper, complete each part.
- graph the line  $\ell$  with the equation  $y = \frac{1}{2}x + 5$ .
  - Define and graph the line  $m$  perpendicular to  $\ell$  and goes through the point  $(10, 0)$ .
  - Label the following points:
    - Lines  $\ell$  and  $m$  intersect at the point  $K$ .
    - Line  $\ell$  has  $y$ -intercept at point  $B$ .
    - Line  $m$  has  $x$ -intercept at point  $A$ .
    - The origin is point  $O$ .
  - Compute the perimeter of  $AOBK$ .
36. On a sheet of graph paper, complete each part.
- Graph the circle  $x^2 + y^2 = 36$ .
  - Plot the points where the circle intersects the  $x$ -axis. Label them  $A$  and  $B$ .
  - Plot the point  $P(4.8, 3.6)$ . Prove  $\triangle ABP$  is a right triangle using coordinate geometry.
37. Use the diagram below.



Segment  $\overline{AB}$  has endpoints on the parabola  $y = \frac{1}{4p}x^2$ , contains the focus, and is parallel to the directrix. Prove that the segment has length  $4p$ .

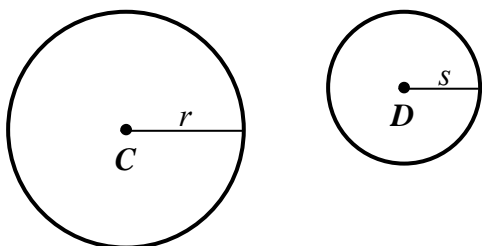
38. On a sheet of graph paper, complete each part

- (a) Sketch the parabola  $y = \frac{x^2}{4}$ .
- (b) Locate and sketch the parabola's focus and directrix. Label the focus  $F$ .
- (c) Plot the point on the parabola when  $x = 3$ . Label the point  $P$ .
- (d) Compare the distance  $PF$  to the distance from  $P$  to the directrix.

39. On a sheet of graph paper, complete each part.

- (a) Locate 3 points in the coordinate plane that form the vertices of a scalene triangle. Label them  $A$ ,  $B$ , and  $C$ .
- (b) Explain how you know your triangle  $ABC$  is scalene.
- (c) Locate the midpoints of  $\overline{AB}$  and  $\overline{AC}$ . Label them  $M$  and  $N$ .
- (d) Prove or disprove  $\overline{MN} \parallel \overline{BC}$ .
- (e) Prove or disprove  $MN = \frac{1}{2}BC$ .

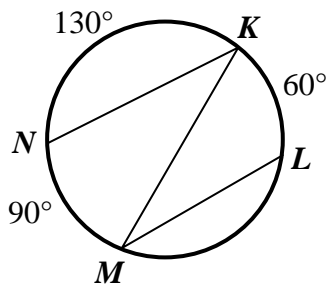
40. Use the diagram.



To show circle  $C$  is similar to circle  $D$ , one would have to translate circle  $C$  by the vector  $\overrightarrow{CD}$ . Then, circle  $C'$  would have to be dilated by what factor?

- (A)  $s - r$
- (B)  $s^2 - r^2$
- (C)  $\frac{s}{r}$
- (D)  $\frac{s^2}{r^2}$

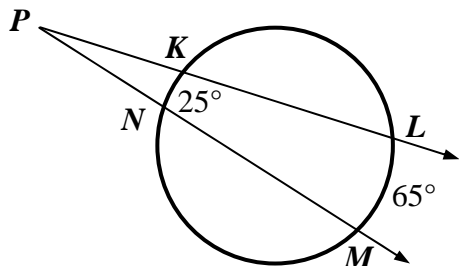
41. Use the diagram.



What is  $m\angle NKM$  ?

- (A)  $30^\circ$
- (B)  $45^\circ$
- (C)  $60^\circ$
- (D)  $90^\circ$

42. In the diagram,  $m\widehat{KN} = 25^\circ$  and  $m\widehat{ML} = 65^\circ$ .

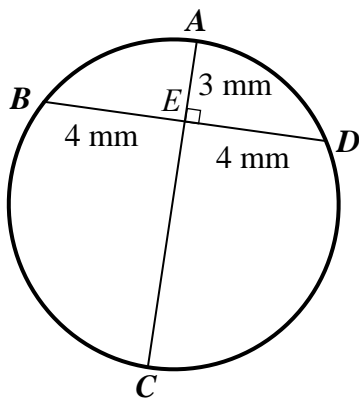


What is  $m\angle KPN$  ?

- (A)  $20^\circ$
- (B)  $25^\circ$
- (C)  $45^\circ$
- (D)  $65^\circ$



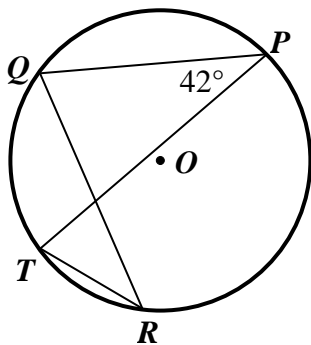
43. Use the diagram.



What is  $AC$ ?

- (A)  $5\frac{1}{3}$  mm
- (B) 8 mm
- (C)  $8\frac{1}{3}$  mm
- (D) 16 mm

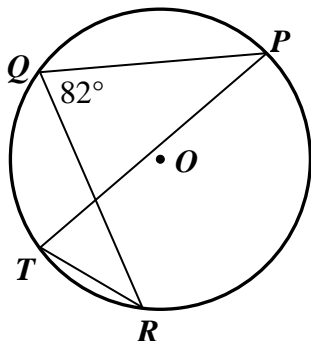
44. In circle  $O$ ,  $m\angle QPT = 42^\circ$ .



What is  $m\angle QRT$  ?

- (A)  $21^\circ$
- (B)  $42^\circ$
- (C)  $63^\circ$
- (D)  $84^\circ$

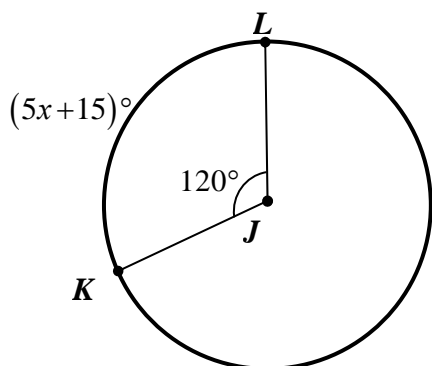
45. In circle  $O$ ,  $m\angle RQP = 82^\circ$ .



What is  $m\angle RTP$ ?

- (A)  $41^\circ$
- (B)  $82^\circ$
- (C)  $98^\circ$
- (D)  $164^\circ$

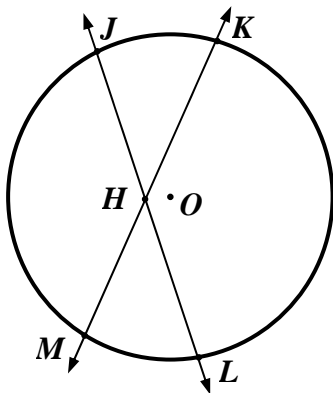
46. Use circle  $J$ .



What is the value of  $x$ ?

- (A) 9
- (B) 21
- (C) 27
- (D) 45

For questions 46–47, use circle  $O$  where  $m\angle MHJ = 120^\circ$ , and points  $H$  and  $O$  are distinct.



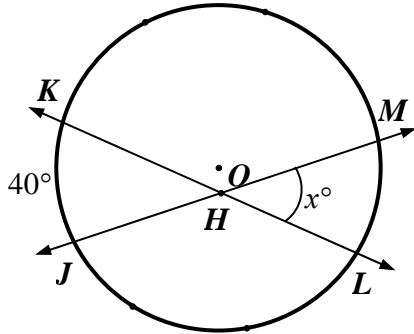
47.  $m\widehat{JM}$  and  $m\widehat{KL}$  could be  $100^\circ$  and  $140^\circ$ , respectively.

- (A) True
- (B) False

48.  $m\widehat{JM}$  and  $m\widehat{KL}$  could both be  $120^\circ$ .

- (A) True
- (B) False

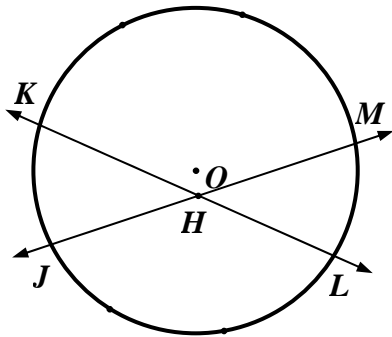
49. In circle  $O$ ,  $m\angle MHL = x^\circ$  and  $m\widehat{JK} = 40^\circ$ .



What is  $m\widehat{LM}$ ?

- (A)  $40^\circ$
- (B)  $(x - 20)^\circ$
- (C)  $(x - 40)^\circ$
- (D)  $(2x - 40)^\circ$

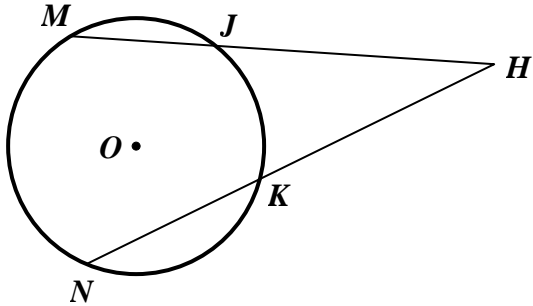
50. In circle  $O$ ,  $m\widehat{JL} = (6x + 5)^\circ$ ,  $m\widehat{KM} = (10x + 3)^\circ$ , and  $m\angle JHL = 140^\circ$ .



What is the value of  $x$ ?

- (A) 8.25
- (B) 9.25
- (C) 17
- (D) 18

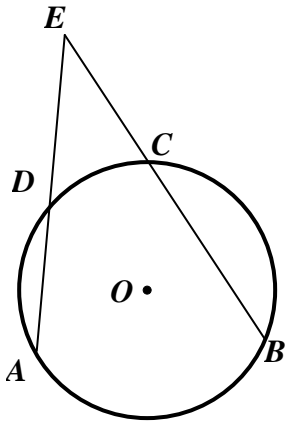
51. In the figure below,  $m\widehat{JK} = 26^\circ$  and  $m\widehat{MN} = 130^\circ$ .



What is  $m\angle H$ ?

- (A)  $52^\circ$
- (B)  $78^\circ$
- (C)  $104^\circ$
- (D)  $128^\circ$

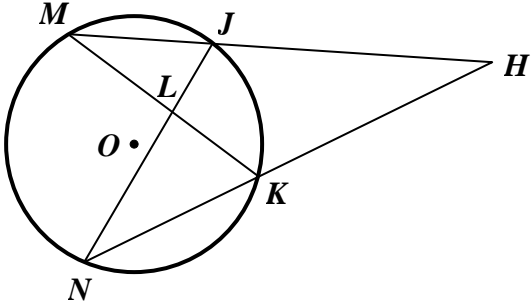
52. In the figure below,  $m\widehat{AB} = m^\circ$  and  $m\widehat{CD} = n^\circ$ .



What is  $m\angle E$ ?

- (A)  $m - n$
- (B)  $\frac{1}{2}(m - n)$
- (C)  $m + n$
- (D)  $\frac{1}{2}(m + n)$

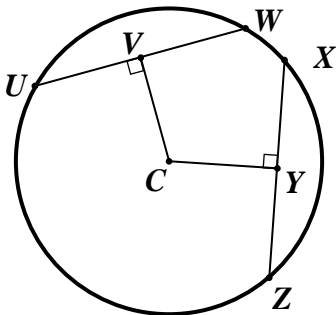
53. In the figure below,  $m\widehat{JK} = 66^\circ$  and  $m\widehat{MN} = 128$ .



