GLENCOE MATHEMATICS

Geometry

Chapter 8
Resource Masters

Glencoe

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 8 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Geometry
Chapter 8 Resource Masters

1 2 3 4 5 6 7 8 9 10 009 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 8 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 8 Resource Masters* includes the core materials needed for Chapter 8. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 8-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 8-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 8 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 458–459. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

8 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 8. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
diagonals		
isosceles trapezoid		
kite		
median		

(continued on the next page)

NAME ______________________________ DATE __________ PERIOD _____

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
parallelogram		
rectangle		
rhombus		
square		
trapezoid		

NAME ____ DATE ____________ PERIOD _____

8 Learning to Read Mathematics

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 8. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 8.1 *Interior Angle Sum Theorem*		
Theorem 8.2 *Exterior Angle Sum Theorem*		
Theorem 8.3		
Theorem 8.4		
Theorem 8.5		
Theorem 8.7		
Theorem 8.8		

(continued on the next page)

NAME ______________________________ DATE __________ PERIOD _____

8

Learning to Read Mathematics

Proof Builder *(continued)*

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 8.12		
Theorem 8.13		
Theorem 8.15		
Theorem 8.17		
Theorem 8.18		
Theorem 8.19		
Theorem 8.20		

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8-1 Study Guide and Intervention

Angles of Polygons

Sum of Measures of Interior Angles

The segments that connect the nonconsecutive sides of a polygon are called **diagonals**. Drawing all of the diagonals from one vertex of an ***n*-gon** separates the polygon into $n - 2$ triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those $n - 2$ triangles.

Interior Angle Sum Theorem	If a convex polygon has n sides, and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

Example 1 **A convex polygon has 13 sides. Find the sum of the measures of the interior angles.**

$S = 180(n - 2)$
$= 180(13 - 2)$
$= 180(11)$
$= 1980$

Example 2 **The measure of an interior angle of a regular polygon is 120. Find the number of sides.**

The number of sides is n, so the sum of the measures of the interior angles is $120n$.

$S = 180(n - 2)$
$120n = 180(n - 2)$
$120n = 180n - 360$
$-60n = -360$
$n = 6$

Exercises

Find the sum of the measures of the interior angles of each convex polygon.

1. 10-gon **2.** 16-gon **3.** 30-gon

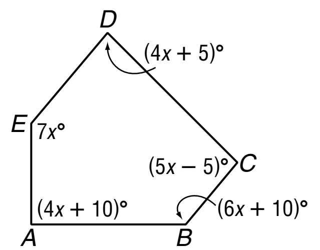
4. 8-gon **5.** 12-gon **6.** $3x$-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

7. 150 **8.** 160 **9.** 175

10. 165 **11.** 168.75 **12.** 135

13. Find x.



Lesson 8-1

8-1 Study Guide and Intervention *(continued)*

Angles of Polygons

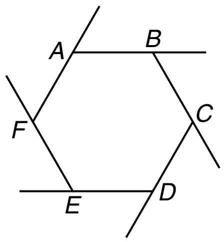
Sum of Measures of Exterior Angles There is a simple relationship among the exterior angles of a convex polygon.

Exterior Angle Sum Theorem	If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

Example 1 **Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.**

For *any* convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example 2 **Find the measure of each exterior angle of regular hexagon *ABCDEF*.**



The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If n is the measure of each exterior angle, then

$6n = 360$

$n = 60$

Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

1. 10-gon **2.** 16-gon **3.** 36-gon

Find the measure of an exterior angle for each convex regular polygon.

4. 12-gon **5.** 36-gon **6.** $2x$-gon

Find the measure of an exterior angle given the number of sides of a regular polygon.

7. 40 **8.** 18 **9.** 12

10. 24 **11.** 180 **12.** 8

8-1 Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. nonagon

2. heptagon

3. decagon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

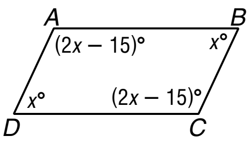
4. 108

5. 120

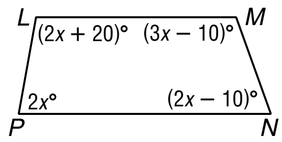
6. 150

Find the measure of each interior angle using the given information.

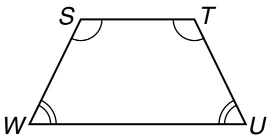
7.



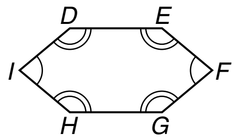
8.



9. quadrilateral $STUW$ with $\angle S \cong \angle T$, $\angle U \cong \angle W$, $m\angle S = 2x + 16$, $m\angle U = x + 14$



10. hexagon $DEFGHI$ with $\angle D \cong \angle E \cong \angle G \cong \angle H$, $\angle F \cong \angle I$, $m\angle D = 7x$, $m\angle F = 4x$



Find the measures of an interior angle and an exterior angle for each regular polygon.

11. quadrilateral

12. pentagon

13. dodecagon

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

14. 8

15. 9

16. 13

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8-1 Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

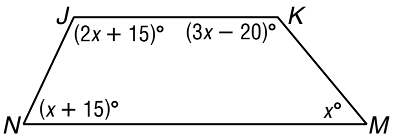
1. 11-gon **2.** 14-gon **3.** 17-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

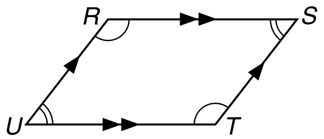
4. 144 **5.** 156 **6.** 160

Find the measure of each interior angle using the given information.

7.



8. quadrilateral $RSTU$ with $m\angle R = 6x - 4$, $m\angle S = 2x + 8$



Find the measures of an interior angle and an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

9. 16-gon **10.** 24-gon **11.** 30-gon

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

12. 14 **13.** 22 **14.** 40

15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.

8-1 Reading to Learn Mathematics

Angles of Polygons

Pre-Activity **How does a scallop shell illustrate the angles of polygons?**

Read the introduction to Lesson 8-1 at the top of page 404 in your textbook.

- How many diagonals of the scallop shell shown in your textbook can be drawn from vertex *A*?
- How many diagonals can be drawn from one vertex of an *n*-gon? Explain your reasoning.

Reading the Lesson

1. Write an expression that describes each of the following quantities for a regular *n*-gon. If the expression applies to regular polygons only, write *regular*. If it applies to all convex polygons, write *all*.

a. the sum of the measures of the interior angles

b. the measure of each interior angle

c. the sum of the measures of the exterior angles (one at each vertex)

d. the measure of each exterior angle

2. Give the measure of an interior angle and the measure of an exterior angle of each polygon.

a. equilateral triangle

b. regular hexagon

c. square

d. regular octagon

3. Underline the correct word or phrase to form a true statement about regular polygons.

a. As the number of sides increases, the sum of the measures of the interior angles (increases/decreases/stays the same).

b. As the number of sides increases, the measure of each interior angle (increases/decreases/stays the same).

c. As the number of sides increases, the sum of the measures of the exterior angles (increases/decreases/stays the same).

d. As the number of sides increases, the measure of each exterior angle (increases/decreases/stays the same).

e. If a regular polygon has more than four sides, each interior angle will be a(n) (acute/right/obtuse) angle, and each exterior angle will be a(n) (acute/right/obtuse) angle.

Helping You Remember

4. A good way to remember a new mathematical idea or formula is to relate it to something you already know. How can you use your knowledge of the Angle Sum Theorem (for a triangle) to help you remember the Interior Angle Sum Theorem?

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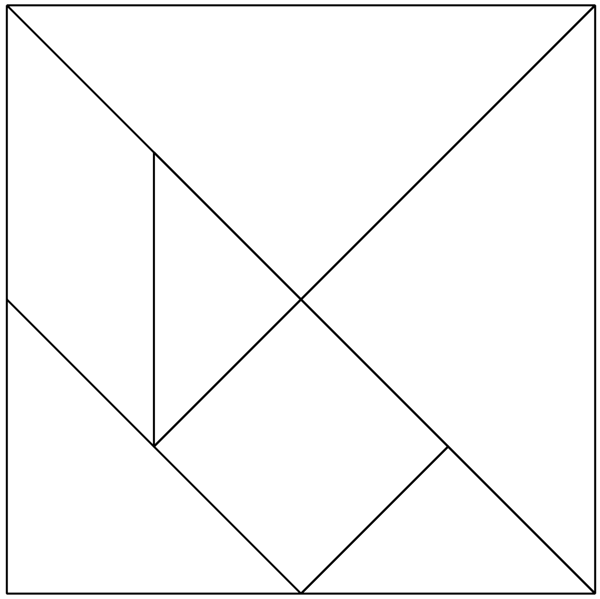
8-1 Enrichment

Tangrams

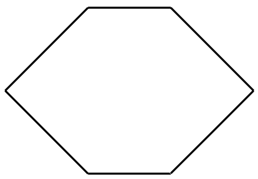
The tangram puzzle is composed of seven pieces that form a square, as shown at the right. This puzzle has been a popular amusement for Chinese students for hundreds and perhaps thousands of years.



Make a careful tracing of the figure above. Cut out the pieces and rearrange them to form each figure below. Record each answer by drawing lines within each figure.

1.

2.



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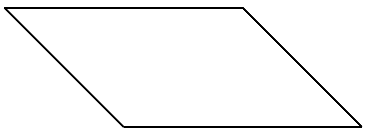

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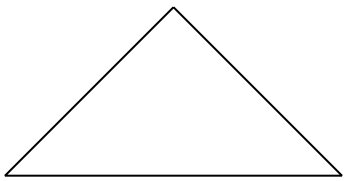
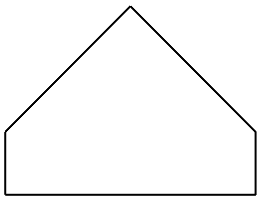
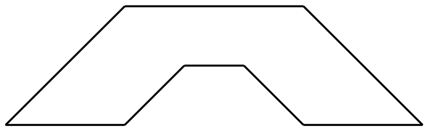


5.



6.

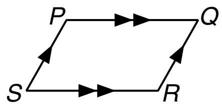


7. Create a different figure using the seven tangram pieces. Trace the outline. Then challenge another student to solve the puzzle.

8-2 Study Guide and Intervention

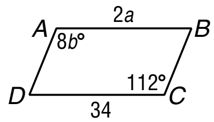
Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If *PQRS* is a parallelogram, then
The opposite sides of a parallelogram are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
The opposite angles of a parallelogram are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
The consecutive angles of a parallelogram are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

Example **If *ABCD* is a parallelogram, find *a* and *b*.**



$\overline{AB}$ and $\overline{CD}$ are opposite sides, so $\overline{AB} \cong \overline{CD}$.

$2a = 34$
$a = 17$

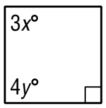
$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

$8b = 112$
$b = 14$

Exercises

Find *x* and *y* in each parallelogram.

1.



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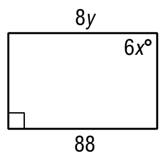
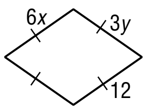
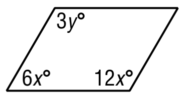


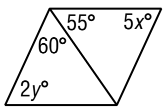
3.



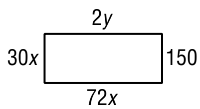
4.



5.



6.

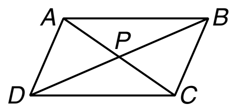


8-2 Study Guide and Intervention *(continued)*

Parallelograms

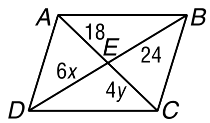
Diagonals of Parallelograms

Two important properties of parallelograms deal with their diagonals.



	If *ABCD* is a parallelogram, then:
The diagonals of a parallelogram bisect each other.	$AP = PC$ and $DP = PB$
Each diagonal separates a parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Example **Find *x* and *y* in parallelogram *ABCD*.**



The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

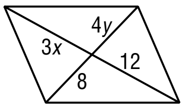
$6x = 24$ $\quad$ $4y = 18$

$x = 4$ $\quad$ $y = 4.5$

Exercises

Find *x* and *y* in each parallelogram.

1.



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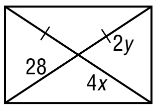

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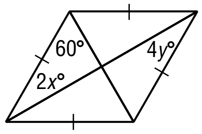

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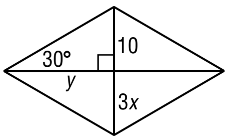

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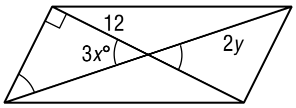
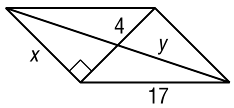
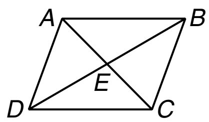


6.



Complete each statement about ▱*ABCD*. Justify your answer.



7. $\angle BAC \cong$

8. $\overline{DE} \cong$

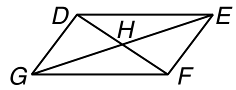
9. $\triangle ADC \cong$

10. $\overline{AD} \parallel$

8-2 Skills Practice

Parallelograms

Complete each statement about ▱*DEFG*. Justify your answer.



1. $\overline{DG} \parallel$ ___?___

2. $\overline{DE} \cong$ ___?___

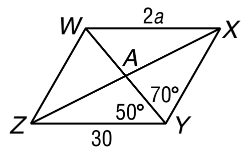
3. $\overline{GH} \cong$ ___?___

4. $\angle DEF \cong$ ___?___

5. $\angle EFG$ is supplementary to ___?___.

6. $\triangle DGE \cong$ ___?___

ALGEBRA Use ▱*WXYZ* to find each measure or value.



7. $m\angle XYZ =$ ______

8. $m\angle WZY =$ ______

9. $m\angle WXY =$ ______

10. $a =$ ______

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of parallelogram *HJKL* given each set of vertices.

11. $H(1, 1)$, $J(2, 3)$, $K(6, 3)$, $L(5, 1)$

12. $H(-1, 4)$, $J(3, 3)$, $K(3, -2)$, $L(-1, -1)$

13. PROOF Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary*.

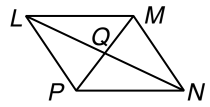
Lesson 8-2

NAME ______________________ DATE __________ PERIOD _____

8-2 Practice

Parallelograms

Complete each statement about ▱*LMNP*. Justify your answer.



1. $\overline{LQ} \cong$ __?__

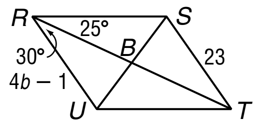
2. $\angle LMN \cong$ __?__

3. $\triangle LMP \cong$ __?__

4. $\angle NPL$ is supplementary to __?__.

5. $\overline{LM} \cong$ __?__

ALGEBRA Use ▱*RSTU* to find each measure or value.



6. $m\angle RST =$ ______

7. $m\angle STU =$ ______

8. $m\angle TUR =$ ______

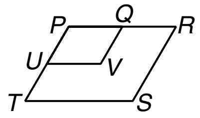
9. $b =$ ______

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of parallelogram *PRYZ* given each set of vertices.

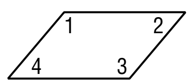
10. $P(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $Z(-3, -1)$

11. $P(2, 3)$, $R(1, -2)$, $Y(-5, -7)$, $Z(-4, -2)$

12. PROOF Write a paragraph proof of the following.
Given: ▱*PRST* and ▱*PQVU*
Prove: $\angle V \cong \angle S$



13. CONSTRUCTION Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



NAME ______________________________ DATE ____________ PERIOD _____

8-2 Reading to Learn Mathematics

Parallelograms

Pre-Activity **How are parallelograms used to represent data?**

Read the introduction to Lesson 8-2 at the top of page 411 in your textbook.

- What is the name of the shape of the top surface of each wedge of cheese?
- Are the three polygons shown in the drawing similar polygons? Explain your reasoning.

Reading the Lesson

1. Underline words or phrases that can complete the following sentences to make statements that are always true. (There may be more than one correct choice for some of the sentences.)

a. Opposite sides of a parallelogram are (congruent/perpendicular/parallel).

b. Consecutive angles of a parallelogram are (complementary/supplementary/congruent).

c. A diagonal of a parallelogram divides the parallelogram into two (acute/right/obtuse/congruent) triangles.

d. Opposite angles of a parallelogram are (complementary/supplementary/congruent).

e. The diagonals of a parallelogram (bisect each other/are perpendicular/are congruent).

f. If a parallelogram has one right angle, then all of its other angles are (acute/right/obtuse) angles.

2. Let $ABCD$ be a parallelogram with $AB \neq BC$ and with no right angles.

a. Sketch a parallelogram that matches the description above and draw diagonal $\overline{BD}$.

In parts **b–f**, complete each sentence.

b. $\overline{AB} \parallel$ ________ and $\overline{AD} \parallel$ ________.

c. $\overline{AB} \cong$ ________ and $\overline{BC} \cong$ ________.

d. $\angle A \cong$ ________ and $\angle ABC \cong$ ________.

e. $\angle ADB \cong$ ________ because these two angles are ________________ ________________ angles formed by the two parallel lines ________ and ________ and the transversal ________.

f. $\triangle ABD \cong$ ________.

Helping You Remember

3. A good way to remember new theorems in geometry is to relate them to theorems you learned earlier. Name a theorem about parallel lines that can be used to remember the theorem that says, "If a parallelogram has one right angle, it has four right angles."

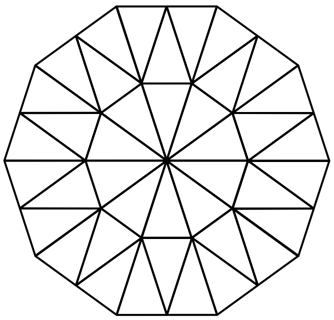
Lesson 8-2

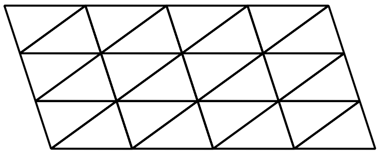
NAME ____ DATE ____________ PERIOD _____

8-2 Enrichment

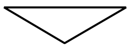
Tessellations

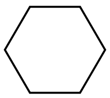
A **tessellation** is a tiling pattern made of polygons. The pattern can be extended so that the polygonal tiles cover the plane completely with no gaps. A checkerboard and a honeycomb pattern are examples of tessellations. Sometimes the same polygon can make more than one tessellation pattern. Both patterns below can be formed from an isosceles triangle.