What is  $m\angle H$  ?

- (A)  $31^\circ$
- (B)  $62^\circ$
- (C)  $64^\circ$
- (D)  $97^\circ$

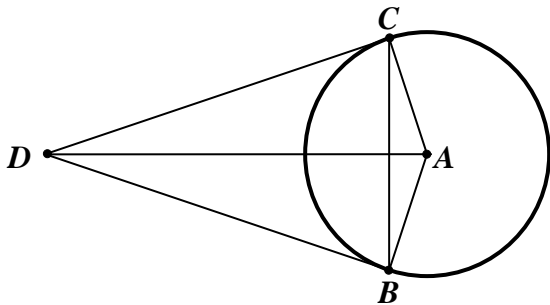
54. In circle  $C$ ,  $UW = XZ$ ,  $VW = 2x + 14$ , and  $YZ = 6x + 2$



What is the value of  $x$ ?

- (A) 2
- (B) 3
- (C) 4
- (D) 5

In questions 54–56, the segments  $\overline{DB}$  and  $\overline{DC}$  are tangent to circle  $A$ .



55.  $\triangle BCD$  is a right triangle

- (A) True
- (B) False

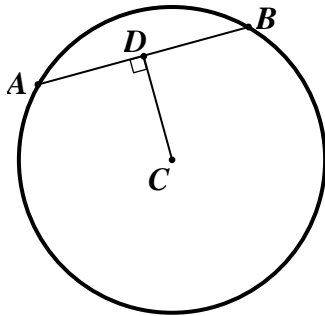
56.  $BD = CD$

- (A) True
- (B) False

57.  $\frac{1}{2}m\widehat{BC} = m\angle BDC$

- (A) True
- (B) False

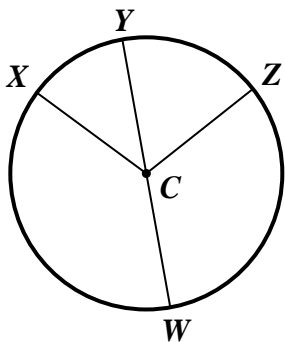
In questions 57–59,  $\overline{CD}$  is perpendicular to chord  $\overline{AB}$  in circle  $C$ .



58.  $AD = BD$
- (A) True  
(B) False
59.  $CD = BD$
- (A) True  
(B) False
60.  $AC = CB$
- (A) True  
(B) False

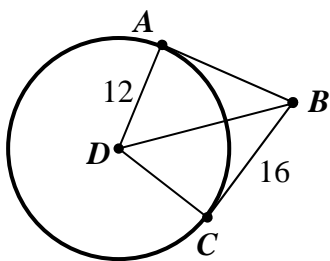


61.  $\overline{WY}$  is a diameter of circle  $C$ .  $m\angle ZCY = (2x + 10)^\circ$ ,  $m\angle WCZ = (4x - 10)^\circ$ , and  $m\angle YCX = (2x - 4)^\circ$ .



What is the value of  $x$ ?

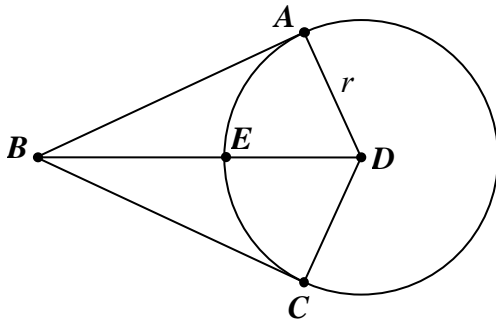
- (A) 22  
 (B) 30  
 (C) 32.5  
 (D) 43.5
62. With respect to circle  $D$ ,  $\overline{AB}$  is tangent at  $A$ , and  $\overline{CB}$  is tangent at  $C$ .



What is the length of  $\overline{BD}$ ?

- (A) 11  
 (B) 14  
 (C) 16  
 (D) 20

In questions 62–64,  $\overline{AB}$  is tangent to circle  $D$  at  $A$ , and  $\overline{BC}$  is tangent to circle  $D$  at  $C$ .



63.  $AB = CB$

- (A) True
- (B) False

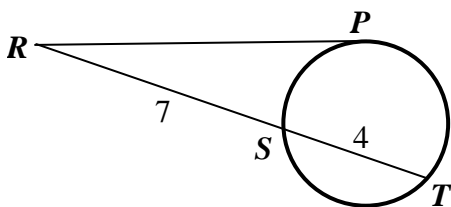
64.  $(AB)^2 = r^2 + (BD)^2$

- (A) True
- (B) False

65.  $(BE)(BD) = (BC)^2$

- (A) True
- (B) False

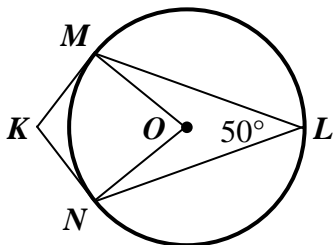
66. Use the figure.



What is the length of  $\overline{RP}$ ?

- (A) 9
- (B) 11
- (C)  $\sqrt{28}$
- (D)  $\sqrt{77}$

67. In the diagram below  $\overline{KM}$  and  $\overline{KN}$  are tangent to circle  $O$ , and  $ML = NL$ .

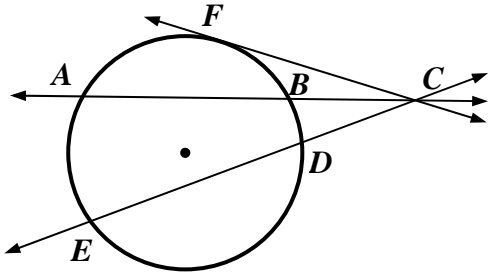


What is  $m\angle ONL$ ?

- (A)  $15^\circ$
- (B)  $25^\circ$
- (C)  $65^\circ$
- (D)  $90^\circ$

In questions 67–68, use the figure.

$\overleftrightarrow{AC}$  and  $\overleftrightarrow{BC}$  are secants of the circle and  $\overleftrightarrow{FC}$  is tangent, where  $AB = 20$ ,  $BC = 4$ , and  $CD = 3$ .



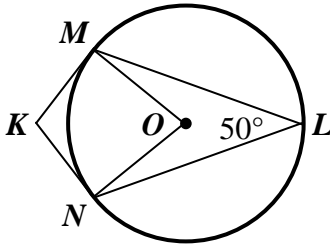
68. What is  $DE$ ?

- (A) 15
- (B) 18
- (C) 29
- (D) 32

69. What is  $FC$ ?

- (A) 12
- (B) 40
- (C)  $\sqrt{80}$
- (D)  $\sqrt{96}$

In questions 69–71, use the diagram below where  $\overline{KM}$  and  $\overline{KN}$  are tangent to circle  $O$ .



70. What is  $m\angle MON$ ?

- (A)  $50^\circ$
- (B)  $80^\circ$
- (C)  $90^\circ$
- (D)  $100^\circ$

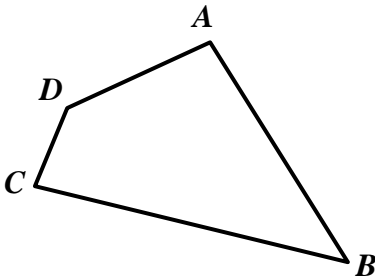
71. What is  $m\angle MKN$ ?

- (A)  $50^\circ$
- (B)  $80^\circ$
- (C)  $90^\circ$
- (D)  $100^\circ$

72. What is  $m\angle KMO$ ?

- (A)  $50^\circ$
- (B)  $80^\circ$
- (C)  $90^\circ$
- (D)  $100^\circ$

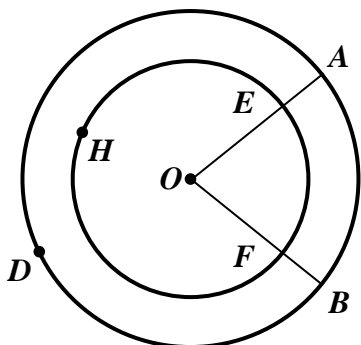
73. Use the figure.



Quadrilateral  $ABCD$  is to be circumscribed by a circle. What must be true?

- (A) Opposite angles are supplementary.
- (B) One of the angles is a right angle.
- (C) Both must be true.
- (D) Neither must be true.

In questions 73–75, use the diagram of two concentric circles centered at  $O$ , and  $m\angle AOB = 70^\circ$ .



74.  $m\widehat{EF} = 70^\circ$

- (A) True
- (B) False

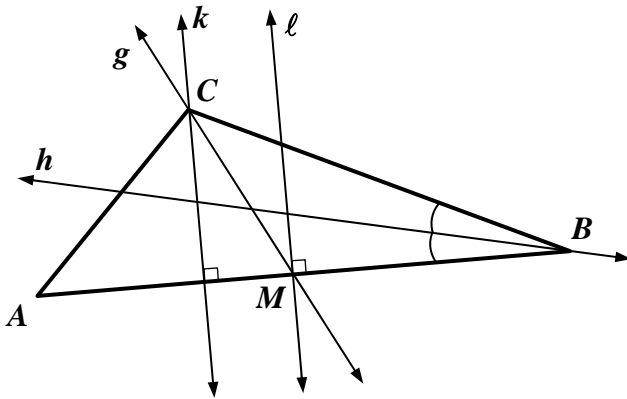
75.  $\widehat{ADB}$  is called a major arc.

- (A) True
- (B) False

76.  $m\widehat{AB} > m\widehat{EF}$

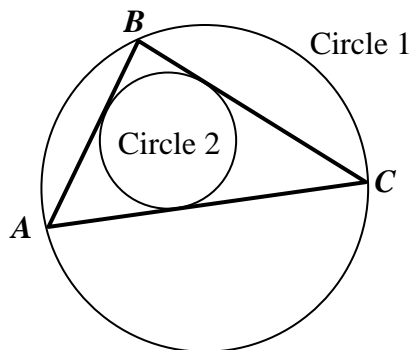
- (A) True
- (B) False

In questions 76–78, use the diagram of a scalene where  $M$  is the midpoint of  $\overline{AB}$ .



77. The circumcenter of  $\triangle ABC$  lies on which line?
- (A)  $g$
  - (B)  $h$
  - (C)  $k$
  - (D)  $\ell$
78. The incenter of  $\triangle ABC$  lies on which line?
- (A)  $g$
  - (B)  $h$
  - (C)  $k$
  - (D)  $\ell$
79. The centroid (center of mass) of  $\triangle ABC$  lies on which line?
- (A)  $g$
  - (B)  $h$
  - (C)  $k$
  - (D)  $\ell$

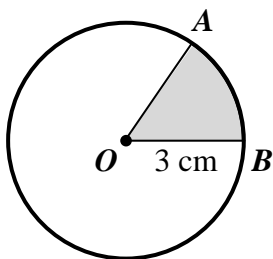
In questions 79–80, use the diagram where Circle 1 is circumscribed about  $\triangle ABC$  and Circle 2 is inscribed in  $\triangle ABC$ .



80. To find the center of Circle 1, what would be constructed on  $\triangle ABC$ ?
- (A) altitudes
  - (B) angle bisectors
  - (C) medians
  - (D) perpendicular bisectors
81. To find the center of Circle 2, what would be constructed on  $\triangle ABC$ ?
- (A) altitudes
  - (B) angle bisectors
  - (C) medians
  - (D) perpendicular bisectors
82. On a circle of radius  $r$ , a central angle of  $x$  radians subtends an arc of length  $r$ . What is the value of  $x$ ?
- (A)  $\pi/6$
  - (B)  $\pi/2$
  - (C) 1
  - (D) 3.14



83. Use the diagram.



What is the area of the shaded region if  $m\angle AOB = 50^\circ$ ?