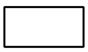
Draw a tessellation using each polygon.

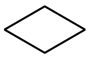
1. 

2. 

3. 

Draw two different tessellations using each polygon.

4. 

5. 

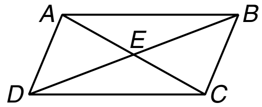
NAME ____ DATE ____________ PERIOD _____

8-3 Study Guide and Intervention

Tests for Parallelograms

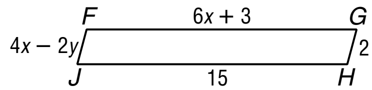
Conditions for a Parallelogram

There are many ways to establish that a quadrilateral is a parallelogram.



If:	**If:**
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	**then:** *ABCD* is a parallelogram.

Example **Find x and y so that *FGHJ* is a parallelogram.**



FGHJ is a parallelogram if the lengths of the opposite sides are equal.

$6x + 3 = 15$ $\quad$ $4x - 2y = 2$

$6x = 12$ $\quad$ $4(2) - 2y = 2$

$x = 2$ $\quad$ $8 - 2y = 2$

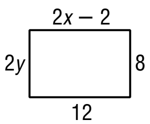
$-2y = -6$

$y = 3$

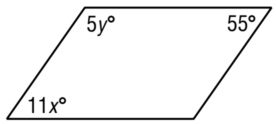
Exercises

Find x and y so that each quadrilateral is a parallelogram.

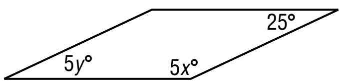
1.



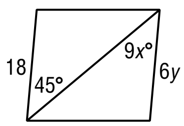
2.



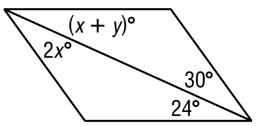
3.



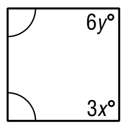
4.



5.



6.



Lesson 8-3

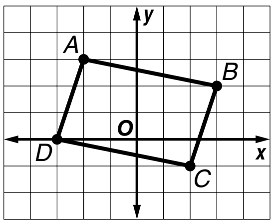
8-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance Formula and the Slope Formula can be used to test if a quadrilateral is a parallelogram.

Example **Determine whether *ABCD* is a parallelogram.**

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of $\overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3$ slope of $\overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$

slope of $\overline{AB} = \frac{2 - 3}{3 - (-2)} = -\frac{1}{5}$ slope of $\overline{CD} = \frac{-1 - 0}{2 - (-3)} = -\frac{1}{5}$

Opposite sides have the same slope, so $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Both pairs of opposite sides are parallel, so *ABCD* is a parallelogram.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

Both pairs of opposite sides have the same length, so *ABCD* is a parallelogram.

Exercises

Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$; Slope Formula

2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$; Slope Formula

3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$; Distance Formula

4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$; Distance and Slope Formulas

5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$; Distance and Slope Formulas

6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$; Midpoint Formula

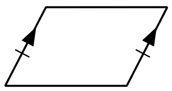
7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.

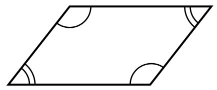
NAME ____ DATE ____________ PERIOD _____

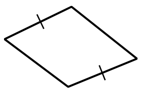
8-3 Skills Practice

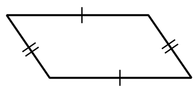
Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

2. 

3. 

4. 

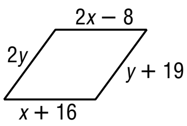
COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

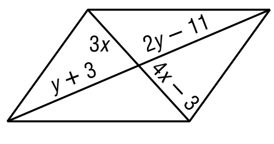
5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $Y(4, 0)$; Slope Formula

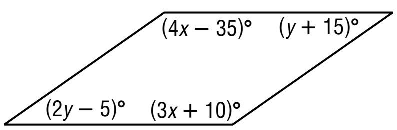
6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formula

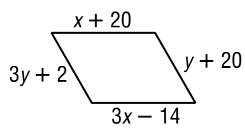
7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $N(-3, 1)$; Midpoint Formula

ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.

8. 

9. 

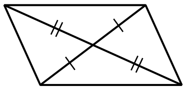
10. 

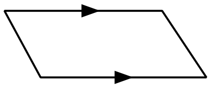
11. 

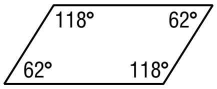
8-3 Practice

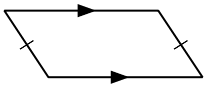
Tests for Parallelograms

Determine whether each quadrilateral is a parallelogram. Justify your answer.

1.


2.


3.


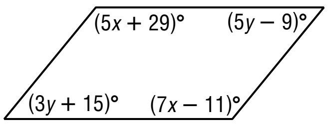
4.


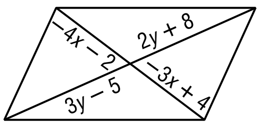
COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

5. $P(-5, 1)$, $S(-2, 2)$, $F(-1, -3)$, $T(2, -2)$; Slope Formula

6. $R(-2, 5)$, $O(1, 3)$, $M(-3, -4)$, $Y(-6, -2)$; Distance and Slope Formula

ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.

7.


8.


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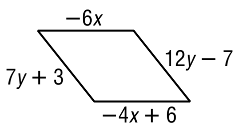
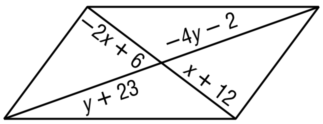
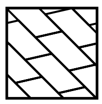


10.


11. TILE DESIGN The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?



NAME ____ DATE ____________ PERIOD _____

8-3 Reading to Learn Mathematics

Tests for Parallelograms

Pre-Activity **How are parallelograms used in architecture?**

Read the introduction to Lesson 8-3 at the top of page 417 in your textbook.

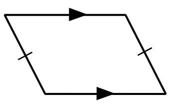
Make two observations about the angles in the roof of the covered bridge.

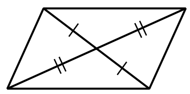
Reading the Lesson

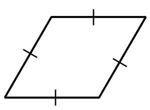
1. Which of the following conditions guarantee that a quadrilateral is a parallelogram?

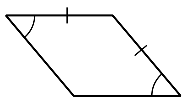
A. Two sides are parallel.
B. Both pairs of opposite sides are congruent.
C. The diagonals are perpendicular.
D. A pair of opposite sides is both parallel and congruent.
E. There are two right angles.
F. The sum of the measures of the interior angles is 360.
G. All four sides are congruent.
H. Both pairs of opposite angles are congruent.
I. Two angles are acute and the other two angles are obtuse.
J. The diagonals bisect each other.
K. The diagonals are congruent.
L. All four angles are right angles.

2. Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

a.


b.


c.


d.


Helping You Remember

3. A good way to remember a large number of mathematical ideas is to think of them in groups. How can you state the conditions as one group about the *sides* of quadrilaterals that guarantee that the quadrilateral is a parallelogram?

8-3 Enrichment

Tests for Parallelograms

By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram?

2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram?

3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram?

4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram?

5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram?

6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram?

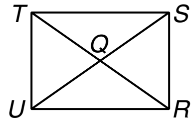
7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram?

NAME ____ DATE ____________ PERIOD _____

8-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1 **In rectangle *RSTU* above, $US = 6x + 3$ and $RT = 7x - 2$. Find x.**

The diagonals of a rectangle bisect each other, so $US = RT$.

$$6x + 3 = 7x - 2$$
$$3 = x - 2$$
$$5 = x$$

Example 2 **In rectangle *RSTU* above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.**

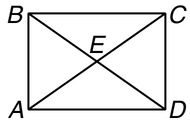
$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$$8x + 3 + 16x - 9 = 90$$
$$24x - 6 = 90$$
$$24x = 96$$
$$x = 4$$

$m\angle STR = 8x + 3 = 8(4) + 3$ or 35

Exercises

***ABCD* is a rectangle.**



1. If $AE = 36$ and $CE = 2x - 4$, find x.

2. If $BE = 6y + 2$ and $CE = 4y + 6$, find y.

3. If $BC = 24$ and $AD = 5y - 1$, find y.

4. If $m\angle BEA = 62$, find $m\angle BAC$.

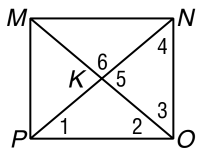
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$.

6. If $BD = 8y - 4$ and $AC = 7y + 3$, find BD.

7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$.

8. If $AB = 6y$ and $BC = 8y$, find BD in terms of y.

9. In rectangle *MNOP*, $m\angle 1 = 40$. Find the measure of each numbered angle.



Lesson 8-4

8-4 Study Guide and Intervention *(continued)*

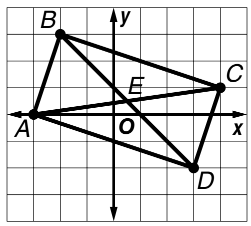
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example **Determine whether $A(-3, 0)$, $B(-2, 3)$, $C(4, 1)$, and $D(3, -2)$ are the vertices of a rectangle.**



Method 1: Use the Slope Formula.

slope of $\overline{AB} = \frac{3-0}{-2-(-3)} = \frac{3}{1}$ or 3 slope of $\overline{AD} = \frac{-2-0}{3-(-3)} = \frac{-2}{6}$ or $-\frac{1}{3}$

slope of $\overline{CD} = \frac{-2-1}{3-4} = \frac{-3}{-1}$ or 3 slope of $\overline{BC} = \frac{1-3}{4-(-2)} = \frac{-2}{6}$ or $-\frac{1}{3}$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so *ABCD* is a rectangle.

Method 2: Use the Midpoint and Distance Formulas.

The midpoint of $\overline{AC}$ is $\left(\frac{-3+4}{2}, \frac{0+1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and the midpoint of $\overline{BD}$ is $\left(\frac{-2+3}{2}, \frac{3-2}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$. The diagonals have the same midpoint so they bisect each other. Thus, *ABCD* is a parallelogram.

$AC = \sqrt{(-3-4)^2 + (0-1)^2} = \sqrt{49+1}$ or $\sqrt{50}$

$BD = \sqrt{(-2-3)^2 + (3-(-2))^2} = \sqrt{25+25}$ or $\sqrt{50}$

The diagonals are congruent. *ABCD* is a parallelogram with diagonals that bisect each other, so it is a rectangle.

Exercises

Determine whether *ABCD* is a rectangle given each set of vertices. Justify your answer.

1. $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$

2. $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$

3. $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$

4. $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$

5. $A(-1, -5)$, $B(-3, 0)$, $C(2, 2)$, $D(4, -3)$

6. $A(-1, -1)$, $B(0, 2)$, $C(4, 3)$, $D(3, 0)$

7. A parallelogram has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(5, -2)$. Find the coordinates of U so that *RSTU* is a rectangle.

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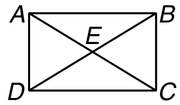
8-4 Skills Practice

Rectangles

ALGEBRA ***ABCD*** **is a rectangle.**



1. If $AC = 2x + 13$ and $DB = 4x - 1$, find x.

2. If $AC = x + 3$ and $DB = 3x - 19$, find AC.

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find AC.

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find DB.

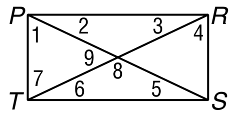
5. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find x.

6. If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$.

7. If $m\angle ABD = x^2 - 7$ and $m\angle CDB = 4x + 5$, find x.

8. If $m\angle BAC = x^2 + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$.

PRST **is a rectangle. Find each measure if** $m\angle 1 = 50$**.**



9. $m\angle 2$

10. $m\angle 3$

11. $m\angle 4$

12. $m\angle 5$

13. $m\angle 6$

14. $m\angle 7$

15. $m\angle 8$

16. $m\angle 9$

COORDINATE GEOMETRY Determine whether ***TUXY*** **is a rectangle given each set of vertices. Justify your answer.**

17. $T(-3, -2)$, $U(-4, 2)$, $X(2, 4)$, $Y(3, 0)$

18. $T(-6, 3)$, $U(0, 6)$, $X(2, 2)$, $Y(-4, -1)$

19. $T(4, 1)$, $U(3, -1)$, $X(-3, 2)$, $Y(-2, 4)$

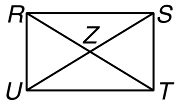
Lesson 8-4

NAME _______________ DATE _______ PERIOD ____

8-4 Practice

Rectangles

ALGEBRA ***RSTU* is a rectangle.**



1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ.

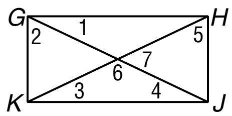
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT.

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$.

5. If $m\angle SRT = x^2 + 9$ and $m\angle UTR = 2x + 44$, find x.

6. If $m\angle RSU = x^2 - 1$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$.

***GHJK* is a rectangle. Find each measure if $m\angle 1 = 37$.**



7. $m\angle 2$

8. $m\angle 3$

9. $m\angle 4$

10. $m\angle 5$

11. $m\angle 6$

12. $m\angle 7$

COORDINATE GEOMETRY Determine whether *BGHL* is a rectangle given each set of vertices. Justify your answer.

13. $B(-4, 3)$, $G(-2, 4)$, $H(1, -2)$, $L(-1, -3)$

14. $B(-4, 5)$, $G(6, 0)$, $H(3, -6)$, $L(-7, -1)$

15. $B(0, 5)$, $G(4, 7)$, $H(5, 4)$, $L(1, 2)$

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

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8-4 Reading to Learn Mathematics

Rectangles

Pre-Activity **How are rectangles used in tennis?**

Read the introduction to Lesson 8-4 at the top of page 424 in your textbook.

Are the singles court and doubles court similar rectangles? Explain your answer.

Reading the Lesson

1. Determine whether each sentence is *always*, *sometimes*, or *never* true.

a. If a quadrilateral has four congruent angles, it is a rectangle.

b. If consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a rectangle.

c. The diagonals of a rectangle bisect each other.

d. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a rectangle.

e. Consecutive angles of a rectangle are complementary.

f. Consecutive angles of a rectangle are congruent.

g. If the diagonals of a quadrilateral are congruent, the quadrilateral is a rectangle.

h. A diagonal of a rectangle bisects two of its angles.

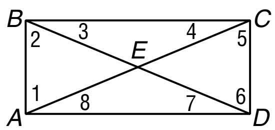
i. A diagonal of a rectangle divides the rectangle into two congruent right triangles.

j. If the diagonals of a quadrilateral bisect each other and are congruent, the quadrilateral is a rectangle.

k. If a parallelogram has one right angle, it is a rectangle.

l. If a parallelogram has four congruent sides, it is a rectangle.

2. *ABCD* is a rectangle with $AD > AB$.
Name each of the following in this figure.



a. all segments that are congruent to $\overline{BE}$

b. all angles congruent to $\angle 1$

c. all angles congruent to $\angle 7$

d. two pairs of congruent triangles

Helping You Remember

3. It is easier to remember a large number of geometric relationships and theorems if you are able to combine some of them. How can you combine the two theorems about diagonals that you studied in this lesson?

Lesson 8-4

NAME ____ DATE ____________ PERIOD _____

8-4 Enrichment

Counting Squares and Rectangles

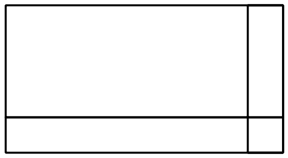
Each puzzle below contains many squares and/or rectangles. Count them carefully. You may want to label each region so you can list all possibilities.

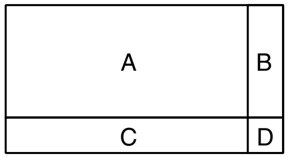
Example **How many rectangles are in the figure at the right?**

Label each small region with a letter. Then list each rectangle by writing the letters of regions it contains.

A, B, C, D, AB, CD, AC, BD, ABCD

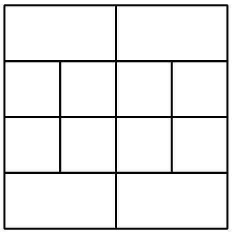
There are 9 rectangles.



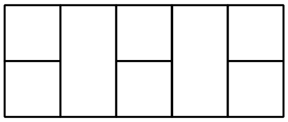


How many squares are in each figure?

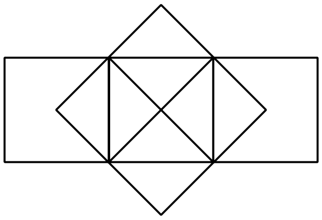
1.



2.

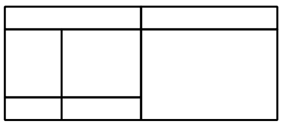


3.

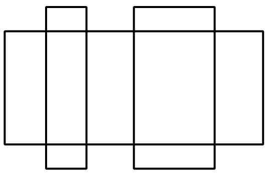


How many rectangles are in each figure?

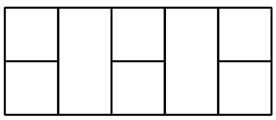
1.



2.



3.



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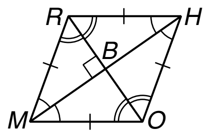
8-5 Study Guide and Intervention

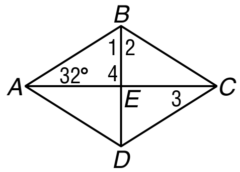
Rhombi and Squares

Properties of Rhombi A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If $RHOM$ is a parallelogram and $\overline{RO} \perp \overline{MH}$, then $RHOM$ is a rhombus.