- (A)  $\frac{5\pi}{2}\text{ cm}^2$
- (B)  $\frac{5\pi}{3}\text{ cm}^2$
- (C)  $\frac{5\pi}{4}\text{ cm}^2$
- (D)  $\frac{5\pi}{6}\text{ cm}^2$

84. Which angle is equivalent to  $\frac{\pi}{2}$  radians?

- (A)  $30^\circ$
- (B)  $45^\circ$
- (C)  $90^\circ$
- (D)  $180^\circ$

85. Which angle is equivalent to  $45^\circ$ ?

- (A)  $\frac{\pi}{8}$  radians
- (B)  $\frac{\pi}{4}$  radians
- (C)  $\frac{\pi}{2}$  radians
- (D)  $2\pi$  radians

86. A circle is centered at  $(-3, 5)$ . The point  $(3, 5)$  is on the circle. What is the equation of the circle?

(A)  $(x-3)^2 + (y+5)^2 = 6$

(B)  $(x+3)^2 + (y-5)^2 = 6$

(C)  $(x-3)^2 + (y+5)^2 = 36$

(D)  $(x+3)^2 + (y-5)^2 = 36$

87. What is the radius of the circle  $9x^2 + 9y^2 = 63$ ?

(A)  $\sqrt{7}$

(B) 7

(C)  $\sqrt{63}$

(D) 63

88. Which is the equation of a circle that passes through at  $(2, 2)$  and centered at  $(5, 6)$ ?

(A)  $(x+5)^2 + (y+6)^2 = 5$

(B)  $(x-5)^2 + (y-6)^2 = 5$

(C)  $(x+5)^2 + (y+6)^2 = 25$

(D)  $(x-5)^2 + (y-6)^2 = 25$

89. The graph of which equation would be a circle with a center at  $(9, -9)$  and a radius of 13?

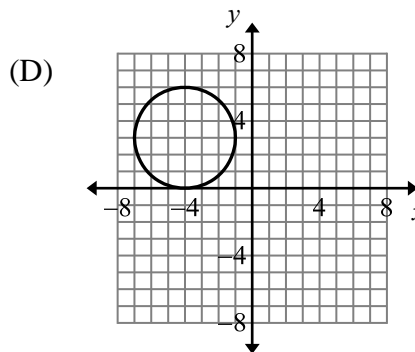
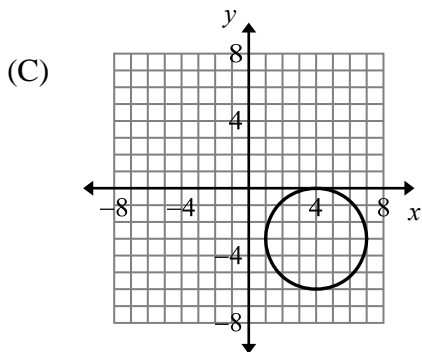
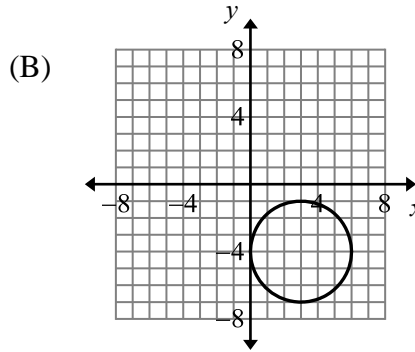
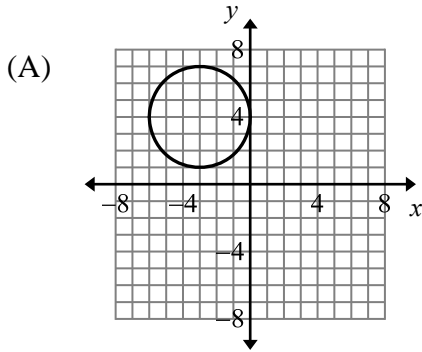
(A)  $(x+9)^2 + (y-9)^2 = 13$

(B)  $(x-9)^2 + (y+9)^2 = 13$

(C)  $(x+9)^2 + (y-9)^2 = 169$

(D)  $(x-9)^2 + (y+9)^2 = 169$

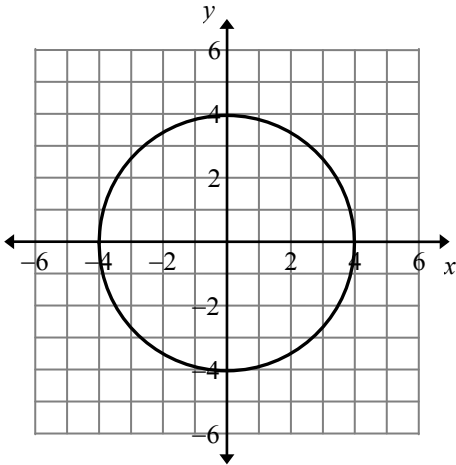
90. What graph represents  $(x+3)^2 + (y-4)^2 = 9$ ?



91. What are the coordinates of the center of the circle  $(x+6)^2 + (y-6)^2 = 36$ ?

- (A)  $(-6, -6)$
- (B)  $(-6, 6)$
- (C)  $(6, -6)$
- (D)  $(6, 6)$

92. Which is the equation of the graph shown below?



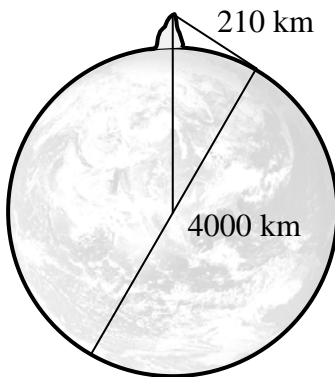
(A)  $x^2 + y^2 = 4$

(B)  $x^2 + y^2 = 16$

(C)  $x + y = 4$

(D)  $x + y = 16$

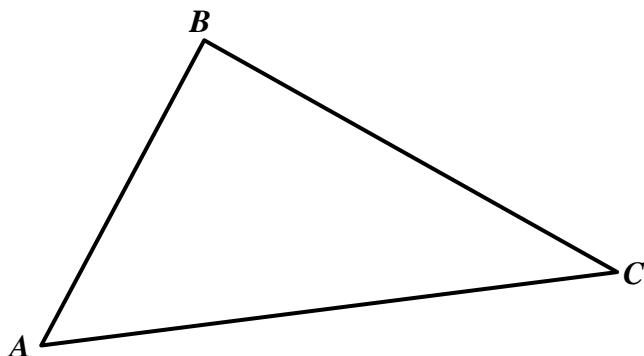
93. An astronaut stands at the peak of a mountain on a distant planet. The planet has a diameter of 4000 km and the distance from the peak to the horizon is about 210 km. (When looking to the horizon, one's line of sight is tangent to the surface of the planet.)



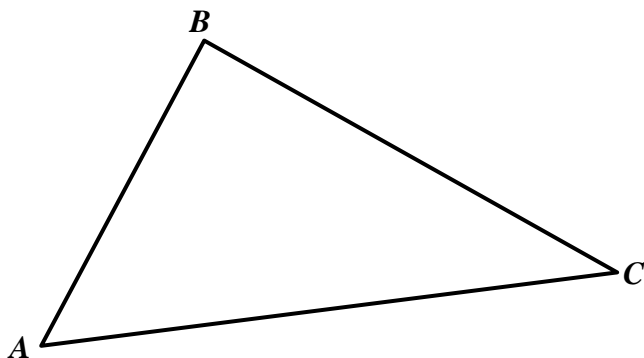
How tall is the mountain? (Diagram not drawn to scale.)

94. Draw two circles of different radii. Prove the circles are similar.

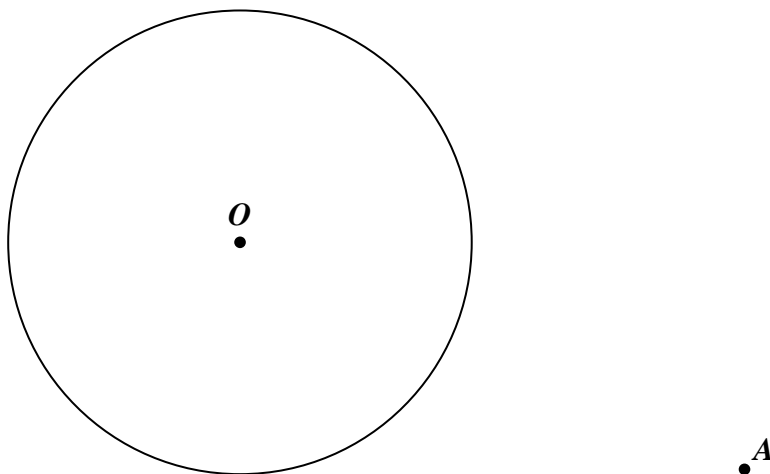
95. Inscribe a circle in the triangle below by construction.



96. Circumscribe a circle in the triangle below by construction.



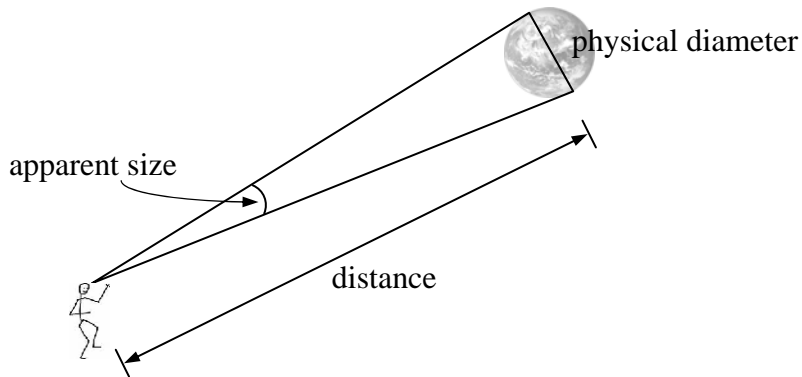
97. Construct a line tangent to circle  $O$  that passes through point  $A$ .



98. A circle passes through the points (1, 3) and (–5, 9).

- (a) Give an equation of the circle.
- (b) Name one other point on the circle.

99. The *apparent size* of an object as seen from a given position is the “visual diameter” of the object measured as an angle. For instance, a thumb held at arm’s length has an apparent size of about 2 degrees. This can be used to approximate a physical diameter by using arc lengths.

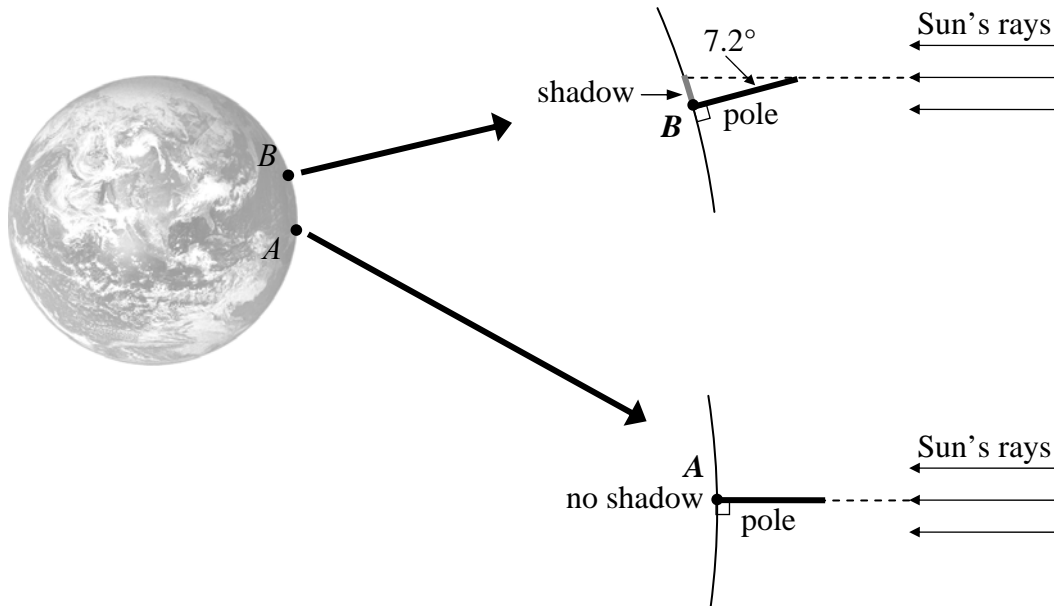


The moon orbits approximately 250,000 miles from the Earth and has an apparent size of about  $0.50^\circ$ . What is the approximate diameter of the moon?

100. Describe the relationship between the distance formula, the Pythagorean Theorem, and the equation of a circle.

101. What is the equation of the line tangent to the circle  $x^2 + y^2 = 32$  at (4, 4)?

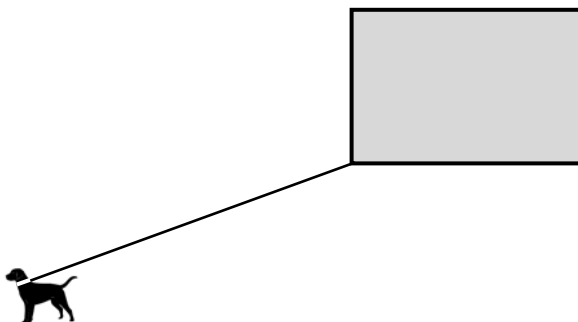
102. At certain places on Earth at certain times, the sun is directly overhead. At those times, a pole perpendicular to the ground will not cast a shadow. If the sun is not directly overhead, a pole will cast a shadow.



At noon on the same day, observers at points  $A$  and  $B$  on the surface of the earth look for shadows cast by a pole. The observer at point  $A$  sees no shadow. The observer at point  $B$ , which is about 500 miles due north of point  $A$ , sees a shadow. The observer measures the length of the shadow and determines the angle between the pole and the sun's rays is  $7.2^\circ$ .

Using this information, determine the approximate circumference of the earth.

103. The diagram below shows dog tethered by a rope to the corner of a 4-foot by 6-foot rectangular shed in a large, open yard. The length of the tether is 9 feet.



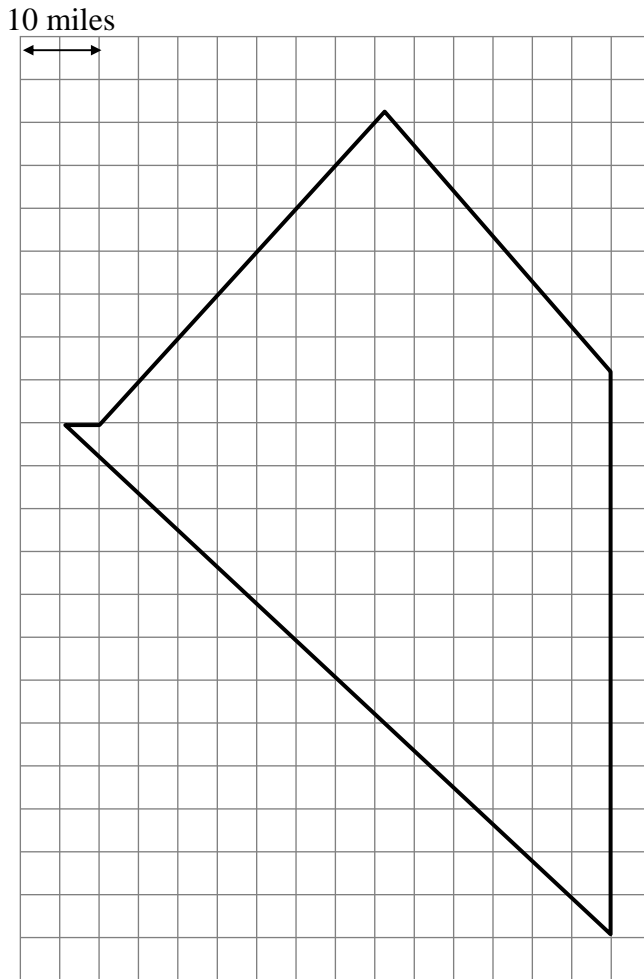
Make a sketch of the places the dog can reach on the tether and compute its area.

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104. The map at right shows the counties in the State of Nevada. The shaded area is Esmeralda County.



This diagram below shows Esmeralda County superimposed on a grid.



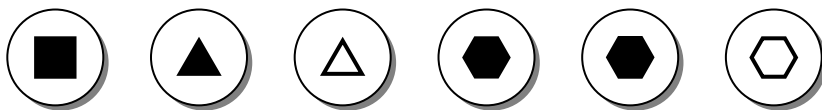
- Approximate the area of Esmeralda County.
- Esmeralda county is the least populated county in Nevada with only 775 people. What is the population density of Esmeralda County?



105. The set of all outcomes of a rolled die is  $\{1, 2, 3, 4, 5, 6\}$ . What is the complement of the subset  $\{1, 2\}$ ?
- (A)  $\{3, 4, 5, 6\}$   
(B)  $\{1, 2\}$   
(C)  $\{5, 6\}$   
(D) There is not enough information to determine.

For questions 104–106, use the following scenario.

The 6 discs shown below are placed in a hat. Each is described by the shape and color of its symbol.



One disc is drawn at random; the shape and color of its symbol is observed.

106. The events “square” and “white” are mutually exclusive.
- (A) True  
(B) False
107. The events “hexagon” and “black” are independent.
- (A) True  
(B) False
108. The probability the shape is a “triangle” given that it is “black” equals  $\frac{1}{3}$ .
- (A) True  
(B) False

109. For two events  $A$  and  $B$ ,  $P(A) = \frac{1}{2}$ , and  $P(B|A) = \frac{2}{3}$ .

What is  $P(B)$ ?

- (A)  $\frac{1}{3}$
- (B)  $\frac{1}{2}$
- (C)  $\frac{2}{3}$
- (D) There is not enough information to determine.

110. A sample of 100 female politicians was asked, “Which ice cream flavor do you prefer: Chocolate or Vanilla?” The respondents were classified by their political parties: Party X or Party Y. The results are shown in the table below.

		Flavor Preference		
		Chocolate	Vanilla	Total
Political Party	Party X		?	40
	Party Y			60
	Total	50	50	100

If ice cream flavor preference is independent of political party, how many female politicians are in Party X and prefer vanilla?

- (A) 20
- (B) 30
- (C) 40
- (D) 50

111. When looking at the association between the events “owns a car” and “owns a pet,” if the events are independent, then the probability

$$P(\text{owns a pet} \mid \text{owns a car})$$

is equal to \_\_\_\_\_.

- (A)  $P(\text{owns a pet})$
- (B)  $P(\text{owns a car})$
- (C)  $P(\text{owns a pet}) \times P(\text{owns a car})$
- (D)  $P(\text{owns a pet}) + P(\text{owns a car})$

In questions 110–111, use the scenario below.

In a study of student part-time job experiences, it was found that 80% of students worked part-time only in the summer. It was also found that 84% of females worked part-time only in the summer. Half of the students in the study were female.

112. Working part-time only in the summer is independent of gender.

- (A) True
- (B) False

113. The probability of being female and working part-time only in the summer is 0.42.

- (A) True
- (B) False

114. The probabilities an adult male has high blood pressure and/or high cholesterol are given below.

		Blood pressure	
		High	Normal
Cholesterol	High	0.10	0.20
	Normal	0.15	0.55

Which event is more likely?

- (A) A man known to have high blood pressure also has high cholesterol.
- (B) A man known to have high cholesterol also has high blood pressure.
- (C) Neither. The probabilities are the same.

115. Park High School has 200 freshmen. Of them, 50 are in an art class and 60 are in a music class. Twenty students are in both art and music.

What is the probability a randomly chosen freshman is in art or music?

- (A)  $\frac{20}{200}$   
(B)  $\frac{70}{200}$   
(C)  $\frac{90}{200}$   
(D)  $\frac{110}{200}$

116. The probabilities an adult male has high blood pressure and/or high cholesterol are given below.

		Blood pressure	
		High	Normal
Cholesterol	High	0.10	0.20
	Normal	0.15	0.55

What the probability a randomly selected adult male has high blood pressure or high cholesterol?

- (A) 0.075  
(B) 0.375  
(C) 0.45  
(D) 0.55
117. Andrew and Lyndon each have a bag of 5 marbles of which 2 are red and 3 are black.

Andrew randomly drew two marbles from his bag without replacement. Both marbles were red.

Lyndon randomly drew two marbles from his bag, with replacement. Both marbles were red.

Which statement is true?

- (A) Andrew had a 6% greater chance of getting both marbles red than Lyndon.  
(B) Lyndon had a 6% greater chance of getting both marbles red than Andrew.  
(C) Andrew had an 8% greater chance of getting both marbles red than Lyndon.  
(D) Lyndon had an 8% greater chance of getting both marbles red than Andrew.

118. A box contains four identically-sized balls, each of different color: red, yellow, green, and blue.

If the four balls are taken from the box one at a time, without replacement, what the probability the 2<sup>nd</sup> ball will be yellow?

(A)  $\frac{1}{4}$

(B)  $\frac{1}{6}$

(C)  $\frac{1}{8}$

(D)  $\frac{1}{24}$

For questions 117–118, consider 8 distinct objects from which 5 are randomly selected.

119. The number of permutations equals:

(A)  $8 \times 7 \times 6$

(B)  $8 \times 7 \times 6 \times 5 \times 4$

(C)  $\frac{8 \times 7 \times 6 \times 5 \times 4}{3 \times 2 \times 1}$

(D)  $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$

120. The number of combinations equals:

(A)  $8 \times 7 \times 6$

(B)  $8 \times 7 \times 6 \times 5 \times 4$

(C)  $\frac{8 \times 7 \times 6 \times 5 \times 4}{3 \times 2 \times 1}$

(D)  $\frac{8 \times 7 \times 6}{3 \times 2 \times 1}$

For questions 119–120, use the scenario below.

An arcade game costs 1 token to play. The player rolls two special dice; the numbers on the dice are multiplied. The table shows the possible outcomes.

		Die 1			
		0	1	2	3
Die 2	1	0	1	2	3
	2	0	2	4	6
	3	0	3	6	9
	4	0	4	8	12

To win the game, the person rolling the dice must get a product that is an odd number. The player then gets his token back plus one more. Otherwise, the player loses his token.

121. This is a fair game.

- (A) True
- (B) False

122. If the numbers on the dice were added instead of multiplied, this would be a fair game.

- (A) True
- (B) False

In questions 121–121, consider the scenario below.

A carnival game is advertised to produce a winner  $\frac{1}{3}$  of the times played. Each game outcome is independent of any other.