Example **In rhombus *ABCD*, $m\angle BAC = 32$. Find the measure of each numbered angle.**

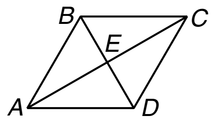


ABCD is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

***ABCD* is a rhombus.**



1. If $m\angle ABD = 60$, find $m\angle BDC$.

2. If $AE = 8$, find AC.

3. If $AB = 26$ and $BD = 20$, find AE.

4. Find $m\angle CEB$.

5. If $m\angle CBD = 58$, find $m\angle ACB$.

6. If $AE = 3x - 1$ and $AC = 16$, find x.

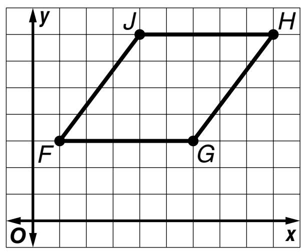
7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y.

8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x.

9. a. What is the midpoint of $\overline{FH}$?

b. What is the midpoint of $\overline{GJ}$?

c. What kind of figure is *FGHJ*? Explain.



d. What is the slope of $\overline{FH}$?

e. What is the slope of $\overline{GJ}$?

f. Based on parts **c**, **d**, and **e**, what kind of figure is *FGHJ*? Explain.

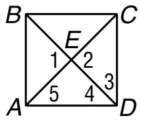
Lesson 8-5

8-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

Example **Find the measure of each numbered angle of square *ABCD*.**



Using properties of rhombi and rectangles, the diagonals are perpendicular and congruent. $\triangle ABE$ is a right triangle, so $m\angle 1 = m\angle 2 = 90$.

Each vertex angle is a right angle and the diagonals bisect the vertex angles, so $m\angle 3 = m\angle 4 = m\angle 5 = 45$.

Exercises

Determine whether the given vertices represent a *parallelogram*, *rectangle*, *rhombus*, or *square*. Explain your reasoning.

1. $A(0, 2)$, $B(2, 4)$, $C(4, 2)$, $D(2, 0)$

2. $D(-2, 1)$, $E(-1, 3)$, $F(3, 1)$, $G(2, -1)$

3. $A(-2, -1)$, $B(0, 2)$, $C(2, -1)$, $D(0, -4)$

4. $A(-3, 0)$, $B(-1, 3)$, $C(5, -1)$, $D(3, -4)$

5. $S(-1, 4)$, $T(3, 2)$, $U(1, -2)$, $V(-3, 0)$

6. $F(-1, 0)$, $G(1, 3)$, $H(4, 1)$, $I(2, -2)$

7. Square *RSTU* has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(2, 0)$. Find the coordinates of vertex *U*.

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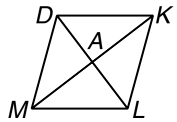
8-5 Skills Practice

Rhombi and Squares

Use rhombus *DKLM* with $AM = 4x$, $AK = 5x - 3$, and $DL = 10$.



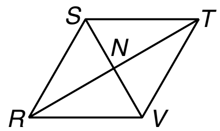
1. Find x.

2. Find AL.

3. Find $m\angle KAL$.

4. Find DM.

Use rhombus *RSTV* with $RS = 5y + 2$, $ST = 3y + 6$, and $NV = 6$.



5. Find y.

6. Find TV.

7. Find $m\angle NTV$.

8. Find $m\angle SVT$.

9. Find $m\angle RST$.

10. Find $m\angle SRV$.

COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*QRST* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.

11. $Q(3, 5)$, $R(3, 1)$, $S(-1, 1)$, $T(-1, 5)$

12. $Q(-5, 12)$, $R(5, 12)$, $S(-1, 4)$, $T(-11, 4)$

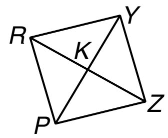
13. $Q(-6, -1)$, $R(4, -6)$, $S(2, 5)$, $T(-8, 10)$

14. $Q(2, -4)$, $R(-6, -8)$, $S(-10, 2)$, $T(-2, 6)$

8-5 Practice

Rhombi and Squares

Use rhombus *PRYZ* with $RK = 4y + 1$, $ZK = 7y - 14$, $PK = 3x - 1$, and $YK = 2x + 6$.



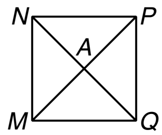
1. Find *PY*.

2. Find *RZ*.

3. Find *RY*.

4. Find $m\angle YKZ$.

Use rhombus *MNPQ* with $PQ = 3\sqrt{2}$, $PA = 4x - 1$, and $AM = 9x - 6$.



5. Find *AQ*.

6. Find $m\angle APQ$.

7. Find $m\angle MNP$.

8. Find *PM*.

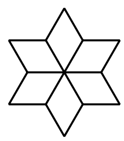
COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*BEFG* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$

10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$

11. $B(-4, -5)$, $E(1, -5)$, $F(-7, -1)$, $G(-2, -1)$

12. TESSELATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.



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8-5 Reading to Learn Mathematics

Rhombi and Squares

Pre-Activity **How can you ride a bicycle with square wheels?**

Read the introduction to Lesson 8-5 at the top of page 431 in your textbook.

If you draw a diagonal on the surface of one of the square wheels shown in the picture in your textbook, how can you describe the two triangles that are formed?

Reading the Lesson

1. Sketch each of the following.

a. a quadrilateral with perpendicular diagonals that is not a rhombus

b. a quadrilateral with congruent diagonals that is not a rectangle

c. a quadrilateral whose diagonals are perpendicular and bisect each other, but are not congruent

2. List all of the following special quadrilaterals that have each listed property: *parallelogram*, *rectangle*, *rhombus*, *square*.

a. The diagonals are congruent.

b. Opposite sides are congruent.

c. The diagonals are perpendicular.

d. Consecutive angles are supplementary.

e. The quadrilateral is equilateral.

f. The quadrilateral is equiangular.

g. The diagonals are perpendicular and congruent.

h. A pair of opposite sides is both parallel and congruent.

3. What is the common name for a regular quadrilateral? Explain your answer.

Helping You Remember

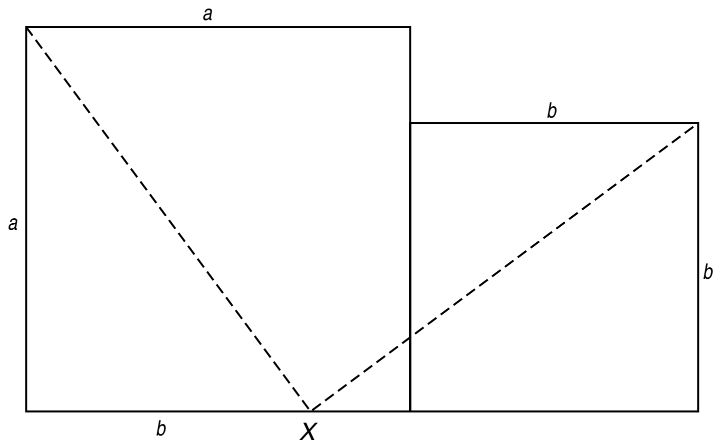
4. A good way to remember something is to explain it to someone else. Suppose that your classmate Luis is having trouble remembering which of the properties he has learned in this chapter apply to squares. How can you help him?

Lesson 8-5

8-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.



1. Carefully construct a square and label the length of a side as a. Then construct a smaller square to the right of it and label the length of a side as b, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X, and from point X to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.

4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

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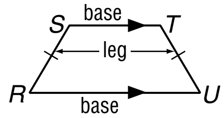
8-6 Study Guide and Intervention

Trapezoids

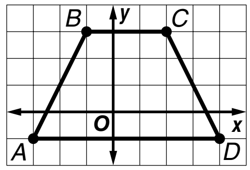
Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases** and the nonparallel sides are called **legs**. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent.



STUR is an isosceles trapezoid. $\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Example

The vertices of *ABCD* are *A*(−3, −1), *B*(−1, 3), *C*(2, 3), and *D*(4, −1). Verify that *ABCD* is a trapezoid.



slope of $\overline{AB} = \dfrac{3 - (-1)}{-1 - (-3)} = \dfrac{4}{2} = 2$

slope of $\overline{AD} = \dfrac{-1 - (-1)}{4 - (-3)} = \dfrac{0}{7} = 0$

slope of $\overline{BC} = \dfrac{3 - 3}{2 - (-1)} = \dfrac{0}{3} = 0$

slope of $\overline{CD} = \dfrac{-1 - 3}{4 - 2} = \dfrac{-4}{2} = -2$

$AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2}$
$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$CD = \sqrt{(2 - 4)^2 + (3 - (-1))^2}$
$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

Exactly two sides are parallel, $\overline{AD}$ and $\overline{BC}$, so *ABCD* is a trapezoid. $AB = CD$, so *ABCD* is an isosceles trapezoid.

Exercises

In Exercises 1–3, determine whether *ABCD* is a trapezoid. If so, determine whether it is an isosceles trapezoid. Explain.

1. *A*(−1, 1), *B*(2, 1), *C*(3, −2), and *D*(2, −2)

2. *A*(3, −3), *B*(−3, −3), *C*(−2, 3), and *D*(2, 3)

3. *A*(1, −4), *B*(−3, −3), *C*(−2, 3), and *D*(2, 2)

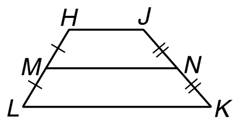
4. The vertices of an isosceles trapezoid are *R*(−2, 2), *S*(2, 2), *T*(4, −1), and *U*(−4, −1). Verify that the diagonals are congruent.

8-6 Study Guide and Intervention *(continued)*

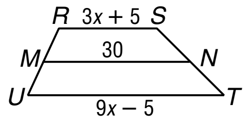
Trapezoids

Medians of Trapezoids The **median** of a trapezoid is the segment that joins the midpoints of the legs. It is parallel to the bases, and its length is one-half the sum of the lengths of the bases.

In trapezoid $HJKL$, $MN = \frac{1}{2}(HJ + LK)$.



Example $\overline{MN}$ **is the median of trapezoid** ***RSTU*****. Find** ***x*****.**



$MN = \frac{1}{2}(RS + UT)$

$30 = \frac{1}{2}(3x + 5 + 9x - 5)$

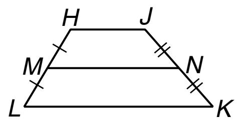
$30 = \frac{1}{2}(12x)$

$30 = 6x$

$5 = x$

Exercises

$\overline{MN}$ **is the median of trapezoid** ***HJKL*****. Find each indicated value.**



1. Find MN if $HJ = 32$ and $LK = 60$.

2. Find LK if $HJ = 18$ and $MN = 28$.

3. Find MN if $HJ + LK = 42$.

4. Find $m\angle LMN$ if $m\angle LHJ = 116$.

5. Find $m\angle JKL$ if $HJKL$ is isosceles and $m\angle HLK = 62$.

6. Find HJ if $MN = 5x + 6$, $HJ = 3x + 6$, and $LK = 8x$.

7. Find the length of the median of a trapezoid with vertices $A(-2, 2)$, $B(3, 3)$, $C(7, 0)$, and $D(-3, -2)$.

8-6 Skills Practice

Trapezoids

COORDINATE GEOMETRY *ABCD* is a quadrilateral with vertices *A*(−4, −3), *B*(3, −3), *C*(6, 4), *D*(−7, 4).

1. Verify that *ABCD* is a trapezoid.

2. Determine whether *ABCD* is an isosceles trapezoid. Explain.

COORDINATE GEOMETRY *EFGH* is a quadrilateral with vertices *E*(1, 3), *F*(5, 0), *G*(8, −5), *H*(−4, 4).

3. Verify that *EFGH* is a trapezoid.

4. Determine whether *EFGH* is an isosceles trapezoid. Explain.

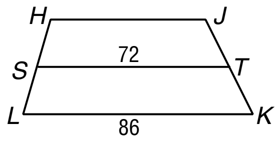
COORDINATE GEOMETRY *LMNP* is a quadrilateral with vertices *L*(−1, 3), *M*(−4, 1), *N*(−6, 3), *P*(0, 7).

5. Verify that *LMNP* is a trapezoid.

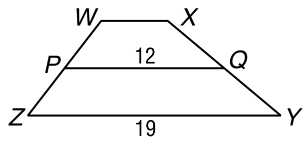
6. Determine whether *LMNP* is an isosceles trapezoid. Explain.

ALGEBRA Find the missing measure(s) for the given trapezoid.

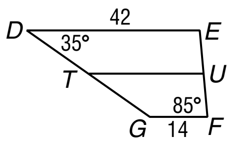
7. For trapezoid *HJKL*, *S* and *T* are midpoints of the legs. Find *HJ*.



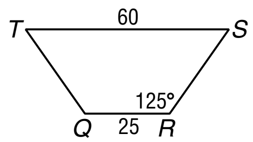
8. For trapezoid *WXYZ*, *P* and *Q* are midpoints of the legs. Find *WX*.



9. For trapezoid *DEFG*, *T* and *U* are midpoints of the legs. Find *TU*, $m\angle E$, and $m\angle G$.



10. For isosceles trapezoid *QRST*, find the length of the median, $m\angle Q$, and $m\angle S$.



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8-6 Practice

Trapezoids

COORDINATE GEOMETRY ***RSTU*** **is a quadrilateral with vertices** ***R***(**−3, −3**), ***S***(**5, 1**), ***T***(**10, −2**), ***U***(**−4, −9**).

1. Verify that *RSTU* is a trapezoid.

2. Determine whether *RSTU* is an isosceles trapezoid. Explain.

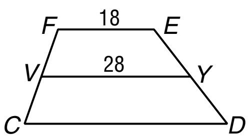
COORDINATE GEOMETRY ***BGHJ*** **is a quadrilateral with vertices** ***B***(**−9, 1**), ***G***(**2, 3**), ***H***(**12, −2**), ***J***(**−10, −6**).

3. Verify that *BGHJ* is a trapezoid.

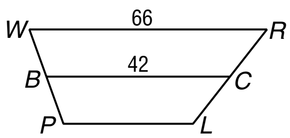
4. Determine whether *BGHJ* is an isosceles trapezoid. Explain.

ALGEBRA Find the missing measure(s) for the given trapezoid.

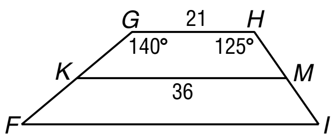
5. For trapezoid *CDEF*, *V* and *Y* are midpoints of the legs. Find *CD*.



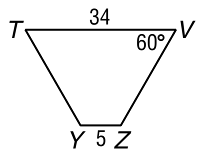
6. For trapezoid *WRLP*, *B* and *C* are midpoints of the legs. Find *LP*.



7. For trapezoid *FGHI*, *K* and *M* are midpoints of the legs. Find *FI*, $m\angle F$, and $m\angle I$.



8. For isosceles trapezoid *TVZY*, find the length of the median, $m\angle T$, and $m\angle Z$.



9. CONSTRUCTION A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14, find the width of the stairs halfway to the top.

10. DESK TOPS A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

8-6 Reading to Learn Mathematics

Trapezoids

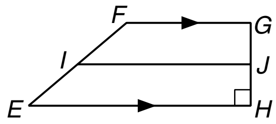
Pre-Activity **How are trapezoids used in architecture?**

Read the introduction to Lesson 8-6 at the top of page 439 in your textbook.

How might trapezoids be used in the interior design of a home?

Reading the Lesson

1. In the figure at the right, *EFGH* is a trapezoid, *I* is the midpoint of $\overline{FE}$, and *J* is the midpoint of $\overline{GH}$. Identify each of the following segments or angles in the figure.



a. the bases of trapezoid *EFGH*

b. the two pairs of base angles of trapezoid *EFGH*

c. the legs of trapezoid *EFGH*

d. the median of trapezoid *EFGH*

2. Determine whether each statement is *true* or *false*. If the statement is false, explain why.

a. A trapezoid is a special kind of parallelogram.

b. The diagonals of a trapezoid are congruent.

c. The median of a trapezoid is parallel to the legs.

d. The length of the median of a trapezoid is the average of the length of the bases.

e. A trapezoid has three medians.

f. The bases of an isosceles trapezoid are congruent.

g. An isosceles trapezoid has two pairs of congruent angles.

h. The median of an isosceles trapezoid divides the trapezoid into two smaller isosceles trapezoids.

Helping You Remember

3. A good way to remember a new geometric theorem is to relate it to one you already know. Name and state in words a theorem about triangles that is similar to the theorem in this lesson about the median of a trapezoid.

8-6 Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges.

a. isosceles triangle

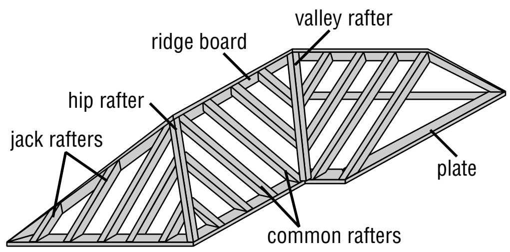
b. scalene triangle

c. rectangle

d. rhombus

e. trapezoid (not isosceles)

f. isosceles trapezoid



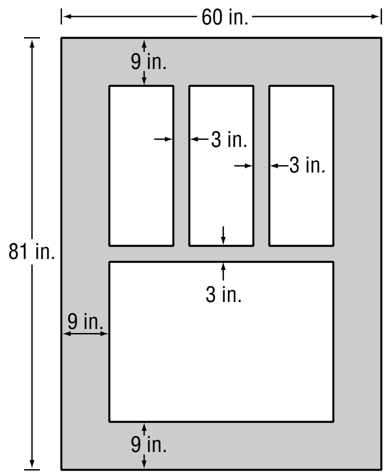
Roof Frame

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.

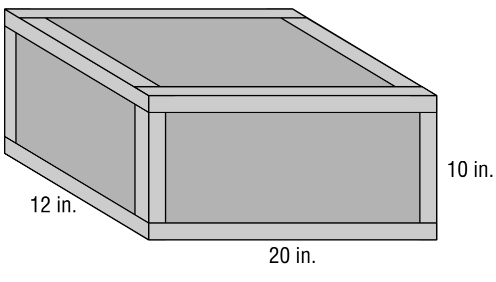
a. How wide is the bottom pane of glass?

b. How wide is each top pane of glass?

c. How high is each pane of glass?