Miguel played the game six times and never won. Assuming the chance of winning is as advertised:

123. The chance of Miguel losing 6 straight games is  $\frac{12}{18}$ .

- (A) True
- (B) False

124. The chance of Miguel winning the next time he plays is  $\frac{1}{3}$ .

- (A) True
  - (B) False
- 

125. Two independent events,  $A$  and  $B$  are independent, have the following probabilities.

$$P(A) = 0.3$$

$$P(B) = 0.4$$

Compute:

- (a)  $P(A \text{ and } B)$
- (b)  $P(A \text{ or } B)$
- (c)  $P(\text{not } A)$
- (d)  $P(A|B)$

126. The network security specialist for a company finds that 15% of incoming email messages is spam. The specialist has created a filter program that correctly labels 80% of incoming spam messages as spam. The program also incorrectly labels 5% of legitimate messages as spam.
- (a) What percentage of all incoming messages is labeled as spam?
  - (b) A worker receives a message that is labeled as spam. What is the probability that it is actually spam?
  - (c) What percentage of all incoming messages are incorrectly classified?
  - (d) Which is more likely: a legitimate message is labeled spam, or a spam message is labeled legitimate? Explain.

127. A survey of 35 randomly-selected students asked whether each has siblings and whether each owns pets. The results are shown at right.

- (a) Construct a two-way table to display the data.
- (b) If one respondent were chosen at random, what is the probability that:
  - i. the respondent has siblings?
  - ii. the respondent has siblings or owns pets?
  - iii. the respondent has siblings given that he/she owns pets?
- (c) Is owning pets independent of having siblings? Explain.

Student Number	Do you have siblings?	Do you own pets?
1	No	No
2	No	No
3	No	No
4	No	No
5	No	No
6	No	No
7	No	No
8	No	No
9	No	Yes
10	No	Yes
11	No	Yes
12	No	Yes
13	No	Yes
14	No	Yes
15	Yes	No
16	Yes	No
17	Yes	No
18	Yes	No
19	Yes	No
20	Yes	No
21	Yes	Yes
22	Yes	Yes
23	Yes	Yes
24	Yes	Yes
25	Yes	Yes
26	Yes	Yes
27	Yes	Yes
28	Yes	Yes
29	Yes	Yes
30	Yes	Yes
31	Yes	Yes
32	Yes	Yes
33	Yes	Yes
34	Yes	Yes
35	Yes	Yes



**GEOMETRY HONORS  
2012–2013 SEMESTER EXAMS  
PRACTICE MATERIALS KEY  
SEMESTER 2**

**Selected Response Key**

#	Question Type	Unit	Common Core State Standard(s)	DOK Level	Key
1	MC	3	G.GMD.1	2	C
2	MC	3	G.GMD.1	2	C
3	MC	3	G.GMD.1	1	A
4	MC	3	G.GMD.3	1	C
5	MC	3	G.GMD.3	1	D
6	MC	3	G.GMD.3	1	B
7	MC	3	G.GMD.3	2	D
8	MC	3	G.GMD.3	2	C
9	MC	3	G.GMD.3	2	B
10	MC	3	G.GMD.3	2	D
11	MC	3	G.GMD.4	2	A
12	MC	3	G.GMD.4	2	D
13	MC	3	G.GMD.4	2	A
14	MC	3	G.MG.1	2	A
15	MC	3	G.MG.2	2	C
16	MC	3	G.MG.2	2	C
17	CR	3	G.GMD.3, G.GMD.4	2	—
18	CR	3	G.GMD.1	3	—
19	ER	3	G.GMD.3, G.MG.1	2	—
20	CR	3	G.GMD.3, G.MG.1	3	—
21	CR	3	G.GMD.3, G.MG.1, G.MG.2	2	—
22	MC	4	G.GPE.2	1	B
23	MC	4	G.GPE.4	2	B
24	MC	4	G.GPE.5	1	A
25	MTF	4	G.GPE.5	1	A
26	MTF	4	G.GPED.5	1	B
27	MC	4	G.GPE.6	1	D
28	MC	4	G.GPE.6	2	A
29	MC	4	G.GPE.6	1	A
30	MC	4	G.GPE.7	2	C
31	MC	4	G.GPE.7	2	B
32	MC	4	G.GPE.7	2	B
33	CR	4	G.GPE.4, G.GPE.6, G.GPE.7	3	—
34	CR	4	G.GPE.6	2	—
35	CR	4	G.GPE.5, G.GPE.7	2	—
36	CR	4	G.GPE.1, G.GPE.4	3	—
37	CR	4	G.GPE.2, G.GPE.4	3	—
38	CR	4	G.GPE.2	2	—
39	CR	4	G.GPE.4, G.GPE.5	3	—
40	MC	5	G.C.1	2	C
41	MC	5	G.C.2	1	B
42	MC	5	G.C.2	1	A
43	MC	5	G.C.2	2	A
44	MC	5	G.C.2	1	B
45	MC	5	G.C.2	1	B
46	MC	5	G.C.2	1	B
47	MTF	5	G.C.2	1	A
48	MTF	5	G.C.2	1	B
49	MC	5	G.C.2	1	D

**GEOMETRY HONORS  
2012–2013 SEMESTER EXAMS  
PRACTICE MATERIALS KEY  
SEMESTER 2**

**Selected Response Key**

#	Question Type	Unit	Common Core State Standard(s)	DOK Level	Key
50	MC	5	G.C.2	1	C
51	MC	5	G.C.2	1	A
52	MC	5	G.C.2	1	B
53	MC	5	G.C.2	1	A
54	MC	5	G.C.2	2	B
55	MTF	5	G.C.2	1	B
56	MTF	5	G.C.2	1	A
57	MTF	5	G.C.2	1	B
58	MTF	5	G.C.2	2	A
59	MTF	5	G.C.2	2	B
60	MTF	5	G.C.2	2	A
61	MC	5	G.C.2	2	B
62	MC	5	G.C.2	2	D
63	MTF	5	G.C.2	1	A
64	MTF	5	G.C.2	1	B
65	MTF	5	G.C.2	1	B
66	MC	5	G.C.2	1	D
67	MC	5	G.C.2	2	B
68	MC	5	G.C.2	1	C
69	MC	5	G.C.2	1	D
70	MC	5	G.C.2	1	D
71	MC	5	G.C.2	1	B
72	MC	5	G.C.2	1	C
73	MC	5	G.C.3	1	A
74	MTF	5	G.C.2	1	A
75	MTF	5	G.C.2	1	A
76	MTF	5	G.C.2	1	B
77	MC	5	G.C.3	1	D
78	MC	5	G.C.3	1	B
79	MC	5	G.C.3	1	A
80	MC	5	G.C.3	1	D
81	MC	5	G.C.3	1	B
82	MC	5	G.C.5	1	C
83	MC	5	G.C.5	2	C
84	MC	5	G.C.5	1	C
85	MC	5	G.C.5	1	B
86	MC	5	G.GPE.1	2	D
87	MC	5	G.GPE.1	2	A
88	MC	5	G.GPE.1	2	D
89	MC	5	G.GPE.1	1	D
90	MC	5	G.GPE.1	1	D
91	MC	5	G.GPE.1	1	B
92	MC	5	G.GPE.1	1	B
93	CR	5	G.C.2	2	—
94	CR	5	G.C.1	3	—
95	CR	5	G.C.3	2	—
96	CR	5	G.C.3	2	—
97	CR	5	G.C.4	2	—
98	CR	5	G.C.5	2	—
99	CR	5	G.C.5, G.MG.1	2	—

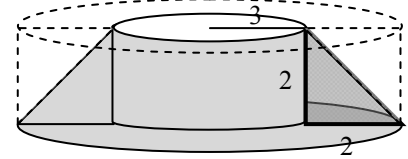
**GEOMETRY HONORS  
2012–2013 SEMESTER EXAMS  
PRACTICE MATERIALS KEY  
SEMESTER 2**

**Selected Response Key**

#	Question Type	Unit	Common Core State Standard(s)	DOK Level	Key
100	CR	5	G.GPE.1	3	—
101	CR	5	G.GPE.5	2	—
102	CR	5	G.C.5, G.MG.1	2	—
103	CR	5	G.GPE.4	2	—
104	CR	5	G.GPE.7	2	—
105	MC	6	S.CP.1	1	A
106	MTF	6	S.CP.1	1	A
107	MTF	6	S.CP.2	1	A
108	MTF	6	S.CP.3	2	B
109	MC	6	S.CP.3	2	D
110	MC	6	S.CP.4	2	A
111	MC	6	S.CP.5	1	A
112	MTF	6	S.CP.5	2	B
113	MTF	6	S.CP.8	2	A
114	MC	6	S.CP.5	2	A
115	MC	6	S.CP.7	1	C
116	MC	6	S.CP.7	1	C
117	MC	6	S.CP.8	2	B
118	MC	6	S.CP.9	2	A
119	MC	6	S.CP.9	1	B
120	MC	6	S.CP.9	1	D
121	MTF	6	S.MD.6	2	B
122	MTF	6	S.MD.6	2	A
123	MTF	6	S.MD.6	2	B
124	MTF	6	S.MD.6	2	A
125	CR	6	S.CP.1, S.CP.2, S.CP.6, S.CP.7	1	—
126	CR	6	S.CP.5, S.CP.6, S.CP.8, S.MD.7	2	—
127	CR	6	S.CP.3, S.CP.4, S.CP.5, S.CP.6	2	—

**Constructed Response Solutions**

17. This question assesses the student’s ability to visualize three-dimensional objects generated by rotations of two-dimensional objects and the application of volume formulas for solids.



(a) The resulting shape is a frustum of a cone with a cylinder cut out of the middle. Essentially it is half of a cylindrical shell.

(b) The volume is half the difference between the volumes of the larger and smaller cylinder, so

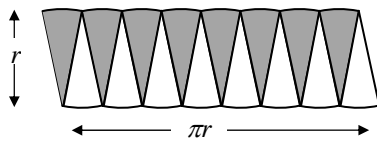
$$V = \frac{1}{2}(\pi(5)^2(2) - \pi(3)^2(2))$$

$$V = \frac{1}{2}(50\pi - 18\pi)$$

$$V = 16\pi$$

18. This question assesses the student’s understanding of the use of limiting principles to derive formulas.

As the circle is cut into increasingly larger numbers of sectors and rearranged as shown, the shape of the resulting figure becomes more like a parallelogram, where the base has a length close to half the circle’s circumference and the height is its radius. The area of the parallelogram is  $(r)(\pi r) = \pi r^2$ .



19. This question assesses the student’s ability to apply volume formulas in modeling situations.

(a) The aquarium has a rectangular cross section 50 cm by 30 cm. The marbles have a diameter of 1 cm, so an array of  $50 \times 30$  marbles will cover the bottom of the tank. To fill the marbles about 5 cm deep will take  $50 \times 30 \times 5 = 7500$  marbles. That is  $7500/50 = 15$  bags of marbles. (Note: if a student realizes that the marbles will probably not form a 3-D array  $50 \times 30 \times 5$ , but have “layers” of different sizes, say  $50 \times 30$  for the first,  $49 \times 29$  for the second,  $50 \times 30$  for the third, etc. this is acceptable. Precisely how the marbles are “packed” needs only be reasonable.)

(b) The total volume of water and marbles in the tank is  $50 \text{ cm} \times 30 \text{ cm} \times 27 \text{ cm} = 40,500 \text{ cm}^3$ . The volume of marbles in the tank is  $7500 \times \frac{4}{3}\pi \left(\frac{1}{2} \text{ cm}\right)^3 \approx 3927 \text{ cm}^3$ . So the volume of water is  $36,573 \text{ cm}^3$  or about 36.6 liters. (Treating the marbles as a solid  $50 \text{ cm} \times 30 \text{ cm} \times 5 \text{ cm}$  block of glass is incorrect; there is space between the marbles. If a student makes the case that 6 layers of marbles is closer to 5 cm tall than 5 layers, this is an acceptable solution.)