3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used?



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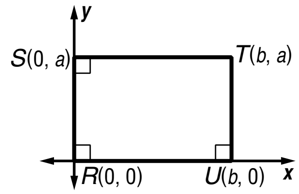
8-7 Study Guide and Intervention

Coordinate Proof with Quadrilaterals

Position Figures Coordinate proofs use properties of lines and segments to prove geometric properties. The first step in writing a coordinate proof is to place the figure on the coordinate plane in a convenient way. Use the following guidelines for placing a figure on the coordinate plane.

1. Use the origin as a vertex, so one set of coordinates is (0, 0), or use the origin as the center of the figure.
2. Place at least one side of the quadrilateral on an axis so you will have some zero coordinates.
3. Try to keep the quadrilateral in the first quadrant so you will have positive coordinates.
4. Use coordinates that make the computations as easy as possible. For example, use even numbers if you are going to be finding midpoints.

Example **Position and label a rectangle with sides a and b units long on the coordinate plane.**

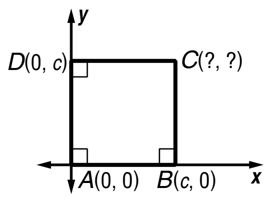


- Place one vertex at the origin for R, so one vertex is $R(0, 0)$.
- Place side $\overline{RU}$ along the x-axis and side $\overline{RS}$ along the y-axis, with the rectangle in the first quadrant.
- The sides are a and b units, so label two vertices $S(0, a)$ and $U(b, 0)$.
- Vertex T is b units right and a units up, so the fourth vertex is $T(b, a)$.

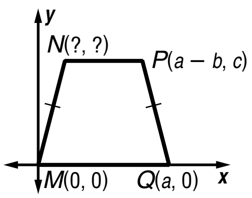
Exercises

Name the missing coordinates for each quadrilateral.

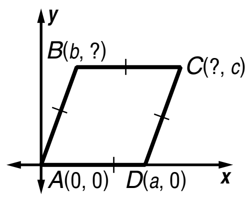
1.



2.

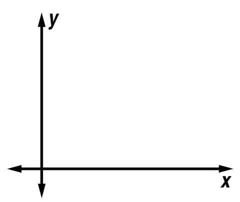


3.

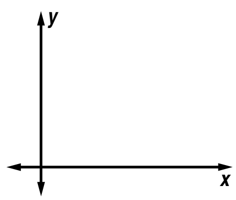


Position and label each quadrilateral on the coordinate plane.

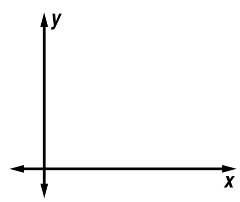
4. square *STUV* with side s units



5. parallelogram *PQRS* with congruent diagonals



6. rectangle *ABCD* with length twice the width

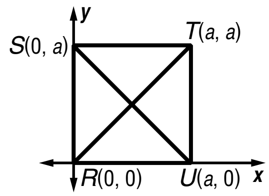


8-7 Study Guide and Intervention *(continued)*

Coordinate Proof With Quadrilaterals

Prove Theorems After a figure has been placed on the coordinate plane and labeled, a coordinate proof can be used to prove a theorem or verify a property. The Distance Formula, the Slope Formula, and the Midpoint Theorem are often used in a coordinate proof.

Example **Write a coordinate proof to show that the diagonals of a square are perpendicular.**



The first step is to position and label a square on the coordinate plane. Place it in the first quadrant, with one side on each axis. Label the vertices and draw the diagonals.

Given: square $RSTU$

Prove: $\overline{SU} \perp \overline{RT}$

Proof: The slope of $\overline{SU}$ is $\frac{0 - a}{a - 0} = -1$, and the slope of $\overline{RT}$ is $\frac{a - 0}{a - 0} = 1$. The product of the two slopes is -1, so $\overline{SU} \perp \overline{RT}$.

Exercise

Write a coordinate proof to show that the length of the median of a trapezoid is half the sum of the lengths of the bases.

8-7 Skills Practice

Coordinate Proof with Quadrilaterals

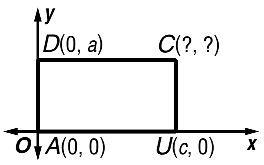
Position and label each quadrilateral on the coordinate plane.

1. rectangle with length $2a$ units and height a units

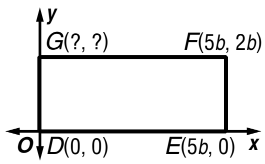
2. isosceles trapezoid with height a units, bases $c - b$ units and $b + c$ units

Name the missing coordinates for each quadrilateral.

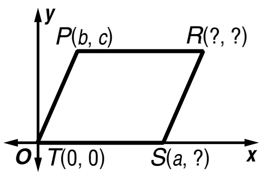
3. rectangle



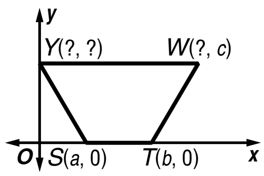
4. rectangle



5. parallelogram



6. isosceles trapezoid



Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

7. The segments joining the midpoints of the sides of a rhombus form a rectangle.

8-7 Practice

Coordinate Proof with Quadrilaterals

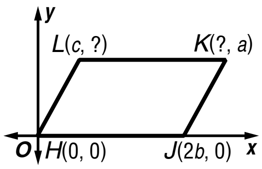
Position and label each quadrilateral on the coordinate plane.

1. parallelogram with side length b units and height a units

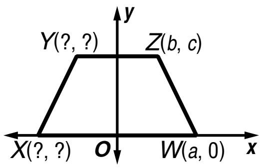
2. isosceles trapezoid with height b units, bases $2c - a$ units and $2c + a$ units

Name the missing coordinates for each quadrilateral.

3. parallelogram



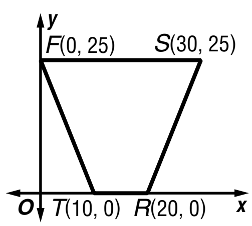
4. isosceles trapezoid



Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

5. The opposite sides of a parallelogram are congruent.

6. THEATER A stage is in the shape of a trapezoid. Write a coordinate proof to prove that $\overline{TR}$ and $\overline{SF}$ are parallel.



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8-7 Reading to Learn Mathematics

Coordinate Proof with Quadrilaterals

Pre-Activity **How can you use a coordinate plane to prove theorems about quadrilaterals?**

Read the introduction to Lesson 8-7 at the top of page 447 in your textbook.

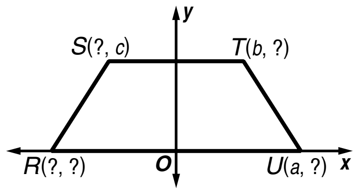
What special kinds of quadrilaterals can be placed on a coordinate system so that two sides of the quadrilateral lie along the axes?

Lesson 8-7

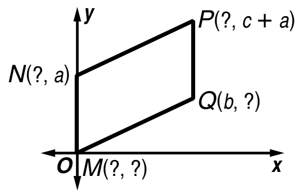
Reading the Lesson

1. Find the missing coordinates in each figure. Then write the coordinates of the four vertices of the quadrilateral.

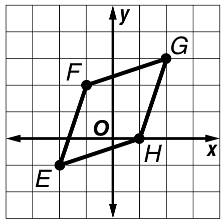
a. isosceles trapezoid



b. parallelogram



2. Refer to quadrilateral *EFGH*.



a. Find the slope of each side.

b. Find the length of each side.

c. Find the slope of each diagonal.

d. Find the length of each diagonal.

e. What can you conclude about the sides of *EFGH*?

f. What can you conclude about the diagonals of *EFGH*?

g. Classify *EFGH* as a *parallelogram*, a *rhombus*, or a *square*. Choose the most specific term. Explain how your results from parts a-f support your conclusion.

Helping You Remember

3. What is an easy way to remember how best to draw a diagram that will help you devise a coordinate proof?

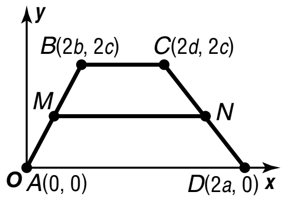
8-7 Enrichment

Coordinate Proofs

An important part of planning a coordinate proof is correctly placing and labeling the geometric figure on the coordinate plane.

Example **Draw a diagram you would use in a coordinate proof of the following theorem.**

The median of a trapezoid is parallel to the bases of the trapezoid.



In the diagram, note that one vertex, A, is placed at the origin. Also, the coordinates of B, C, and D use $2a$, $2b$, and $2c$ in order to avoid the use of fractions when finding the coordinates of the midpoints, M and N.

When doing coordinate proofs, the following strategies may be helpful.

1. If you are asked to prove that segments are parallel or perpendicular, use slopes.

2. If you are asked to prove that segments are congruent or have related measures, use the distance formula.

3. If you are asked to prove that a segment is bisected, use the midpoint formula.

For each of the following theorems, a diagram has been provided to be used in a formal proof. Name the missing coordinates in the diagram. Then, using the *Given* and the *Prove* statements, prove the theorem.

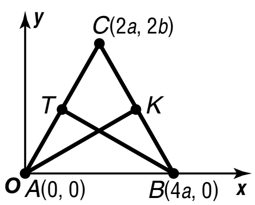
1. The median of a trapezoid is parallel to the bases of the trapezoid. (Use the diagram given in the example above.)

Coordinates of M and N:

Given: Trapezoid $ABCD$ has median $\overline{MN}$.
Prove: $\overline{BC} \parallel \overline{MN}$ and $\overline{AD} \parallel \overline{MN}$

Proof:

2. The medians to the legs of an isosceles triangle are congruent.



Coordinates of T and K:

Given: $\triangle ABC$ is isosceles with medians $\overline{TB}$ and $\overline{KA}$.
Prove: $\overline{TB} \cong \overline{KA}$

Proof:

NAME ________________________ DATE ____________ PERIOD ______

8

Chapter 8 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 30-gon. **1.** ______

A. 5400 **B.** 5040 **C.** 360 **D.** 168

2. Find the sum of the measures of the exterior angles of a convex 21-gon. **2.** ______

A. 21 **B.** 180 **C.** 360 **D.** 3420

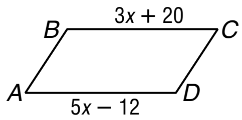
3. If the measure of each interior angle of a regular polygon is 108, find the measure of each exterior angle. **3.** ______

A. 18 **B.** 72 **C.** 90 **D.** 108

4. For parallelogram *ABCD*, find *x*. **4.** ______

A. 4 **B.** 10.25

C. 16 **D.** 21.5



5. Which of the following is a property of a parallelogram? **5.** ______

A. The diagonals are congruent. **B.** The diagonals bisect the angles.

C. The diagonals are perpendicular. **D.** The diagonals bisect each other.

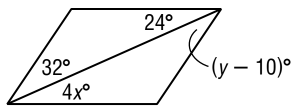
6. Find *x* and *y* so that *ABCD* will be a parallelogram. **6.** ______

A. $x = 6, y = 42$

B. $x = 6, y = 22$

C. $x = 20, y = 42$

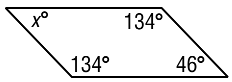
D. $x = 20, y = 22$



7. Find *x* so that this quadrilateral is a parallelogram. **7.** ______

A. 44 **B.** 46

C. 90 **D.** 134



8. *ABCD* is a parallelogram with $A(0, 0)$, $B(2, 4)$, and $C(10, 4)$. Find the possible coordinates of *D*. **8.** ______

A. (8, 0) **B.** (10, 0) **C.** (0, 4) **D.** (10, 8)

9. Which of the following is a property of all rectangles? **9.** ______

A. four congruent sides **B.** diagonals bisect the angles

C. diagonals are perpendicular **D.** four right angles

10. *ABCD* is a rectangle with diagonals $\overline{AC}$ and $\overline{BD}$. If $AC = 2x + 10$ and $BD = 56$, find *x*. **10.** ______

A. 23 **B.** 33 **C.** 78 **D.** 122

11. *ABCD* is a rectangle with $B(-5, 0)$, $C(7, 0)$ and $D(7, 3)$. Find the coordinates of *A*. **11.** ______

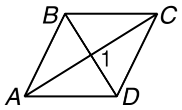
A. $(-5, 7)$ **B.** $(3, 5)$

C. $(-5, 3)$ **D.** $(7, -3)$

Assessments

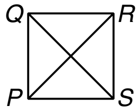
8 Chapter 8 Test, Form 1 *(continued)*

12. Given rhombus *ABCD*, find $m\angle 1$. **12.** ______



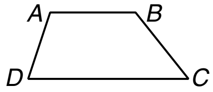
A. 45 **B.** 60
C. 90 **D.** 120

13. Find $m\angle PRS$ in square *PQRS*. **13.** ______



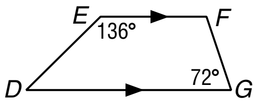
A. 30 **B.** 45
C. 60 **D.** 90

14. Choose a pair of base angles of trapezoid *ABCD*. **14.** ______



A. $\angle A$, $\angle C$ **B.** $\angle B$, $\angle D$
C. $\angle A$, $\angle D$ **D.** $\angle D$, $\angle C$

15. In trapezoid *DEFG*, find $m\angle D$. **15.** ______



A. 44 **B.** 72
C. 108 **D.** 136

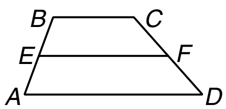
16. The bases of a trapezoid are 12 and 26. Find the length of the median. **16.** ______

A. 12 **B.** 19 **C.** 26 **D.** 38

17. If the length of one base of a trapezoid is 44, the median is 36, and the other base is $2x + 10$, find x. **17.** ______

A. 9 **B.** 17 **C.** 21 **D.** 40

18. Given trapezoid *ABCD* with median $\overline{EF}$, which of the following is true? **18.** ______



A. $EF = \frac{1}{2}AD$ **B.** $AE = FD$
C. $AB = EF$ **D.** $EF = \frac{BC + AD}{2}$

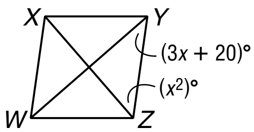
19. *ABCD* is a rectangle with $A(0, 0)$, $B(b, 0)$, and $D(0, a)$. Find the coordinates of *C*. **19.** ______

A. $C(a, b)$ **B.** $C(b, a)$ **C.** $C(-b, a)$ **D.** $C(a + b, a)$

20. To prove that the diagonals of a square bisect each other, you would position and label a square in the coordinate plane and then find which of the following? **20.** ______

A. measures of the angles **B.** midpoints of the diagonals
C. lengths of the diagonals **D.** slopes of the diagonals

Bonus Find x and $m\angle WYZ$ in rhombus *XYZW*. **B:** ____________



8 Chapter 8 Test, Form 2A

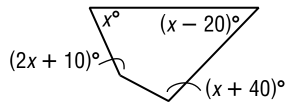
SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the sum of the measures of the interior angles of a convex 45-gon. **1.** ______

A. 8100 **B.** 7740 **C.** 360 **D.** 172

2. Find x. **2.** ______



A. 30 **B.** 66

C. 102 **D.** 138

3. Find the sum of the measures of the exterior angles of a convex 39-gon. **3.** ______

A. 39 **B.** 90 **C.** 180 **D.** 360

4. Which of the following is a property of a parallelogram? **4.** ______

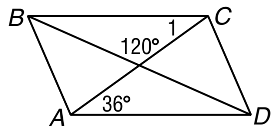
A. Each pair of opposite sides is congruent.

B. Only one pair of opposite angles is congruent.

C. Each pair of opposite angles is supplementary.

D. There are four right angles.

5. Find $m\angle 1$ in parallelogram $ABCD$. **5.** ______



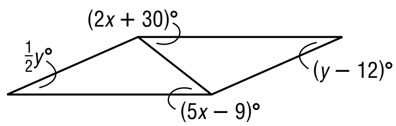
A. 60 **B.** 54

C. 36 **D.** 18

6. $ABCD$ is a parallelogram with diagonals intersecting at E. If $AE = 3x + 12$ and $EC = 27$, find x. **6.** ______

A. 5 **B.** 17 **C.** 27 **D.** 47

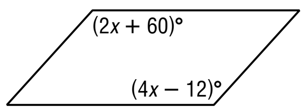
7. Find x and y so that this quadrilateral is a parallelogram. **7.** ______



A. $x = 13, y = 24$ **B.** $x = 13, y = 6$

C. $x = 7, y = 24$ **D.** $x = 7, y = 6$

8. Find x so that this quadrilateral is a parallelogram. **8.** ______



A. 12 **B.** 24

C. 36 **D.** 132

9. Given $A(8, 2)$, $B(6, -4)$, $C(-5, -4)$, find the coordinates of D so that $ABCD$ is a parallelogram. **9.** ______

A. $D(-5, 2)$ **B.** $D(-3, 2)$ **C.** $D(-2, 2)$ **D.** $D(-4, 8)$

10. $ABCD$ is a rectangle. If $AC = 5x + 2$ and $BD = x + 22$, find x. **10.** ______

A. 5 **B.** 6 **C.** 11 **D.** 26

11. Which of the following is true for all rectangles? **11.** ______

A. The diagonals are perpendicular.

B. The diagonals bisect the angles.

C. The consecutive sides are congruent.

D. The consecutive sides are perpendicular.

Assessments

8 Chapter 8 Test, Form 2A *(continued)*

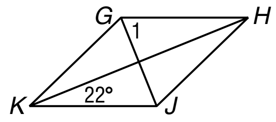
12. *ABCD* is a rectangle with $B(-4, 6)$, $C(-4, 2)$, and $D(10, 2)$. Find the coordinates of *A*. **12.** ______

A. $A(6, 4)$ **B.** $A(10, 4)$ **C.** $A(2, 6)$ **D.** $A(10, 6)$

13. Find $m\angle 1$ in rhombus *GHJK*. **13.** ______

A. 22 **B.** 44

C. 68 **D.** 90



14. The diagonals of square *ABCD* intersect at *E*. If $AE = 2x + 6$ and $BD = 6x - 10$, find *AC*. **14.** ______

A. 11 **B.** 28 **C.** 56 **D.** 90

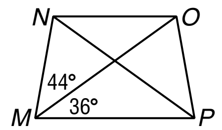
15. *ABCD* is an isosceles trapezoid with $A(10, -1)$, $B(8, 3)$, and $C(-1, 3)$. Find the coordinates of *D*. **15.** ______

A. $(-3, -1)$ **B.** $(-10, -11)$ **C.** $(-1, 8)$ **D.** $(-3, 3)$

16. For isosceles trapezoid *MNOP*, find $m\angle MNP$. **16.** ______

A. 44 **B.** 64

C. 80 **D.** 116



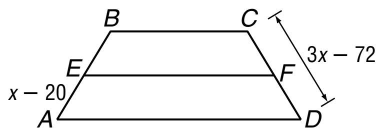
17. The length of one base of a trapezoid is 19 inches and the length of the median is 16 inches. Find the length of the other base. **17.** ______

A. 35 in. **B.** 19 in. **C.** 17.5 in. **D.** 13 in.

18. $\overline{EF}$ is the median of isosceles trapezoid *ABCD*. Find *x*. **18.** ______

A. $2x - 46$ **B.** 32

C. 46 **D.** 68



19. What type of quadrilateral has vertices at $(0, 0)$, (a, b), (c, b), and $(c + a, 0)$? **19.** ______

A. parallelogram **B.** rectangle

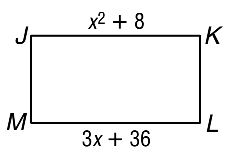
C. rhombus **D.** trapezoid

20. To prove that the diagonals of a rhombus are perpendicular to each other, you would position and label a rhombus on a coordinate plane and then find which of the following? **20.** ______

A. measures of the angles **B.** slopes of the diagonals

C. lengths of the diagonals **D.** midpoints of the diagonals

Bonus Find the possible value(s) of *x* in rectangle *JKLM*. **B:** ____________________



NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

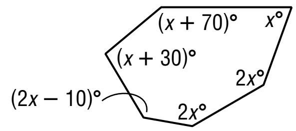
1. Find the sum of the measures of the interior angles of a convex 50-gon. **1.** ______

A. 9000 **B.** 8640 **C.** 360 **D.** 172.8

2. Find x. **2.** ______

A. 16 **B.** 34

C. 50 **D.** 70



3. Find the sum of the measures of the exterior angles of a convex 65-gon. **3.** ______

A. 5.54 **B.** 90 **C.** 180 **D.** 360

4. Which of the following is a property of all parallelograms? **4.** ______

A. Each pair of opposite angles is congruent.

B. Only one pair of opposite sides is congruent.

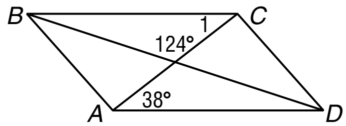
C. Each pair of opposite angles is supplementary.

D. There are four right angles.

5. Find $m\angle 1$ in parallelogram $ABCD$. **5.** ______

A. 19 **B.** 38

C. 52 **D.** 56



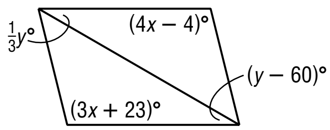
6. $ABCD$ is a parallelogram with diagonals intersecting at E. If $AE = 4x - 8$ and $EC = 36$, find x. **6.** ______

A. 7 **B.** 11 **C.** 15.5 **D.** 38

7. Find x and y so that this quadrilateral is a parallelogram. **7.** ______

A. $x = 27, y = 90$ **B.** $x = 27, y = 40$

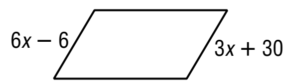
C. $x = 13, y = 90$ **D.** $x = 13, y = 40$



8. Find x so that this quadrilateral is a parallelogram. **8.** ______

A. $7\frac{1}{3}$ **B.** 8

C. 12 **D.** 66



9. $ABCD$ is a parallelogram with $A(5, 4)$, $B(-1, -2)$, $C(8, -2)$. Find the coordinates of D. **9.** ______

A. $D(-5, 4)$ **B.** $D(8, 2)$ **C.** $D(14, 4)$ **D.** $D(4, 1)$

10. $ABCD$ is a rectangle. If $AB = 7x - 6$ and $CD = 5x + 30$, find x. **10.** ______

A. $5\frac{1}{3}$ **B.** 12 **C.** 13 **D.** 18

11. Which of the following is true for all rectangles? **11.** ______

A. The diagonals are perpendicular.

B. The consecutive angles are supplementary.

C. The opposite sides are supplementary.

D. The opposite angles are complementary.

8 Chapter 8 Test, Form 2B *(continued)*

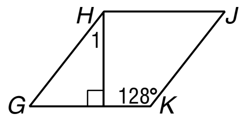
12. *ABCD* is a rectangle with $B(-7, 3)$, $C(5, 3)$, and $D(5, -8)$. Find the coordinates of *A*. **12.** ______

A. $(-8, -7)$ **B.** $(-7, -8)$ **C.** $(-5, -3)$ **D.** $(-8, -5)$

13. Find $m\angle 1$ in rhombus *GHJK*. **13.** ______

A. 90 **B.** 64

C. 52 **D.** 38



14. The diagonals of square *ABCD* intersect at *E*. If $AE = 3x - 4$ and $BD = 10x - 48$, find *AC*. **14.** ______

A. 90 **B.** 52 **C.** 26 **D.** 10

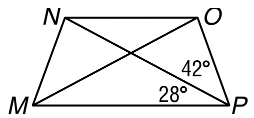
15. *ABCD* is an isosceles trapezoid with $A(0, -1)$, $B(-2, 3)$, and $D(6, -1)$. Find the coordinates of *C*. **15.** ______

A. $C(6, 1)$ **B.** $C(9, 4)$ **C.** $C(2, 3)$ **D.** $C(8, 3)$

16. Find $m\angle MNP$ in isosceles trapezoid *MNOP*. **16.** ______

A. 42 **B.** 70

C. 82 **D.** 98



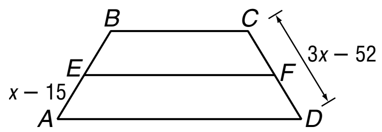
17. The length of one base of a trapezoid is 19 meters and the length of the median is 23 meters. Find the length of the other base. **17.** ______

A. 15 m **B.** 21 m **C.** 27 m **D.** 42 m

18. $\overline{EF}$ is the median of isosceles trapezoid *ABCD*. Find *x*. **18.** ______

A. 22 **B.** 18.5

C. 42.5 **D.** 82



19. What type of quadrilateral has vertices at $(0, 0)$, (a, b), $(a + c, b)$, and $(c, 0)$? **19.** ______

A. parallelogram **B.** rectangle

C. rhombus **D.** trapezoid

20. To prove that the diagonals of a rectangle are congruent, you would position and label a rectangle on a coordinate plane and then find which of the following? **20.** ______

A. measures of the angles **B.** slopes of the diagonals

C. lengths of the diagonals **D.** midpoints of the diagonals

Bonus The sum of the measures of the interior angles of a convex polygon is ten times the sum of the measures of its exterior angles. Find the number of sides of the polygon. **B:** ____________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 2C

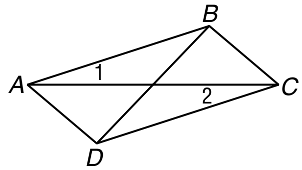
SCORE ______

1. Find the sum of the measures of the interior angles of a convex 60-gon. **1.** ________

2. A convex pentagon has interior angles with measures $(5x - 12)°$, $(2x + 100)°$, $(4x + 16)°$, $(6x + 15)°$, and $(3x + 41)°$. Find x. **2.** ________

3. If the measure of each interior angle of a regular polygon is 171, find the number of sides of the polygon. **3.** ________

4. In parallelogram $ABCD$, $m\angle 1 = x + 12$, and $m\angle 2 = 6x - 18$. Find $m\angle 1$. **4.** ________

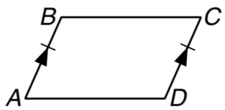


5. Find the measure of each exterior angle of a regular 45-gon. **5.** ________

6. In parallelogram $ABCD$, $m\angle A = 58$. Find $m\angle B$. **6.** ________

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(2, 2)$, $Y(3, 6)$, $Z(10, 6)$, and $W(9, 2)$. **7.** ________

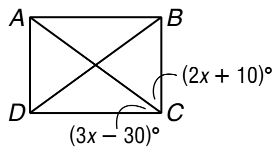
8. Determine whether $ABCD$ is a parallelogram. Justify your answer. **8.** ________



9. Use the Slope Formula to determine whether $A(5, 7)$, $B(1, -2)$, $C(-6, -3)$, and $D(2, 5)$ are the coordinates of the vertices of parallelogram $ABCD$. **9.** ________

10. If the slope of $\overline{AB}$ is $\frac{1}{4}$, the slope of $\overline{BC}$ is $-\frac{2}{3}$, and the slope of $\overline{CD}$ is $\frac{1}{4}$, find the slope of $\overline{DA}$ so that $ABCD$ will be a parallelogram. **10.** ________

11. Given rectangle $ABCD$, find x. **11.** ________



12. $ABCD$ is a parallelogram and $\overline{AC} \cong \overline{BD}$. Determine whether $ABCD$ is a rectangle. Justify your answer. **12.** ________

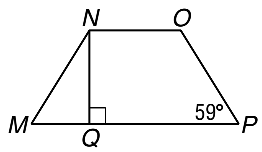
13. $ABCD$ is a rhombus with diagonals intersecting at E. If $m\angle ABC = 3m\angle BAD$, find $m\angle EBC$. **13.** ________

Assessments

8 Chapter 8 Test, Form 2C *(continued)*

14. *TUVW* is a square with *U*(10, 2), *V*(8, 8), and *W*(2, 6). Find the coordinates of *T*. **14.** ________________

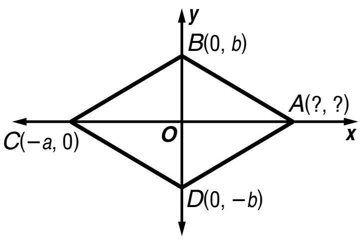
15. Find $m\angle MNQ$ in isosceles trapezoid *MNOP*. **15.** ________________



16. *ABCD* is a quadrilateral with *A*(8, 3), *B*(6, 7), *C*(−1, 5), and *D*(−6, −1). Determine whether *ABCD* is a trapezoid. Justify your answer. **16.** ________________

17. The length of the median of trapezoid *EFGH* is 13 feet. If the bases have lengths $2x + 4$ and $10x - 50$, find x. **17.** ________________

18. Name the missing coordinates for rhombus *ABCD*. Then determine the relationship between $\overline{AC}$ and $\overline{BD}$. **18.** ________________



For Questions 19–25, write *true* or *false*.

19. A rectangle is always a parallelogram. **19.** ________________

20. The diagonals of a rhombus are always perpendicular. **20.** ________________

21. The diagonals of a square always bisect each other. **21.** ________________

22. A trapezoid always has two congruent sides. **22.** ________________

23. The median of a trapezoid is always parallel to the bases. **23.** ________________

24. A quadrilateral with vertices $(a, 0)$, (b, c), $(-b, c)$, and $(-a, 0)$ is an isosceles trapezoid. **24.** ________________

25. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle. **25.** ________________

Bonus In parallelogram *ABCD*, $AB = 2x - 7$, $BC = x + 3y$, $CD = x + y$, and $AD = 2x - y - 1$. Find x and y. **B:** ________________

NAME ________________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 2D

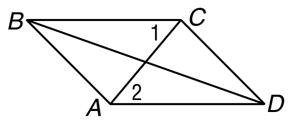
SCORE ______

1. Find the sum of the measures of the interior angles of a convex 36-gon. **1.** ____________

2. A convex octagon has interior angles with measures $(x + 55)°$, $(3x + 20)°$, $4x°$, $(4x - 10)°$, $(6x - 55)°$, $(3x + 52)°$, $3x°$, and $(2x + 30)°$. Find x. **2.** ____________

3. If the measure of each interior angle of a regular polygon is 176 find the number of sides on the polygon. **3.** ____________

4. In parallelogram $ABCD$, $m\angle 1 = x + 25$, and $m\angle 2 = 2x$. Find $m\angle 2$. **4.** ____________

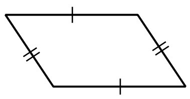


5. Find the measure of each exterior angle of a regular 100-gon. **5.** ____________

6. If $m\angle A = 63$ in parallelogram $ABCD$, find $m\angle B$. **6.** ____________

7. Find the coordinates of the intersection of the diagonals of parallelogram $XYZW$ with vertices $X(3, 0)$, $Y(3, 8)$, $Z(-2, 6)$, and $W(-2, -2)$. **7.** ____________

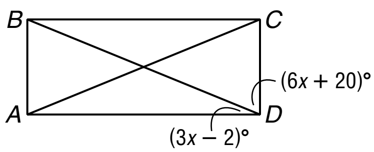
8. Determine whether this quadrilateral is a parallelogram. Justify your answer. **8.** ____________



9. Use the Slope Formula to determine whether $A(5, 7)$, $B(1, -1)$, $C(-6, -3)$, and $D(-2, 5)$ are the coordinates of the vertices of a parallelogram. **9.** ____________

10. If the slope of $\overline{AB}$ is $\frac{1}{2}$, the slope of $\overline{BC}$ is -4, and the slope of $\overline{CD}$ is $\frac{1}{2}$, find the slope of $\overline{DA}$ so that $ABCD$ is a parallelogram. **10.** ____________

11. Given rectangle $ABCD$, find x. **11.** ____________



12. $ABCD$ is a parallelogram and $m\angle A = 90$. Determine whether $ABCD$ is a rectangle. Justify your answer. **12.** ____________

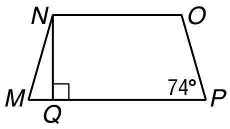
Assessments

8 Chapter 8 Test, Form 2D *(continued)*

13. *ABCD* is a rhombus with diagonals intersecting at *E*. If $m\angle ABC = 4m\angle BAD$, find $m\angle EBC$. **13.** ____________

14. *PQRS* is a square with $Q(-2, 8)$, $R(5, 7)$, and $S(4, 0)$. Find the coordinates of *P*. **14.** ____________

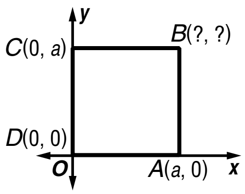
15. Find $m\angle MNQ$ in isosceles trapezoid *MNOP*. **15.** ____________



16. *ABCD* is a quadrilateral with $A(8, -1)$, $B(10, -7)$, $C(-6, -1)$, and $D(0, 2)$. Determine whether *ABCD* is a trapezoid. Justify your answer. **16.** ____________

17. The length of the median of trapezoid *EFGH* is 17 centimeters. If the bases have lengths $2x + 4$ and $8x - 50$, find x. **17.** ____________

18. Name the missing coordinates for square *ABCD*. Then determine the coordinates of the midpoints of the diagonals. **18.** ____________



For Questions 19–25, write *true* or *false*.

19. A parallelogram always has four right angles. **19.** ____________

20. The diagonals of a rhombus always bisect the angles. **20.** ____________

21. A rhombus is always a square. **21.** ____________

22. A rectangle is always a square. **22.** ____________

23. The diagonals of an isosceles trapezoid are always congruent. **23.** ____________

24. The median of a trapezoid always bisects the angles. **24.** ____________

25. A quadrilateral with vertices $(0, 0)$, $(a, 0)$, $(a + c, b)$, and (b, c) is a parallelogram. **25.** ____________

Bonus The measure of each interior angle of a regular polygon is 24 more than 38 times the measure of each exterior angle. Find the number of sides of the polygon. **B:** ____________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Test, Form 3

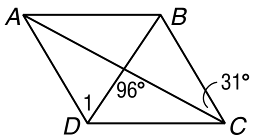
SCORE ______

1. Find the sum of the measures of the interior angles of a convex 24-gon. **1.** ____________

2. A convex hexagon has interior angles with measures $x°$, $(5x - 103)°$, $(2x + 60)°$, $(7x - 31)°$, $(6x - 6)°$, and $(9x - 100)°$. Find x and the measure of each angle. **2.** ____________

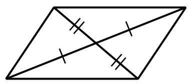
3. Find the measure of each exterior angle of a regular $2x$-gon. **3.** ____________

4. Find $m\angle 1$ in parallelogram *ABCD*. **4.** ____________



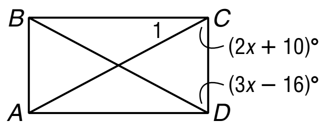
5. *ABCD* is a parallelogram with diagonals that intersect each other at *E*. If $AE = x^2$ and $EC = 6x - 8$, find all possible values of *AC*. **5.** ____________

6. Determine whether this quadrilateral is a parallelogram. Justify your answer. **6.** ____________



7. Given the slope of $\overline{AB}$ is $\frac{2}{3}$ and the slope of $\overline{BC}$ is -2, find the slopes of $\overline{CD}$ and $\overline{DA}$ so that *ABCD* will be a parallelogram. **7.** ____________

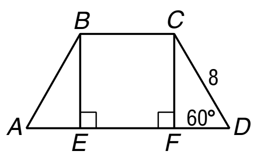
8. In rectangle *ABCD*, find $m\angle 1$. **8.** ____________



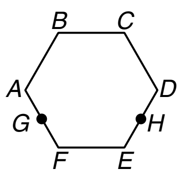
9. The diagonals of rhombus *ABCD* intersect at *E*. If $m\angle BAE = \frac{2}{3}(m\angle ABE)$, find $m\angle BCD$. **9.** ____________

10. The diagonals of square *ABCD* intersect at *E*. If $AE = 2$, find the perimeter of *ABCD*. **10.** ____________

11. Find *AE* in isosceles trapezoid *ABCD*. **11.** ____________



12. Points *G* and *H* are midpoints of $\overline{AF}$ and $\overline{DE}$ in regular hexagon *ABCDEF*. If $AB = 6$ find *GH*. **12.** ____________



Assessments

8 Chapter 8 Test, Form 3 *(continued)*

13. If the vertices of trapezoid $ABCD$ are $A(10, -1)$, $B(6, 6)$, $C(-8, -1)$, and $D(-2, 6)$, find the length of the median.