**Constructed Response Solutions**

20. This question assesses the student’s ability to apply volume formulas in modeling situations.

(a) The beaker is well approximated by a cylinder:

$$400 \text{ cm}^3 = \pi(4 \text{ cm})^2 h$$

$$h \approx 8 \text{ cm}$$

(b) The Erlenmeyer Flask well approximated by a cone:

$$400 \text{ cm}^3 = \frac{1}{3}\pi(4 \text{ cm})^2 h$$

$$h \approx 24 \text{ cm}$$

(c) The Florence Flask is well approximated by a sphere:

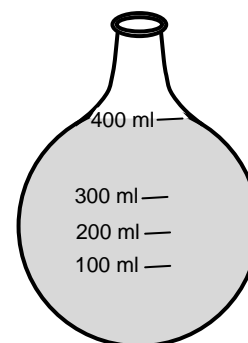
$$400 \text{ cm}^3 = \frac{4}{3}\pi r^3$$

$$r \approx 4.6 \text{ cm}$$

This is larger than the beaker’s radius, so it will NOT fit inside the beaker.

(d) As one moves upward from the bottom of the flask, the cross-sectional area decreases. Therefore, greater height (of the cone) is needed to hold an equivalent volume.

(e) The 200-ml mark must be roughly half way between the bottom of the flask and the 400-ml mark. The 100-ml and 300-ml must be closer to the 200-ml mark than the bottom and 400-ml mark, respectively.



Florence Flask

21. This question assesses the student’s ability to apply volume formulas and the concept of density in modeling situations.

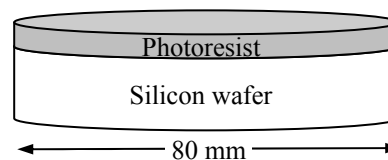
$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$1.2 \frac{\text{mg}}{\text{mm}^3} = \frac{0.06 \text{ mg}}{V}$$

(a)

$$V = \frac{0.06 \text{ mg}}{1.2 \frac{\text{mg}}{\text{mm}^3}}$$

$$V = 0.05 \text{ mm}^3$$



(b) The photoresist covers the circular wafer with a uniform thickness, essentially making it a cylinder with a height equal to the thickness:

$$V = \pi r^2 h$$

$$0.05 \text{ mm}^3 = \pi(40 \text{ mm})^2 h$$

$$h = \frac{0.05 \text{ mm}^3}{\pi(40 \text{ mm})^2}$$

$$h = 0.000009947 \dots \text{ mm}$$

$$h \approx 10^{-5} \text{ mm} \approx 10 \text{ nm}$$

**Constructed Response Solutions**

33. This question assesses the student’s ability to locate a point on a directed line segment, apply transformations, use coordinate proofs, and apply area formulas.

(a)  $P$  is  $\frac{3}{4}$  of the way from  $M$  to  $N$  or  $(-1, 2)$

(b) See diagram.

(c) The slope of  $\overline{MN} = \text{slope of } \overline{M'N'} = 1$ .

The slope of  $\overline{MM'} = \text{slope of } \overline{NN'} = 0$ .

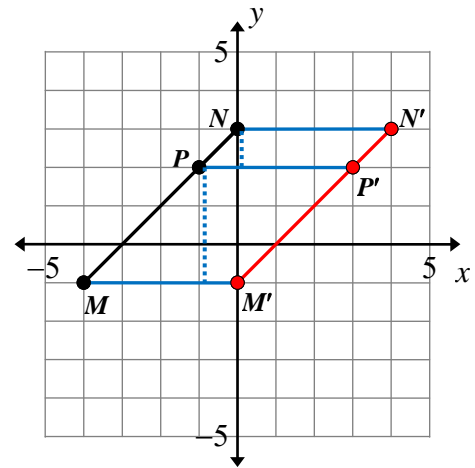
Since opposite sides are parallel (same slope),  $MM'N'N$  is a parallelogram.

(d) The area of a parallelogram is equal to the product of the base length and the height.

For parallelogram  $PP'N'N$ ,  $A = 4 \times 1 = 4$  square units.

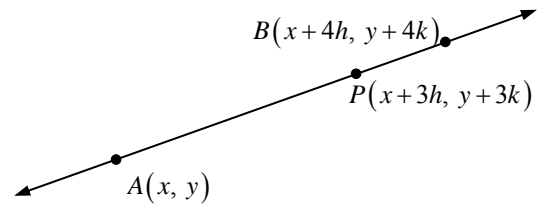
For parallelogram  $MM'P'P$ ,  $A = 4 \times 3 = 12$  square units  $= 3 \times 4$  square units.

Thus, the area of  $MM'P'P$  is 3 times the area of  $PP'N'N$ .



34. This question assesses the student’s understanding of directed line segments.

$$\begin{aligned} \frac{AP}{BP} &= \frac{(x+3h)-x}{(x+4h)-(x+3h)} \\ &= \frac{3h}{h} \\ &= 3 \end{aligned}$$



35. This question assesses the student’s to graph linear functions, construct parallel and/or perpendicular lines to given lines on the coordinate plane, and compute the perimeter of a figure using coordinate geometry.

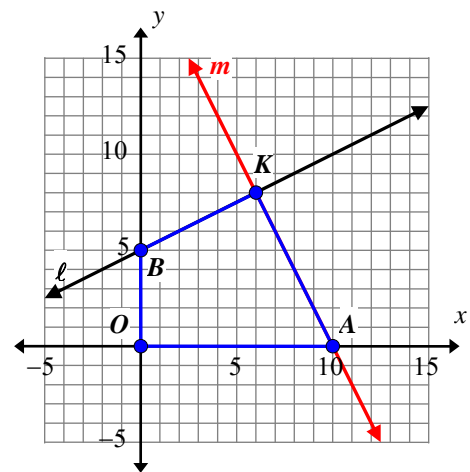
(a) See graph.

(b) The line  $m$  has equation  $y = -2x + 20$ . See graph.

(c) See graph.

(d)  $AO = 10$ .  $OB = 5$ .  $BK = \sqrt{45} = 3\sqrt{5}$ .  $KA = \sqrt{80} = 4\sqrt{5}$ .

Perimeter  $= 15 + 7\sqrt{5}$ .



**Constructed Response Solutions**

36. This question assesses the student's to graph linear functions, construct parallel and/or perpendicular lines to given lines on the coordinate plane, and compute the perimeter of a figure using coordinate geometry.

(a) See graph.

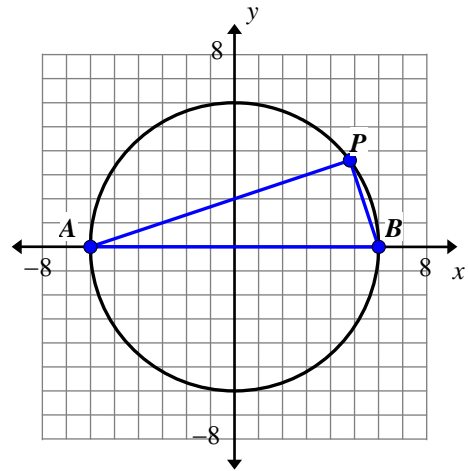
(b)  $A$  is  $(-6, 0)$  and  $B(6, 0)$ . See graph.

$$m_{\overline{AP}} = \frac{3.6 - 0}{4.8 - (-6)} = \frac{3.6}{10.8} = \frac{1}{3}$$

(c)  $m_{\overline{BP}} = \frac{3.6 - 0}{4.8 - 6} = \frac{3.6}{-1.2} = -3$

$$(m_{\overline{AP}})(m_{\overline{BP}}) = \left(\frac{1}{3}\right)(-3) = -1$$

Since the product of the slopes of  $\overline{AP}$  and  $\overline{BP}$  equal  $-1$ , the segments are perpendicular and, therefore,  $\angle APB$  is a right angle. Thus,  $\triangle ABP$  is a right triangle.



37. This question assesses the student's ability to apply the geometric definition of a parabola on the coordinate plane.

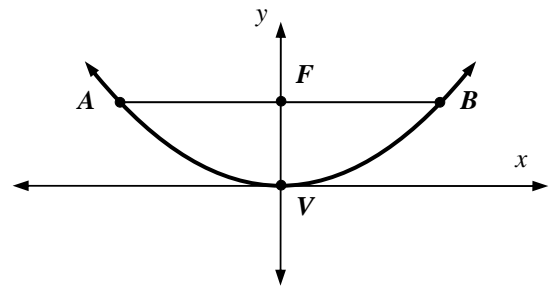
The focus is  $p$  units from the vertex, therefore, points  $A$  and  $B$  have a  $y$ -

$$p = \frac{1}{4p}x^2$$

coordinate equal to  $p$ . Thus,  $4p^2 = x^2$ . The distance from  $A(-2p, 0)$

$$\pm 2p = x$$

to  $B(2p, 0)$  is  $4p$ .



38. This question assesses the student's ability to apply the geometric definition of a parabola on the coordinate plane.

(a) See graph.

(b) See graph. The focus is at  $(0, 1)$

(c) See graph. Point  $P$  is at  $(3, 2.25)$ .

$$PF = \sqrt{(3-0)^2 + (2.25-1)^2}$$

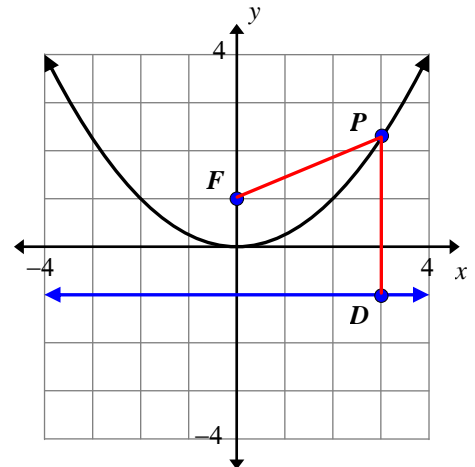
(d)  $= 3.25$

. The distances are the same (which is how

$$PD = 2.25 - (-1)$$

$$= 3.25$$

the parabola is defined.)



**Constructed Response Solutions**

39. This question assesses the student’s ability to use coordinate geometry to solve problems.

(a) See graph for example.

(b)  $AB = 2$ ,  $AC = 4$ , and  $BC = \sqrt{20}$ . Since all sides are different lengths, the triangle is scalene.

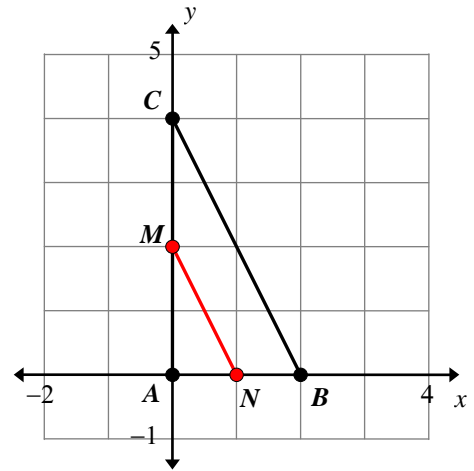
(c) See graph.

$$m_{\overline{MN}} = -2$$

(d)  $m_{\overline{BC}} = -2$ . Since the slopes are equal, the segments are parallel.

$$m_{\overline{MN}} = m_{\overline{BC}}$$

(e)  $MN = \sqrt{5} = \frac{1}{2}\sqrt{20} = \frac{1}{2}BC$ .