13. ____________________

14. A parallelogram has three vertices $(0, 0)$, (b, c), and $(d, 0)$. What are the possible coordinates of the fourth vertex?

14. ____________________

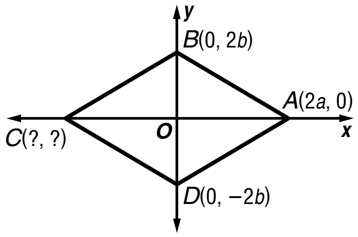
15. Determine whether $ABCD$ is a rectangle given $A(6, 2)$, $B(2, 10)$, $C(-6, 6)$, and $D(-2, -2)$. Justify your answer.

15. ____________________

16. Use the Distance Formula to determine whether $ABCD$ is a parallelogram given $A(1, 6)$, $B(7, 6)$, $C(2, -3)$, and $D(-4, -3)$.

16. ____________________

17. Name the missing coordinates for $ABCD$. Then determine the relationship between AC and the segment formed by connecting the midpoints of $\overline{BC}$ and $\overline{AB}$.

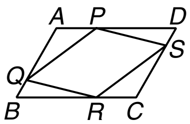


17. ____________________

For Questions 18 and 19, complete the two-column proof by supplying the missing information for each corresponding location.

Given: $ABCD$ is a parallelogram.
$\overline{BQ} \cong \overline{DS}$, $\overline{PA} \cong \overline{RC}$

Prove: $PQRS$ is a parallelogram.

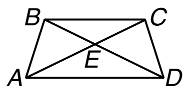


Statements	Reasons
1. $ABCD$ is a $\square$.	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. (Question 18)
3. $\overline{PA} \cong \overline{RC}$	3. Given
4. $\overline{PD} \cong \overline{RB}$	4. Seg. Sub. Prop.
5. $\overline{AB} \cong \overline{CD}$	5. Opp. sides of a $\square$ are $\cong$.
6. $\overline{BQ} \cong \overline{DS}$	6. Given
7. $\overline{AQ} \cong \overline{CS}$	7. Seg. Sub. Prop.
8. $\angle B \cong \angle D$, $\angle A \cong \angle C$	8. Opp. $\angle$s of a $\square$ are $\cong$.
9. $\triangle QBR \cong \triangle SDP$, $\triangle PAQ \cong \triangle RCS$	9. SAS
10. $\overline{QP} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$	10. CPCTC
11. $PQRS$ is a parallelogram.	11. (Question 19)

18. ____________________

19. ____________________

20. In isosceles trapezoid $ABCD$, $AE = 2x + 5$, $EC = 3x - 12$, and $BD = 4x + 20$. Find x.



20. ____________________

Bonus If three of the interior angles of a convex hexagon each measure 140, a fourth angle measures 84, and the measure of the fifth angle is 3 times the measure of the sixth angle, find the measure of the sixth angle.

B: ____________________

8 Chapter 8 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Draw a regular convex polygon and a convex polygon that is not regular, each with the same number of sides.

b. Label the measures of each exterior angle on your figures.

c. Find the sum of the exterior angles for each figure. What conjecture can be made?

2. Draw a rectangle. Connect the midpoints of the consecutive sides. What type of quadrilateral is formed? How do you know?

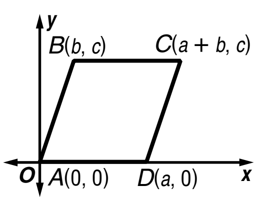
3. Draw an example to show why *one pair of opposite sides congruent and the other pair of opposite sides parallel* is not sufficient to form a parallelogram.

4. a. Name a property that is true for a square and not always true for a rectangle.

b. Name a property that is true for a square and not always true for a rhombus.

c. Name a property that is true for a rectangle and not always true for a parallelogram.

5. John drew this figure to use in a coordinate proof. Explain whether John's diagram could actually be used to prove that the diagonals of a rhombus are perpendicular.



Assessments

NAME ______________________________ DATE ____________ PERIOD _____

8 Chapter 8 Vocabulary Test/Review

SCORE _____

bases	kite	rectangle	square
diagonals	median	rhombus	trapezoid
isosceles trapezoid	parallelogram		

Choose from the terms above to complete each sentence.

1. A quadrilateral with only one pair of opposite sides parallel and the other pair of opposite sides congruent is a(n) __?__. **1.** ______________

2. A quadrilateral with two pairs of opposite sides parallel is a(n) __?__. **2.** ______________

3. A quadrilateral with only one pair of opposite sides parallel is a(n) __?__. **3.** ______________

4. A quadrilateral that is both a rectangle and a rhombus is a(n) __?__. **4.** ______________

5. A quadrilateral with four congruent sides is a(n) __?__. **5.** ______________

6. A quadrilateral with four right angles is a(n) __?__. **6.** ______________

7. A quadrilateral with two pairs of congruent consecutive sides is a(n) __?__. **7.** ______________

8. Segments that join opposite vertices in a quadrilateral are called __?__. **8.** ______________

9. The segment joining the midpoints of the nonparallel sides of a trapezoid is called the __?__. **9.** ______________

10. The parallel sides of a trapezoid are called the __?__. **10.** ______________

Define each term.

11. base angles of an isosceles trapezoid **11.** ______________

12. legs of a trapezoid **12.** ______________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Quiz

SCORE ______

(Lessons 8–1 and 8–2)

1. Find the sum of the measures of the interior angles of a convex 70-gon. **1.** ____________

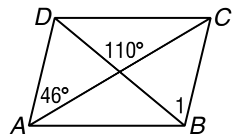
2. If the measure of each interior angle of a regular polygon is 172, find the number of sides of the polygon. **2.** ____________

3. If the measure of each exterior angle of a regular polygon is 18, find the number of sides of the polygon. **3.** ____________

4. Given parallelogram *ABCD* with *C*(5, 4), find the coordinates of *A* if the diagonals intersect at (2, 7). **4.** ____________

5. STANDARDIZED TEST PRACTICE Find $m\angle 1$ in parallelogram *ABCD*. **5.** ____________

A. 64 **B.** 58
C. 46 **D.** 36



Assessments

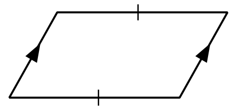
NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Quiz

SCORE ______

(Lessons 8–3 and 8–4)

1. Determine whether this quadrilateral is a parallelogram. Justify your answer. **1.** ____________



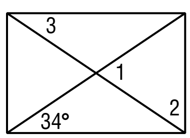
For Questions 2–4, write *true* or *false*.

2. A quadrilateral with two pairs of parallel sides is always a rectangle. **2.** ____________

3. The diagonals of a parallelogram are always perpendicular. **3.** ____________

4. The slope of $\overline{AB}$ and $\overline{CD}$ is $\frac{3}{5}$ and the slope of $\overline{BC}$ and $\overline{AD}$ is $-\frac{5}{3}$. *ABCD* is a parallelogram. **4.** ____________

5. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the rectangle at the right. **5.** ____________



NAME ______________________ DATE ____________ PERIOD ______

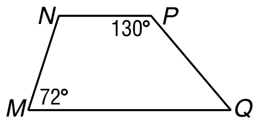
8 Chapter 8 Quiz

SCORE ______

(Lessons 8–5 and 8–6)

1. Given $M(4, 0)$, $N(0, 6)$, $P(-4, 0)$ and $Q(0, -6)$, determine whether $MNPQ$ is a *trapezoid*, a *parallelogram*, a *square*, a *rhombus*, or a *quadrilateral*. Choose the most specific term. Explain. **1.** ____________

For Questions 2 and 3, use trapezoid *MNPQ*.



2. Find $m\angle N$. **2.** ____________

3. Find $m\angle Q$. **3.** ____________

4. *True* or *false*. A quadrilateral that is a rectangle and a rhombus is a square. **4.** ____________

5. Find x if the bases of a trapezoid have lengths $2x + 4$ and $8x - 10$ and the length of the median is $3x + 21$. **5.** ____________

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Quiz

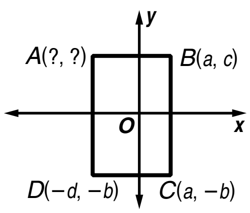
SCORE ______

(Lesson 8–7)

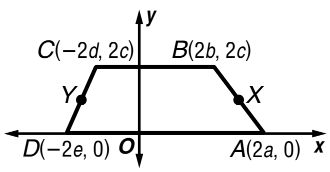
1. $ABCD$ is a rhombus with $A(a, 0)$, $B(0, b)$, and $D(0, -b)$ Find the possible coordinates of C. **1.** ____________

2. $ABCD$ is a square with $A(a, 0)$, $B(0, a)$, and $C(-a, 0)$. Find the possible coordinates of D. **2.** ____________

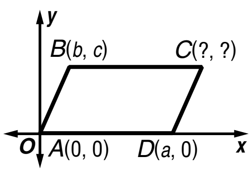
3. Given rectangle $ABCD$, find the coordinates of A. **3.** ____________



4. Name the coordinates of the endpoints of the median of trapezoid $ABCD$. **4.** ____________



5. Name the missing coordinates for parallelogram $ABCD$. Then determine the lengths of each of the four sides. **5.** ____________



NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Mid-Chapter Test

(Lessons 8–1 through 8–4)

SCORE ______

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

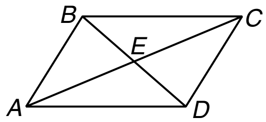
1. Find the measure of each exterior angle of a regular 56-gon. Round to the nearest tenth. **1.** ______

A. 3.2 **B.** 6.4 **C.** 173.6 **D.** 9720

2. Given $BE = 2x + 6$ and $ED = 5x - 12$ in parallelogram $ABCD$, find BD. **2.** ______

A. 6 **B.** 12

C. 18 **D.** 36



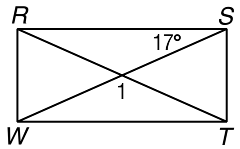
3. If the slope of $\overline{PQ}$ is $\frac{2}{3}$ and the slope of $\overline{RS}$ is $\frac{2}{3}$, find the slope of $\overline{QR}$ and $\overline{SP}$ so that $PQRS$ is a rectangle. **3.** ______

A. $-\frac{3}{2}$ **B.** $\frac{3}{2}$ **C.** $-\frac{2}{3}$ **D.** $\frac{2}{3}$

4. Find $m\angle 1$ in rectangle $RSTW$. **4.** ______

A. 163 **B.** 146

C. 34 **D.** 17

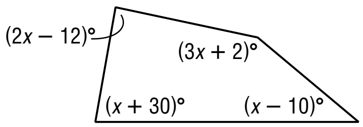


5. Find the sum of the measures of the interior angles of a convex 48-gon. **5.** ______

A. 172.5 **B.** 360 **C.** 8280 **D.** 8640

Part II

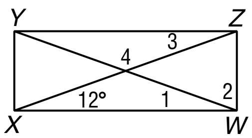
6. Find x. **6.** ____________



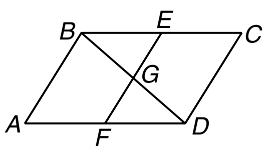
7. $ABCD$ is a parallelogram with $m\angle A = 138$. Find $m\angle B$. **7.** ____________

8. Determine whether $ABCD$ is a parallelogram if $AB = 6$, $BC = 12$, $CD = 6$, and $DA = 12$. **8.** ____________

9. In rectangle $XYZW$, find $m\angle 1$, $m\angle 2$, $m\angle 3$, and $m\angle 4$. **9.** ____________



10. In quadrilateral $ABCD$, $\overline{BC} \cong \overline{AD}$ and $\overline{BD}$ and $\overline{EF}$ bisect each other at G. By what theorem is $ABCD$ a parallelogram? **10.** ____________



Assessments

NAME ______________________ DATE ____________ PERIOD ______

8 Chapter 8 Cumulative Review

SCORE ______

(Chapters 1–8)

1. Point B lies on $\overleftrightarrow{RT}$ between points R and T. Name two opposite rays. (Lesson 1-4)

1. ____________________

2. Write an equation in slope-intercept form for the line that contains $(-12, 9)$ and is perpendicular to the line $y = \frac{2}{3}x + 5$. (Lesson 3-4)

2. ____________________

3. If $\triangle ABC \cong \triangle WXY$, $AB = 72$, $BC = 65$, $CA = 13$, $XY = 7x - 12$, and $WX = 19y + 34$, find x and y. (Lesson 4-3)

3. ____________________

4. Freda bought two bells for just over \$90 before tax. State the assumption you would make to write an indirect proof to show that at least one of the bells costs more than \$45. (Lesson 5-3)

4. ____________________

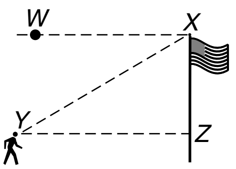
5. A plot of land has dimensions 82 feet, 132 feet, 75 feet, and 120 feet. A draftsman applies a scale of 2 inches = 8 feet to make a scale drawing of the land. Find the dimensions of the drawing. (Lesson 6-2)

5. ____________________

6. Triangle QRT has vertices $Q(1, 2)$, $R(5, -1)$, and $T(-2, -4)$, and $\overline{AB}$ is a midsegment parallel to $\overline{RT}$. Find the coordinates of A and B. (Lesson 6-4)

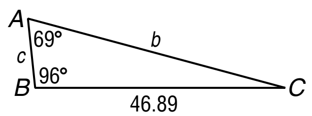
6. ____________________

7. Name the angles of elevation and depression. (Lesson 7-5)



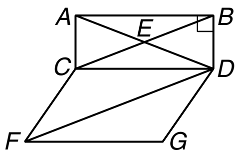
7. ____________________

8. Use the Law of Sines to find c to the nearest whole number. (Lesson 7-6)



8. ____________________

For Questions 10 and 11, use rectangle *ABDC* and parallelogram *CDGF*.



9. If $m\angle CDF = 21$, $m\angle FDG = 34$, $CF = 3x - 8$, and $DG = 7$, find $m\angle GFC$, $m\angle FCD$, and x. (Lesson 8-2)

9. ____________________

10. Find AE if $AD = 12y - 7$ and $BC = 21 - 2y$. (Lesson 8-4)

10. ____________________

11. Quadrilateral $LMNP$ has vertices $L(3, 0)$, $M(7, -3)$, $N(-7, -9)$, and $P(-4, -3)$. Verify that $LMNP$ is a trapezoid. Explain. (Lesson 8-6)

11. ____________________

NAME ____________________ DATE __________ PERIOD ______

8 Standardized Test Practice

(Chapters 1–8)

SCORE ______

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Find the coordinates of X if $V(0.5, 5)$ is the midpoint of $\overline{UX}$ with $U(15, -10)$. (Lesson 1-3)

A. $(-14, 20)$ **B.** $(7.75, -2.5)$
C. $(0, 0)$ **D.** $(15.5, -5)$

1. Ⓐ Ⓑ Ⓒ Ⓓ

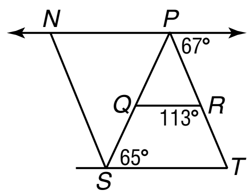
2. Which of the following are possible measures for vertical angles G and H? (Lesson 2-8)

E. $m\angle G = 125$ and $m\angle H = 55$
F. $m\angle G = 125$ and $m\angle H = 125$
G. $m\angle G = 55$ and $m\angle H = 45$
H. $m\angle G = 55$ and $m\angle H = 152.5$

2. Ⓔ Ⓕ Ⓖ Ⓗ

3. Determine which lines are parallel. (Lesson 3-5)

A. $\overleftrightarrow{NS} \parallel \overleftrightarrow{PT}$ **B.** $\overleftrightarrow{NP} \parallel \overleftrightarrow{ST}$
C. $\overleftrightarrow{QR} \parallel \overleftrightarrow{ST}$ **D.** $\overleftrightarrow{NP} \parallel \overleftrightarrow{QR}$



3. Ⓐ Ⓑ Ⓒ Ⓓ

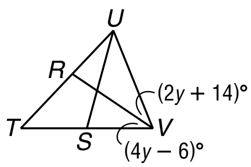
4. Find the coordinates of B, the midpoint of $\overline{AC}$, if $A(2a, b)$ and $C(0, -b)$. (Lesson 4-7)

E. $(2a, -b)$ **F.** (a, b) **G.** $(a, 0)$ **H.** $(0, b)$

4. Ⓔ Ⓕ Ⓖ Ⓗ

5. If $\overline{RV}$ is an angle bisector, find $m\angle UVT$. (Lesson 5-1)

A. 10 **B.** 34
C. 68 **D.** 136



5. Ⓐ Ⓑ Ⓒ Ⓓ

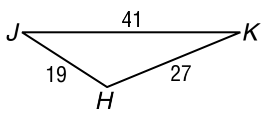
6. Which is not a valid postulate or theorem for proving two triangles similar? (Lesson 6-3)

E. SSA Theorem **F.** SAS Theorem
G. SSS Theorem **H.** AA Postulate

6. Ⓔ Ⓕ Ⓖ Ⓗ

7. Find $m\angle K$ to the nearest tenth. (Lesson 7-7)

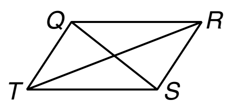
A. 22.3 **B.** 32.6
C. 54.8 **D.** 125.2



7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Which statement ensures that quadrilateral $QRST$ is a parallelogram? (Lesson 8-3)

E. $\angle Q \cong \angle S$ **F.** $\overline{QR} \cong \overline{TS}$ and $\overline{QR} \parallel \overline{TS}$
G. $\overline{QT} \parallel \overline{RS}$ **H.** $m\angle Q + m\angle S = 180$



8. Ⓔ Ⓕ Ⓖ Ⓗ

Assessments

NAME ______________________ DATE __________ PERIOD ______

8 Standardized Test Practice *(continued)*

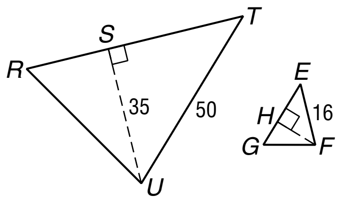
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

9. $\overleftrightarrow{AB} \parallel \overleftrightarrow{JK}$. Find the slope of $\overleftrightarrow{AB}$ if $J(14, 8)$ and $K(-7, 5)$. (Lesson 3-3)

10. If $\triangle UVW$ is an isosceles triangle, $\overline{UV} \cong \overline{WU}$, $UV = 16b - 40$, $VW = 6b$, and $WU = 10b + 2$, find b. (Lesson 4-1)

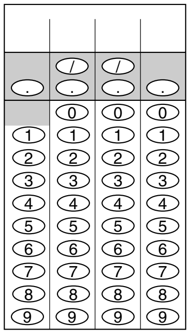
11. If $\triangle RTU \sim \triangle GEF$ and $\overline{SU}$ and $\overline{FH}$ are altitudes of $\triangle RTU$ and $\triangle GEF$ respectively, find FH. (Lesson 6-5)



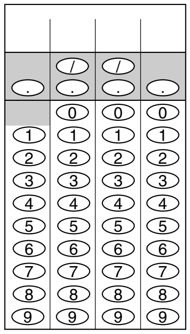
12. Find the geometric mean between 25 and 4. (Lesson 7-1)

13. Find the sum of the measures of the interior angles for a convex heptagon. (Lesson 8-1)

9.



11.



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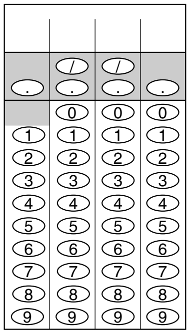
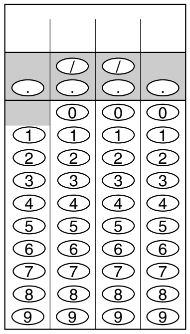
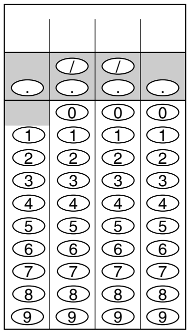


12.



13.



Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

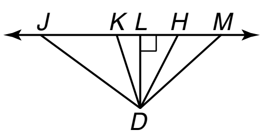
14. A polygon has six congruent sides. Lines containing two of its sides contain points in its interior. Name the polygon by its number of sides, and then classify it as *convex* or *concave* and *regular* or *irregular*. (Lesson 1-6)

14. ______________

15. If $\overline{RT} \cong \overline{QM}$ and $RT = 88.9$ centimeters, find QM. (Lesson 2-7)

15. ______________

16. Which segment is the shortest segment from D to $\overleftrightarrow{JM}$? (Lesson 5-4)



16. ______________

8 Standardized Test Practice

Student Record Sheet *(Use with pages 458–459 of the Student Edition.)*

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

2 (A) (B) (C) (D)

3 (A) (B) (C) (D)

4 (A) (B) (C) (D)

5 (A) (B) (C) (D)

6 (A) (B) (C) (D)

7 (A) (B) (C) (D)

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Question 11, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

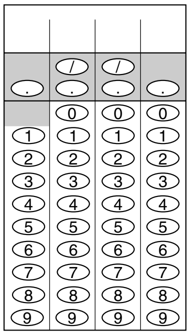
8 ______________

9 ______________

10 ______________

11 ______________ (grid in)

11



Part 3 Open-Ended

Record your answers for Questions 12–13 on the back of this paper.

Answers

NAME ______________________ DATE __________ PERIOD _____

8-1 Study Guide and Intervention

Angles of Polygons

Sum of Measures of Interior Angles The segments that connect the nonconsecutive sides of a polygon are called **diagonals**. Drawing all of the diagonals from one vertex of an ***n*-gon** separates the polygon into $n - 2$ triangles. The sum of the measures of the interior angles of the polygon can be found by adding the measures of the interior angles of those $n - 2$ triangles.

Interior Angle Sum Theorem	If a convex polygon has n sides, and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.

Example 1 **A convex polygon has 13 sides. Find the sum of the measures of the interior angles.**

$S = 180(n - 2)$
$= 180(13 - 2)$
$= 180(11)$
$= 1980$

Example 2 **The measure of an interior angle of a regular polygon is 120. Find the number of sides.**

The number of sides is n, so the sum of the measures of the interior angles is $120n$.

$S = 180(n - 2)$
$120n = 180(n - 2)$
$120n = 180n - 360$
$-60n = -360$
$n = 6$

Exercises

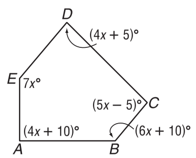
Find the sum of the measures of the interior angles of each convex polygon.

1. 10-gon **1440**
2. 16-gon **2520**
3. 30-gon **5040**
4. 8-gon **1080**
5. 12-gon **1800**
6. $3x$-gon **$180(3x - 2)$**

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

7. 150 **12**
8. 160 **18**
9. 175 **72**
10. 165 **24**
11. 168.75 **32**
12. 135 **8**

13. Find x. **20**



Lesson 8-1

NAME ______________________ DATE __________ PERIOD _____

8-1 Study Guide and Intervention *(continued)*

Angles of Polygons

Sum of Measures of Exterior Angles There is a simple relationship among the exterior angles of a convex polygon.

Exterior Angle Sum Theorem	If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

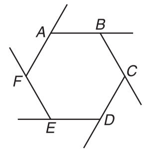
Example 1 **Find the sum of the measures of the exterior angles, one at each vertex, of a convex 27-gon.**

For *any* convex polygon, the sum of the measures of its exterior angles, one at each vertex, is 360.

Example 2 **Find the measure of each exterior angle of regular hexagon *ABCDEF*.**

The sum of the measures of the exterior angles is 360 and a hexagon has 6 angles. If n is the measure of each exterior angle, then

$6n = 360$
$n = 60$



Exercises

Find the sum of the measures of the exterior angles of each convex polygon.

1. 10-gon **360**
2. 16-gon **360**
3. 36-gon **360**

Find the measure of an exterior angle for each convex regular polygon.

4. 12-gon **30**
5. 36-gon **10**
6. $2x$-gon **$\frac{180}{x}$**

Find the measure of an exterior angle given the number of sides of a regular polygon.

7. 40 **9**
8. 18 **20**
9. 12 **30**
10. 24 **15**
11. 180 **2**
12. 8 **45**

NAME ______________________ DATE __________ PERIOD _____

8-1 Skills Practice

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. nonagon
1260

2. heptagon
900

3. decagon
1440

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

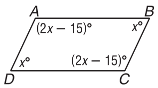
4. 108
5

5. 120
6

6. 150
12

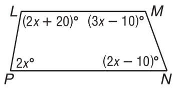
Find the measure of each interior angle using the given information.

7.



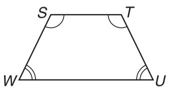
$m\angle A = 115$, $m\angle B = 65$, $m\angle C = 115$, $m\angle D = 65$

8.



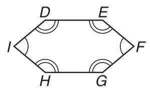
$m\angle L = 100$, $m\angle M = 110$, $m\angle N = 70$, $m\angle P = 80$

9. quadrilateral $STUW$ with $\angle S \cong \angle T$, $\angle U \cong \angle W$, $m\angle S = 2x + 16$, $m\angle U = x + 14$



$m\angle S = 116$, $m\angle T = 116$, $m\angle U = 64$, $m\angle W = 64$

10. hexagon $DEFGHI$ with $\angle D \cong \angle E \cong \angle G \cong \angle H$, $\angle F \cong \angle I$, $m\angle D = 7x$, $m\angle F = 4x$



$m\angle D = 140$, $m\angle E = 140$, $m\angle F = 80$, $m\angle G = 140$, $m\angle H = 140$, $m\angle I = 80$

Find the measures of an interior angle and an exterior angle for each regular polygon.

11. quadrilateral
90, 90

12. pentagon
108, 72

13. dodecagon
150, 30

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

14. 8
135, 45

15. 9
140, 40

16. 13
152.3, 27.7

Lesson 8-1

NAME ______________________ DATE __________ PERIOD _____

8-1 Practice (Average)

Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon
1620

2. 14-gon
2160

3. 17-gon
2700

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

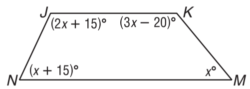
4. 144
10

5. 156
15

6. 160
18

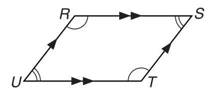
Find the measure of each interior angle using the given information.

7.



$m\angle J = 115$, $m\angle K = 130$, $m\angle M = 50$, $m\angle N = 65$

8. quadrilateral $RSTU$ with $m\angle R = 6x - 4$, $m\angle S = 2x + 8$



$m\angle R = 128$, $m\angle S = 52$, $m\angle T = 128$, $m\angle U = 52$

Find the measures of an interior angle and an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

9. 16-gon
157.5, 22.5

10. 24-gon
165, 15

11. 30-gon
168, 12

Find the measures of an interior angle and an exterior angle given the number of sides of each regular polygon. Round to the nearest tenth if necessary.

12. 14
154.3, 25.7

13. 22
163.6, 16.4

14. 40
171, 9

15. CRYSTALLOGRAPHY Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.
360

Answers

NAME ______________________ DATE ________ PERIOD _____

8-1 Reading to Learn Mathematics

Angles of Polygons

Pre-Activity **How does a scallop shell illustrate the angles of polygons?**

Read the introduction to Lesson 8-1 at the top of page 404 in your textbook.

- How many diagonals of the scallop shell shown in your textbook can be drawn from vertex *A*? **9**
- How many diagonals can be drawn from one vertex of an *n*-gon? Explain your reasoning. **$n - 3$; Sample answer: An *n*-gon has *n* vertices, but diagonals cannot be drawn from a vertex to itself or to either of the two adjacent vertices. Therefore, the number of diagonals from one vertex is 3 less than the number of vertices.**

Reading the Lesson

1. Write an expression that describes each of the following quantities for a regular *n*-gon. If the expression applies to regular polygons only, write *regular*. If it applies to all convex polygons, write *all*.

a. the sum of the measures of the interior angles **$180(n - 2)$; all**

b. the measure of each interior angle **$\frac{180(n-2)}{n}$; regular**

c. the sum of the measures of the exterior angles (one at each vertex) **360; all**

d. the measure of each exterior angle **$\frac{360}{n}$; regular**

2. Give the measure of an interior angle and the measure of an exterior angle of each polygon.

a. equilateral triangle **60; 120**

b. regular hexagon **120; 60**

c. square **90; 90**

d. regular octagon **135; 45**

3. Underline the correct word or phrase to form a true statement about regular polygons.

a. As the number of sides increases, the sum of the measures of the interior angles (increases/decreases/stays the same).

b. As the number of sides increases, the measure of each interior angle (increases/decreases/stays the same).

c. As the number of sides increases, the sum of the measures of the exterior angles (increases/decreases/stays the same).

d. As the number of sides increases, the measure of each exterior angle (increases/decreases/stays the same).

e. If a regular polygon has more than four sides, each interior angle will be a(n) (acute/right/obtuse) angle, and each exterior angle will be a(n) (acute/right/obtuse) angle.

Helping You Remember

4. A good way to remember a new mathematical idea or formula is to relate it to something you already know. How can you use your knowledge of the Angle Sum Theorem (for a triangle) to help you remember the Interior Angle Sum Theorem? **Sample answer: The sum of the measures of the (interior) angles of a triangle is 180. Each time another side is added, the polygon can be subdivided into one more triangle, so 180 is added to the interior angle sum.**

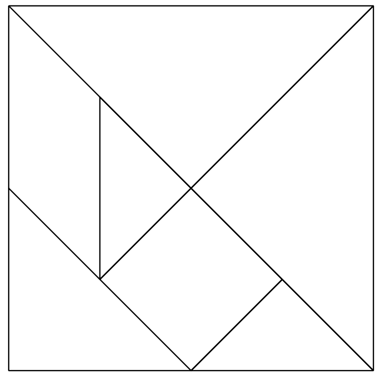
Lesson 8-1

NAME ______________________ DATE ________ PERIOD _____

8-1 Enrichment

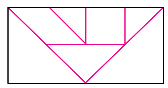
Tangrams

The tangram puzzle is composed of seven pieces that form a square, as shown at the right. This puzzle has been a popular amusement for Chinese students for hundreds and perhaps thousands of years.

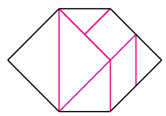


Make a careful tracing of the figure above. Cut out the pieces and rearrange them to form each figure below. Record each answer by drawing lines within each figure.

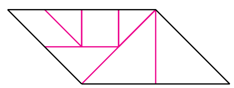
1.



2.



3.



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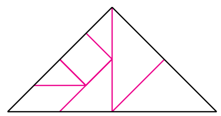
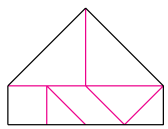
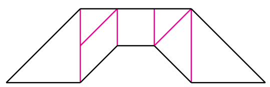


5.



6.



7. Create a different figure using the seven tangram pieces. Trace the outline. Then challenge another student to solve the puzzle. **See students' work.**

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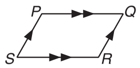
8-2 Study Guide and Intervention

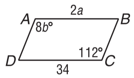
Parallelograms

Sides and Angles of Parallelograms A quadrilateral with both pairs of opposite sides parallel is a **parallelogram**. Here are four important properties of parallelograms.



	If *PQRS* is a parallelogram, then
The opposite sides of a parallelogram are congruent.	$\overline{PQ} \cong \overline{SR}$ and $\overline{PS} \cong \overline{QR}$
The opposite angles of a parallelogram are congruent.	$\angle P \cong \angle R$ and $\angle S \cong \angle Q$
The consecutive angles of a parallelogram are supplementary.	$\angle P$ and $\angle S$ are supplementary; $\angle S$ and $\angle R$ are supplementary; $\angle R$ and $\angle Q$ are supplementary; $\angle Q$ and $\angle P$ are supplementary.
If a parallelogram has one right angle, then it has four right angles.	If $m\angle P = 90$, then $m\angle Q = 90$, $m\angle R = 90$, and $m\angle S = 90$.

Example **If *ABCD* is a parallelogram, find *a* and *b*.**



$\overline{AB}$ and $\overline{CD}$ are opposite sides, so $\overline{AB} \cong \overline{CD}$.

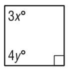
$2a = 34$
$a = 17$

$\angle A$ and $\angle C$ are opposite angles, so $\angle A \cong \angle C$.

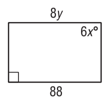
$8b = 112$
$b = 14$

Exercises

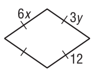
Find *x* and *y* in each parallelogram.

1. 

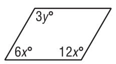
$x = 30$; $y = 22.5$

2. 

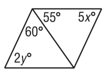
$x = 15$; $y = 11$

3. 

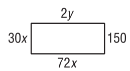
$x = 2$; $y = 4$

4. 

$x = 10$; $y = 40$

5. 

$x = 13$; $y = 32.5$

6. 

$x = 5$; $y = 180$

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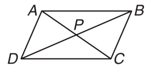
8-2 Study Guide and Intervention *(continued)*

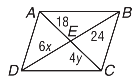
Parallelograms

Diagonals of Parallelograms Two important properties of parallelograms deal with their diagonals.



	If *ABCD* is a parallelogram, then:
The diagonals of a parallelogram bisect each other.	$AP = PC$ and $DP = PB$
Each diagonal separates a parallelogram into two congruent triangles.	$\triangle ACD \cong \triangle CAB$ and $\triangle ADB \cong \triangle CBD$

Example **Find *x* and *y* in parallelogram *ABCD*.**

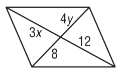


The diagonals bisect each other, so $AE = CE$ and $DE = BE$.

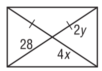
$6x = 24$ $\quad 4y = 18$
$x = 4$ $\quad y = 4.5$

Exercises

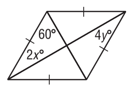
Find *x* and *y* in each parallelogram.

1. 

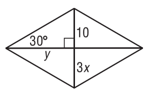
$x = 4$; $y = 2$

2. 

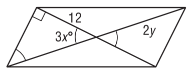
$x = 7$; $y = 14$

3. 

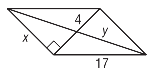
$x = 15$; $y = 7.5$

4. 

$x = 3\frac{1}{3}$; $y = 10\sqrt{3}$

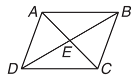
5. 

$x = 15$; $y = 6\sqrt{2}$

6. 

$x = 15$; $y = \sqrt{241}$

Complete each statement about ▱*ABCD*. Justify your answer.



7. $\angle BAC \cong$ **$\angle ACD$**
If lines are ∥, then alt. int. ∠s are ≅.

8. $\overline{DE} \cong$ **$\overline{BE}$**
The diags. of a parallelogram bisect each other.

9. $\triangle ADC \cong$ **$\triangle CBA$**
The diagonal of a parallelogram divides the parallelogram into 2 ≅ △s.

10. $\overline{AD} \parallel$ **$\overline{CB}$**
Opposite sides of a parallelogram are ∥.