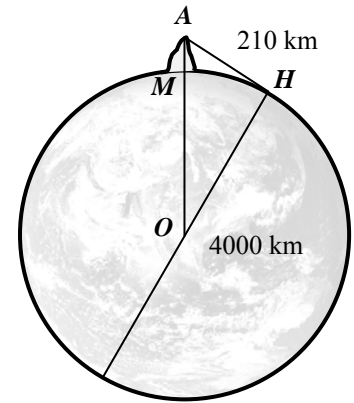
93. This question assesses the student’s ability apply relationships between lines and angles in circles.

Triangle  $AHO$  is a right triangle.

$$AO = \sqrt{210^2 + 2000^2}$$

$$AO \approx 2011 \text{ km}$$

The mountain’s height,  $AM = AO - 2000 \text{ km} \approx 11 \text{ km}$ .



94. This question assesses the student’s ability prove all circles are similar.

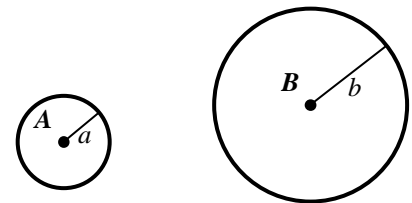
To prove circles  $A$  and  $B$  are similar, there must be a sequence of similarity transformations that map circle  $A$  to circle  $B$ .

Translate circle  $A$  by vector  $\overrightarrow{AB}$ . Under the translation,  $A' = B$ .

Dilate circle  $A'$  by a factor  $\frac{b}{a}$ . Since circle  $A$  was the set of all points distance  $a$  from

point  $A$ , circle  $A'$  is the set of all points distance  $a \cdot \frac{b}{a} = b$  from point  $A'$ . That is the same set of points as circle  $B$ . The

composition of the transformation and dilation, both similarity transformations, maps circle  $A$  to circle  $B$ . Thus, circles  $A$  and  $B$  are similar.

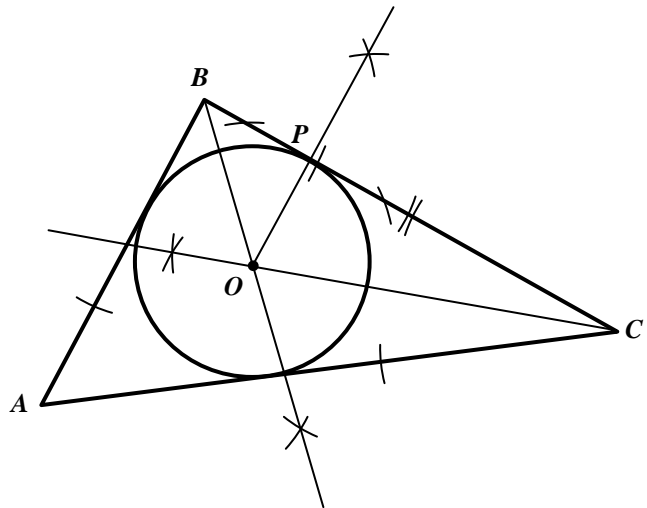




**Constructed Response Solutions**

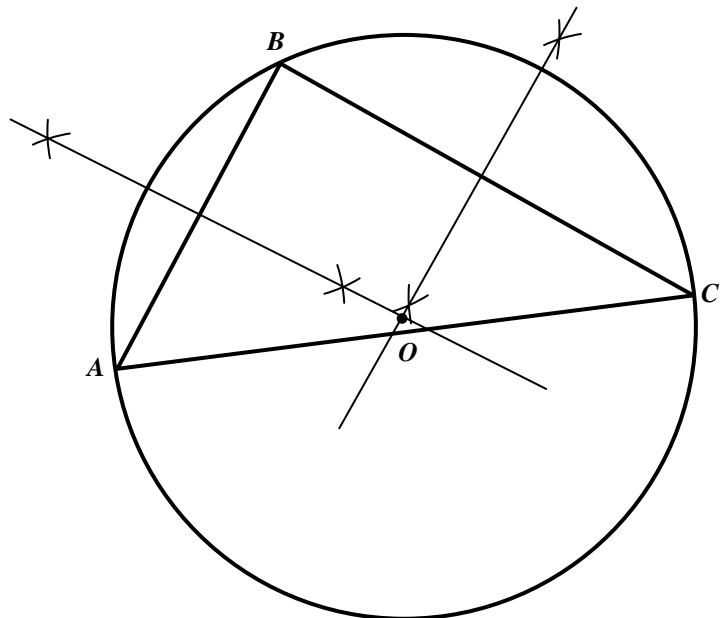
95. This question assesses the student's ability to construct a circle inside a triangle.

The student must (1) construct at least two angle bisectors, (2) locate the point of intersection of the bisectors (point  $O$ ), (3) construct a perpendicular to one side through  $O$ , (4) locate the point of intersection of the side and the perpendicular line (point  $P$ ), and (5) construct a circle centered at  $O$  whose radius is  $OP$ .



96. This question assesses the student's ability to construct a circle circumscribed about a triangle.

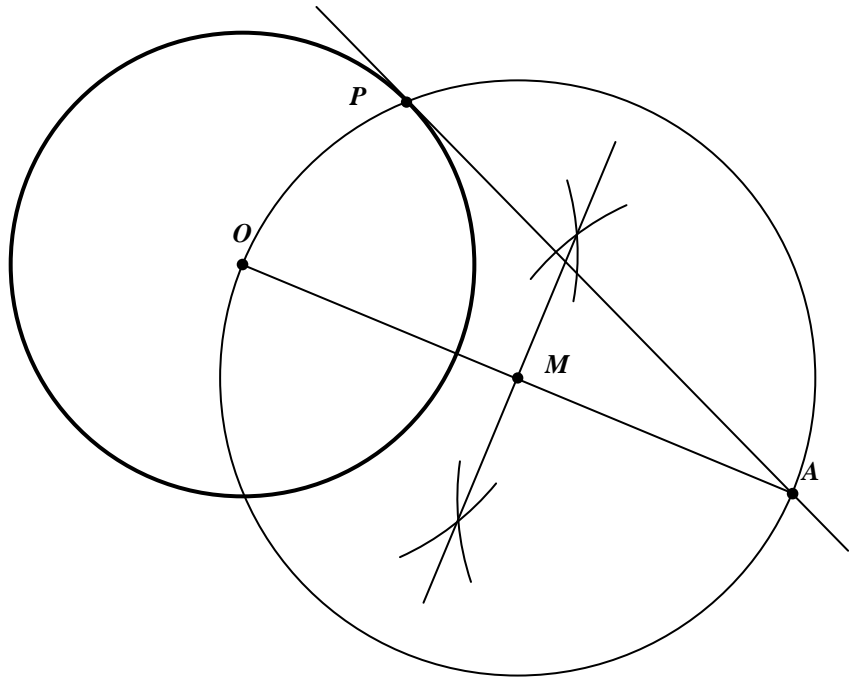
The student must (1) construct at least two perpendicular bisectors, (2) locate the point of intersection of the bisectors (point  $O$ ), and (3) construct a circle centered at  $O$  whose radius is  $OA$  (or  $OB$  or  $OC$ ).



Constructed Response Solutions

97. This question assesses the student’s ability to construct a tangent to a circle from a point.

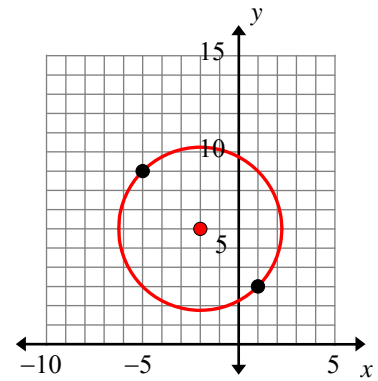
The student must (1) construct  $\overline{AO}$ , (2) construct the midpoint of  $\overline{AO}$  at  $M$ , (3) construct a circle of radius  $OM$  centered at  $M$ , (4) locate one intersection of circle  $M$  and circle  $O$  (point  $P$ ), and construct  $\overleftrightarrow{AP}$  which is tangent to circle  $O$ .



98. This question assesses the student’s ability to apply circles and their equations in the coordinate plane.

(a) The most obvious choice is to center the circle at the midpoint of  $(1, 3)$  and  $(-5, 9)$ , or  $(-2, 6)$ . The radius of the circle will be  $\sqrt{(1-(-2))^2 + (3-6)^2} = \sqrt{18}$ . The equation is then  $(x+2)^2 + (y-6)^2 = 18$ . Any circle that goes through the points  $(1, 3)$  and  $(-5, 9)$  is correct.

(b) Answers will vary depending on the equation, but obvious choices are  $y$ -intercepts  $(0, 6 + \sqrt{14})$  or  $(0, 6 - \sqrt{14})$ , or the lattice points  $90^\circ$  from the given points i.e.  $(1, 9)$  or  $(-5, 3)$ .



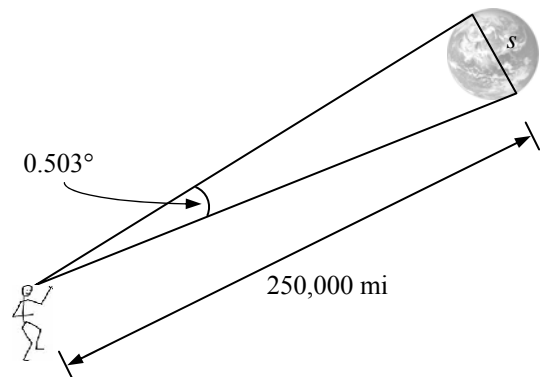
99. This questions assesses the student’s ability apply the measurement of arc lengths.

$$s = 2\pi r \frac{\theta}{360^\circ}$$

$$= 2\pi (250000 \text{ miles}) \frac{0.50^\circ}{360^\circ}$$

$$= 2181.661... \text{ miles}$$

The approximate diameter of the moon is 2200 miles.



**Constructed Response Solutions**

100. This questions assesses the student’s understanding of the relationship between the distance formula, Pythagorean Theorem, and the equation of a circle.

Triangle  $ABC$  is a right triangle, so by the Pythagorean Theorem

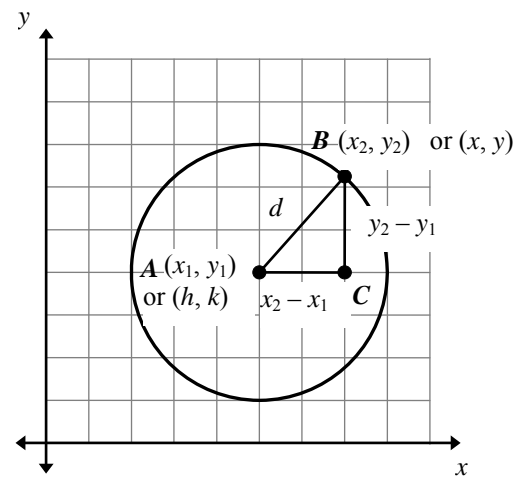
$$AB^2 = AC^2 + BC^2 .$$

The distance  $d$  from  $A$  to  $B$  can be rewritten at  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ .

The circle centered at  $A$  is the set of all points a distance  $d$  from point  $A$ . Using  $B$  as an arbitrary point, the equation  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$  holds. If

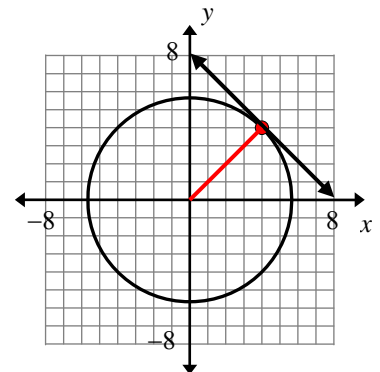
points  $A$  and  $B$  are described as having coordinates  $(h, k)$  and  $(x, y)$ , respectively, and the circle is described as having a radius of  $r$ , then

$$r^2 = (x - h)^2 + (y - k)^2 .$$



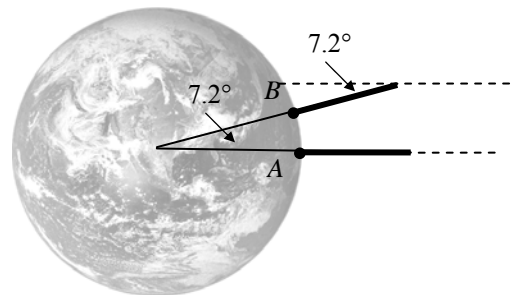
101. This question assesses the student’s ability to graph a circle, apply properties of parts of circles, and use coordinate geometry to find equations of lines.

The radius shown has a slope of 1. Thus, the slope of the tangent line is  $-1$ . The equation of the tangent line is  $y - 4 = -1(x - 4)$  or  $y = -x + 8$ .



102. This questions assesses the student’s ability apply the measurement of arc lengths.

The arc length from  $A$  to  $B$  is given as 500 miles. It is  $\frac{7.2}{360}$  of the circumference of the earth. So, the approximate circumference of the earth is  $500 \text{ miles} \times \frac{360^\circ}{7.2^\circ} = 25000 \text{ miles}$ .



**Constructed Response Solutions**

103. This questions assesses the student’s ability to apply the area of sectors of circles.

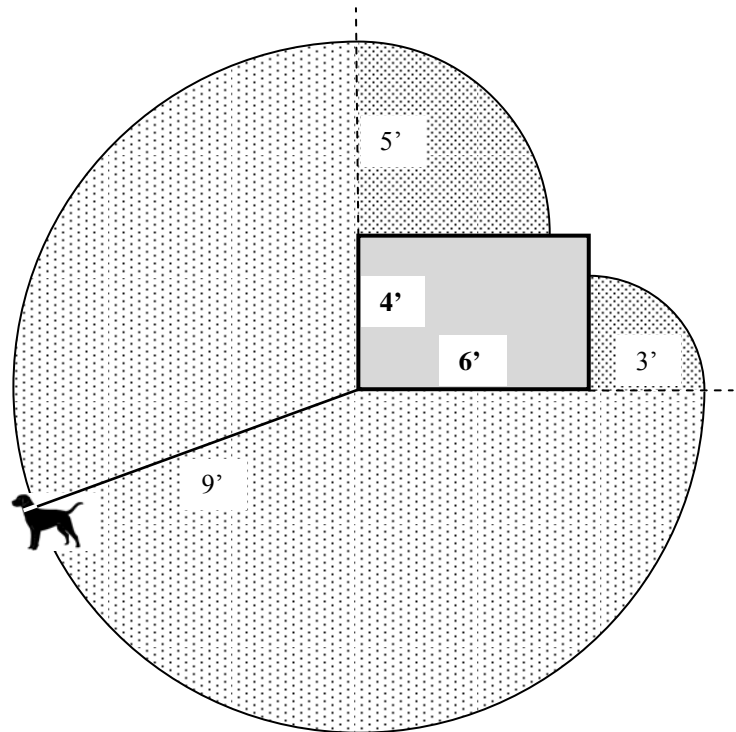
See diagram.

The dog’s area is three parts:  $\frac{3}{4}$  of a circle of radius 9 feet,  $\frac{1}{4}$  of a circle of radius 5 feet, and  $\frac{1}{4}$  of a circle of radius 3 feet.

$$A = \frac{3}{4}\pi(9 \text{ ft})^2 + \frac{1}{4}\pi(5 \text{ ft})^2 + \frac{1}{4}\pi(3 \text{ ft})^2$$

$$A = 69.25\pi \text{ ft}^2$$

$$A \approx 218 \text{ ft}^2$$



**Constructed Response Solutions**

104. This questions assesses the student’s ability to apply the area of triangles and quadrilaterals.

(a) Esmeralda county can be divided into a triangle (south of dotted line) and a quadrilateral.

The quadrilateral north of the dotted line has an area equal to the large triangle north of the dotted line minus the small shaded triangle added to the figure.

South triangle: base is about 70 miles and height is about 60 miles. Area equals

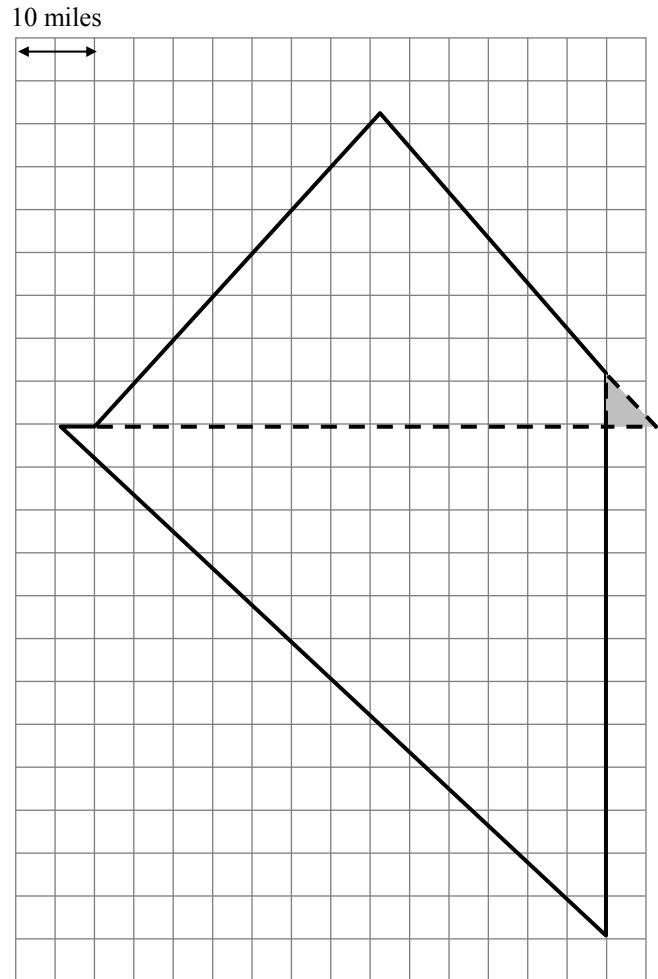
$$\frac{1}{2}(60 \text{ mi})(70 \text{ mi}) = 2100 \text{ mi}^2.$$

North quadrilateral is the difference between the large triangle with base 70 miles and height about 36 miles, and the shaded triangle with base and height of 6 miles. Area equals

$$\frac{1}{2}(36 \text{ mi})(70 \text{ mi}) - \frac{1}{2}(6 \text{ mi})(6 \text{ mi}) = 1242 \text{ mi}^2.$$

The area of Esmeralda County is approximately 3342 square miles.

(b) The population density of Esmeralda County is  $\frac{775 \text{ people}}{3342 \text{ mi}^2} \approx 0.23$  people per square mile.



**Constructed Response Solutions**

125. This questions assesses the student’s understanding of probability rules.

- (a)  $P(A \text{ and } B) = P(A) \cdot P(B) = (0.3)(0.4) = 0.12$
- (b)  $P(A \text{ or } B) = P(A) + (B) - P(A \text{ and } B) = 0.3 + 0.4 - 0.12 = 0.58$
- (c)  $P(\text{not } A) = 1 - P(A) = 1 - 0.3 = 0.7$
- (d) Since  $A$  and  $B$  are independent,  $P(A | B) = P(A) = 0.3$

126. This questions assesses the student’s ability to apply rules for probability to compound events and real-world situations.

This question can be solved using multiple methods, including formulas, a tree diagram, or a table (as shown). Values based on given information is **bold**.

- (a)  $P(\text{Classified as spam?}) = 16.25\%$
- (b)  $P(\text{Spam} | \text{Classified spam}) = \frac{12\%}{16.25\%} \approx 74\%$
- (c)  $P(\text{Legitimate and classified Spam}) + P(\text{Spam and classified Legitimate}) = 4.25\% + 3\% = 7.25\%$
- (d)  $P(\text{Classified Spam} | \text{Legitimate}) = 5\%$  (given)

$P(\text{Not Classified Spam} | \text{Spam}) = \frac{3\%}{15\%} = 20\%$ . It is 4 times more likely that a spam message will get through the filter than a legitimate message will be classified as spam.

		Message Classified as		
		Spam	Legitimate	Total
Incoming Message Type	Legitimate	5% of 85% = <b>4.25%</b>	<b>80.75%</b>	<b>85%</b>
	Spam	80% of 15% = <b>12%</b>	<b>3%</b>	<b>15%</b>
	Total	16.25%	83.75%	<b>100%</b>

127. This questions assesses the student’s ability to compile data into a two way table and apply rules of probability, and understanding of the concept of independence.

- (a) See table.
- (b, i)  $P(\text{siblings yes}) = \frac{21}{35} = \frac{3}{5}$
- (b, ii)  $P(\text{siblings yes or pets yes}) = \frac{21}{35} + \frac{21}{35} - \frac{15}{35} = \frac{27}{35}$
- (b, iii)  $P(\text{siblings yes} | \text{pets yes}) = \frac{15}{21} = \frac{5}{7}$

(e) No, because  $P(\text{siblings yes}) \neq P(\text{siblings yes} | \text{pets yes})$

$$\frac{3}{5} \neq \frac{5}{7}$$

		Have siblings?		
		Yes	No	Total
Own pets?	Yes	15	6	21
	No	6	8	14
	Total	21	14	35

### **Notes on Practice Materials**

The Geometry Honors Practice Materials are provided to help teachers and students prepare for the CCSD Semester Exams in Geometry Honors. School choosing to teach regular Geometry using the CCSS should also use these materials.

The questions are representative of the style, format, and type that will be on the exams. They are not, however, completely parallel in construction. That is, practice questions on a particular standard show how that standard may be assessed, but the questions on the actual exam could assess that standard in a different way. Teachers must provide students with opportunities to explore all aspects of a standard, and not simply focus on those addressed by the practice materials.

There are 3 types of questions in the practice materials that will appear on the semester exams.

- **MC – Multiple Choice.** This is a traditional selected-response type of question. Each item will have 3 or 4 possible responses. Some multiple choice questions may have a common lead-in statement and should remain grouped together when provided for practice.
- **MTF – Multiple True/False.** These items will have 2–4 true/false questions based on a common lead-in statement or concept. These should remain grouped together when provided for practice.
- **CR – Constructed response.** These items may have multiple parts that address one or more standards. The DOK level is the overall level of the item, though some parts may be at lower levels. Short CR items average about 6–8 minutes to complete; longer CR items average 10–15 minutes to complete.

A fourth type of question is the Extended Response (ER). These will not appear on the semester exam at this time, but are indicative of longer, performance-type tasks that will appear on the Smarter Balanced Assessments beginning in 2013–2014.

Sample solutions are provided for CR and ER questions. Student methods may vary and any logical, mathematically correct approach, including different types of proof, should be accepted as correct.

It is expected that students will have access to mathematical tools for these practice questions and the semester exam. Tools include compass, protractor, ruler, patty paper, and graph paper. Students may use calculators, including graphing calculators, constructed response questions.