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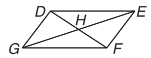
8-2 Skills Practice

Parallelograms

Complete each statement about ▱*DEFG*. Justify your answer.



1. $\overline{DG} \parallel$ _?_ **$\overline{EF}$; opp. sides of ▱ are ∥.**

2. $\overline{DE} \cong$ _?_ **$\overline{GF}$; opp. sides of ▱ are ≅.**

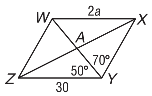
3. $\overline{GH} \cong$ _?_ **$\overline{EH}$; diag. of ▱ bisect each other.**

4. $\angle DEF \cong$ _?_ **∠*FGD*; opp. ∠s of ▱ are ≅.**

5. ∠*EFG* is supplementary to _?_. **∠*DEF* or ∠*FGD*; cons. ∠s in ▱ are suppl.**

6. △*DGE* ≅ _?_ **△*FEG*; diag. of ▱ separates ▱ into 2 ≅ △s.**

ALGEBRA **Use ▱*WXYZ* to find each measure or value.**



7. $m\angle XYZ =$ **120**

8. $m\angle WZY =$ **60**

9. $m\angle WXY =$ **60**

10. $a =$ **15**

COORDINATE GEOMETRY **Find the coordinates of the intersection of the diagonals of parallelogram *HJKL* given each set of vertices.**

11. $H(1, 1), J(2, 3), K(6, 3), L(5, 1)$
(3.5, 2)

12. $H(-1, 4), J(3, 3), K(3, -2), L(-1, -1)$
(1, 1)

13. **PROOF** Write a paragraph proof of the theorem *Consecutive angles in a parallelogram are supplementary.*

Given: ▱*ABCD*
Prove: ∠*A* and ∠*B* are supplementary.
∠*B* and ∠*C* are supplementary.
∠*C* and ∠*D* are supplementary.
∠*D* and ∠*A* are supplementary.

Proof: We are given ▱*ABCD*, so we know that $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DA}$ by the definition of a parallelogram. We also know that if two parallel lines are cut by a transversal, then consecutive interior angles are supplementary. So, ∠*A* and ∠*B*, ∠*B* and ∠*C*, ∠*C* and ∠*D*, and ∠*D* and ∠*A* are pairs of supplementary angles.

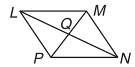
Lesson 8-2

NAME ______________ DATE ________ PERIOD _____

8-2 Practice (Average)

Parallelograms

Complete each statement about ▱*LMNP*. Justify your answer.



1. $\overline{LQ} \cong$ _?_ **$\overline{NQ}$; diag. of ▱ bisect each other.**

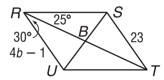
2. $\angle LMN \cong$ _?_ **∠*NPL*; opp. ∠s of ▱ are ≅.**

3. △*LMP* ≅ _?_ **△*NPM*; diag. of ▱ separates ▱ into 2 ≅ △s.**

4. ∠*NPL* is supplementary to _?_. **∠*MNP* or ∠*PLM*; cons. ∠s in ▱ are suppl.**

5. $\overline{LM} \cong$ _?_ **$\overline{NP}$; opp. sides of ▱ are ≅.**

ALGEBRA **Use ▱*RSTU* to find each measure or value.**



6. $m\angle RST =$ **125**

7. $m\angle STU =$ **55**

8. $m\angle TUR =$ **125**

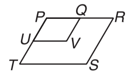
9. $b =$ **6**

COORDINATE GEOMETRY **Find the coordinates of the intersection of the diagonals of parallelogram *PRYZ* given each set of vertices.**

10. $P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1)$
(0, 1)

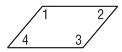
11. $P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2)$
(−1.5, −2)

12. **PROOF** Write a paragraph proof of the following.
Given: ▱*PRST* and ▱*PQVU*
Prove: ∠*V* ≅ ∠*S*



Proof: We are given ▱*PRST* and ▱*PQVU*. Since opposite angles of a parallelogram are congruent, ∠*P* ≅ ∠*V* and ∠*P* ≅ ∠*S*. Since congruence of angles is transitive, ∠*V* ≅ ∠*S* by the Transitive Property of Congruence.

13. **CONSTRUCTION** Mr. Rodriquez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If $m\angle 1$ is 130, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.
50, 130, 50



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8-2 Reading to Learn Mathematics

Parallelograms

Pre-Activity **How are parallelograms used to represent data?**

Read the introduction to Lesson 8-2 at the top of page 411 in your textbook.

- What is the name of the shape of the top surface of each wedge of cheese? **parallelogram**
- Are the three polygons shown in the drawing similar polygons? Explain your reasoning. **No; sample answer: Their sides are not proportional.**

Reading the Lesson

1. Underline words or phrases that can complete the following sentences to make statements that are always true. (There may be more than one correct choice for some of the sentences.)

a. Opposite sides of a parallelogram are (congruent/perpendicular/parallel).

b. Consecutive angles of a parallelogram are (complementary/supplementary/congruent).

c. A diagonal of a parallelogram divides the parallelogram into two (acute/right/obtuse/congruent) triangles.

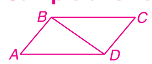
d. Opposite angles of a parallelogram are (complementary/supplementary/congruent).

e. The diagonals of a parallelogram (bisect each other/are perpendicular/are congruent).

f. If a parallelogram has one right angle, then all of its other angles are (acute/right/obtuse) angles.

2. Let $ABCD$ be a parallelogram with $AB \neq BC$ and with no right angles.

a. Sketch a parallelogram that matches the description above and draw diagonal $\overline{BD}$. **Sample answer:**



In parts **b–f**, complete each sentence.

b. $\overline{AB} \parallel$ **$\overline{CD}$** and $\overline{AD} \parallel$ **$\overline{BC}$**.

c. $\overline{AB} \cong$ **$\overline{CD}$** and $\overline{BC} \cong$ **$\overline{AD}$**.

d. $\angle A \cong$ **$\angle C$** and $\angle ABC \cong$ **$\angle CDA$**.

e. $\angle ADB \cong$ **$\angle CBD$** because these two angles are **alternate** **interior** angles formed by the two parallel lines **$\overleftrightarrow{AD}$** and **$\overleftrightarrow{BC}$** and the transversal **$\overleftrightarrow{BD}$**.

f. $\triangle ABD \cong$ **$\triangle CDB$**.

Helping You Remember

3. A good way to remember new theorems in geometry is to relate them to theorems you learned earlier. Name a theorem about parallel lines that can be used to remember the theorem that says, "If a parallelogram has one right angle, it has four right angles." **Perpendicular Transversal Theorem**

Lesson 8-2

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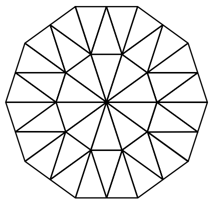
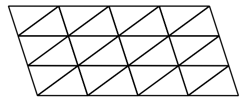
8-2 Enrichment

Tessellations

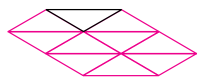
A **tessellation** is a tiling pattern made of polygons. The pattern can be extended so that the polygonal tiles cover the plane completely with no gaps. A checkerboard and a honeycomb pattern are examples of tessellations. Sometimes the same polygon can make more than one tessellation pattern. Both patterns below can be formed from an isosceles triangle.



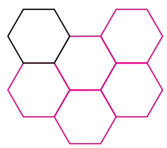


Draw a tessellation using each polygon.

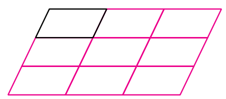
1.



2.

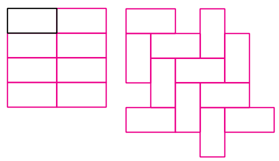


3.

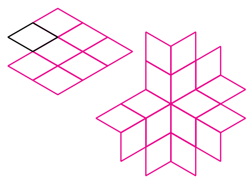


Draw two different tessellations using each polygon.

4.



5.



Answers

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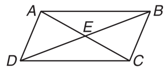
8-3 Study Guide and Intervention

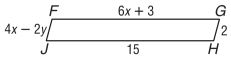
Tests for Parallelograms

Conditions for a Parallelogram There are many ways to establish that a quadrilateral is a parallelogram.



If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	**then:** *ABCD* is a parallelogram.

Example **Find *x* and *y* so that *FGHJ* is a parallelogram.**

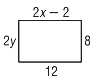


FGHJ is a parallelogram if the lengths of the opposite sides are equal.

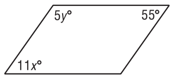
$6x + 3 = 15$ $\quad$ $4x - 2y = 2$
$6x = 12$ $\quad$ $4(2) - 2y = 2$
$x = 2$ $\quad$ $8 - 2y = 2$
$-2y = -6$
$y = 3$

Exercises

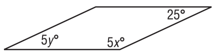
Find *x* and *y* so that each quadrilateral is a parallelogram.

1.


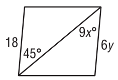
$x = 7$; $y = 4$

2.


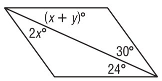
$x = 5$; $y = 25$

3.


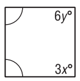
$x = 31$; $y = 5$

4.


$x = 5$; $y = 3$

5.


$x = 15$; $y = 9$

6.


$x = 30$; $y = 15$

Lesson 8-3

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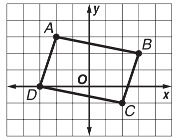
8-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance Formula and the Slope Formula can be used to test if a quadrilateral is a parallelogram.

Example **Determine whether *ABCD* is a parallelogram.**



The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.

Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

slope of $\overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3$ $\quad$ slope of $\overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$

slope of $\overline{AB} = \frac{2 - 3}{3 - (-2)} = -\frac{1}{5}$ $\quad$ slope of $\overline{CD} = \frac{-1 - 0}{2 - (-3)} = -\frac{1}{5}$

Opposite sides have the same slope, so $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Both pairs of opposite sides are parallel, so *ABCD* is a parallelogram.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1}$ or $\sqrt{26}$

$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9}$ or $\sqrt{10}$

Both pairs of opposite sides have the same length, so *ABCD* is a parallelogram.

Exercises

Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$; Slope Formula
yes

2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$; Slope Formula
yes

3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$; Distance Formula
no

4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$; Distance and Slope Formulas
yes

5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$; Distance and Slope Formulas
yes

6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$; Midpoint Formula
no

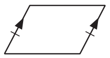
7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.
(4, −1), (0, 3), or (−4, −5)

NAME ______________________ DATE ____________ PERIOD _____

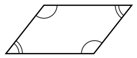
8-3 Skills Practice

Tests for Parallelograms

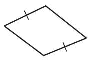
Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

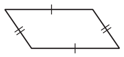
Yes; a pair of opposite sides is parallel and congruent.

2. 

Yes; both pairs of opposite angles are congruent.

3. 

No; none of the tests for parallelograms is fulfilled.

4. 

Yes; both pairs of opposite sides are congruent.

COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

5. $P(0, 0)$, $Q(3, 4)$, $S(7, 4)$, $Y(4, 0)$; Slope Formula

yes

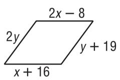
6. $S(-2, 1)$, $R(1, 3)$, $T(2, 0)$, $Z(-1, -2)$; Distance and Slope Formula

yes

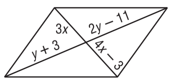
7. $W(2, 5)$, $R(3, 3)$, $Y(-2, -3)$, $N(-3, 1)$; Midpoint Formula

no

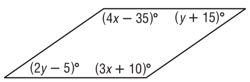
ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.

8. 

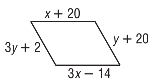
$x = 24$, $y = 19$

9. 

$x = 3$, $y = 14$

10. 

$x = 45$, $y = 20$

11. 

$x = 17$, $y = 9$

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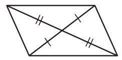
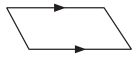
8-3 Practice (Average)

Tests for Parallelograms

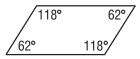
Determine whether each quadrilateral is a parallelogram. Justify your answer.

1. 

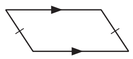
Yes; the diagonals bisect each other.

2. 

No; none of the tests for parallelograms is fulfilled.

3. 

Yes; both pairs of opposite angles are congruent.

4. 

No; none of the tests for parallelograms is fulfilled.

COORDINATE GEOMETRY Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

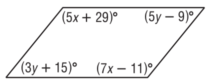
5. $P(-5, 1)$, $S(-2, 2)$, $F(-1, -3)$, $T(2, -2)$; Slope Formula

no

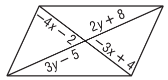
6. $R(-2, 5)$, $O(1, 3)$, $M(-3, -4)$, $Y(-6, -2)$; Distance and Slope Formula

yes

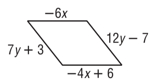
ALGEBRA Find *x* and *y* so that each quadrilateral is a parallelogram.

7. 

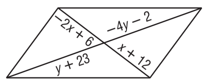
$x = 20$, $y = 12$

8. 

$x = -6$, $y = 13$

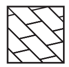
9. 

$x = -3$, $y = 2$

10. 

$x = -2$, $y = -5$

11. TILE DESIGN The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?



Sample answer: Confirm that both pairs of opposite ∠s are ≅.

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8-3 Reading to Learn Mathematics

Tests for Parallelograms

Pre-Activity **How are parallelograms used in architecture?**

Read the introduction to Lesson 8-3 at the top of page 417 in your textbook.

Make two observations about the angles in the roof of the covered bridge.
Sample answer: Opposite angles appear congruent. Consecutive angles appear supplementary.

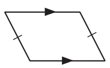
Reading the Lesson

1. Which of the following conditions guarantee that a quadrilateral is a parallelogram? **B, D, G, H, J, L**

- **A.** Two sides are parallel.
- **B.** Both pairs of opposite sides are congruent.
- **C.** The diagonals are perpendicular.
- **D.** A pair of opposite sides is both parallel and congruent.
- **E.** There are two right angles.
- **F.** The sum of the measures of the interior angles is 360.
- **G.** All four sides are congruent.
- **H.** Both pairs of opposite angles are congruent.
- **I.** Two angles are acute and the other two angles are obtuse.
- **J.** The diagonals bisect each other.
- **K.** The diagonals are congruent.
- **L.** All four angles are right angles.

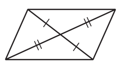
2. Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

a.



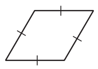
no

b.



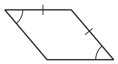
Yes; if the diagonals bisect each other, then the quadrilateral is a parallelogram.

c.



Yes; if both pairs of opposite sides are ≅, then quadrilateral is ▱.

d.



no

Helping You Remember

3. A good way to remember a large number of mathematical ideas is to think of them in groups. How can you state the conditions as one group about the *sides* of quadrilaterals that guarantee that the quadrilateral is a parallelogram? **Sample answer: both pairs of opposite sides parallel, both pairs of opposite sides congruent, or one pair of opposite sides both parallel and congruent**

Lesson 8-3

NAME ______________________ DATE ____________ PERIOD _____

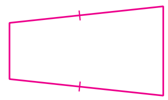
8-3 Enrichment

Tests for Parallelograms

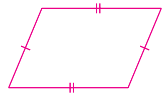
By definition, a quadrilateral is a parallelogram *if and only if* both pairs of opposite sides are parallel. What conditions other than both pairs of opposite sides parallel will guarantee that a quadrilateral is a parallelogram? In this activity, several possibilities will be investigated by drawing quadrilaterals to satisfy certain conditions. Remember that any test that seems to work is not guaranteed to work unless it can be formally proven.

Complete.

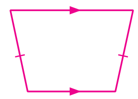
1. Draw a quadrilateral with one pair of opposite sides congruent. Must it be a parallelogram? **no**



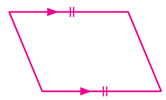
2. Draw a quadrilateral with both pairs of opposite sides congruent. Must it be a parallelogram? **yes**



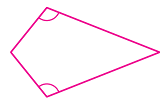
3. Draw a quadrilateral with one pair of opposite sides parallel and the other pair of opposite sides congruent. Must it be a parallelogram? **no**



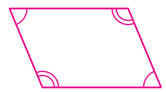
4. Draw a quadrilateral with one pair of opposite sides both parallel and congruent. Must it be a parallelogram? **yes**



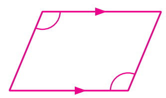
5. Draw a quadrilateral with one pair of opposite angles congruent. Must it be a parallelogram? **no**



6. Draw a quadrilateral with both pairs of opposite angles congruent. Must it be a parallelogram? **yes**



7. Draw a quadrilateral with one pair of opposite sides parallel and one pair of opposite angles congruent. Must it be a parallelogram? **yes**

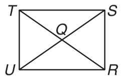


NAME ______________________ DATE __________ PERIOD _____

8-4 Study Guide and Intervention

Rectangles

Properties of Rectangles A **rectangle** is a quadrilateral with four right angles. Here are the properties of rectangles.



A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. $\angle UTS$, $\angle TSR$, $\angle SRU$, and $\angle RUT$ are right angles.
- The diagonals are congruent. $\overline{TR} \cong \overline{US}$

Example 1 **In rectangle *RSTU* above, $US = 6x + 3$ and $RT = 7x - 2$. Find *x*.**

The diagonals of a rectangle bisect each other, so $US = RT$.

$6x + 3 = 7x - 2$
$3 = x - 2$
$5 = x$

Example 2 **In rectangle *RSTU* above, $m\angle STR = 8x + 3$ and $m\angle UTR = 16x - 9$. Find $m\angle STR$.**

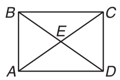
$\angle UTS$ is a right angle, so $m\angle STR + m\angle UTR = 90$.

$8x + 3 + 16x - 9 = 90$
$24x - 6 = 90$
$24x = 96$
$x = 4$

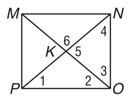
$m\angle STR = 8x + 3 = 8(4) + 3$ or 35

Exercises

***ABCD* is a rectangle.**



1. If $AE = 36$ and $CE = 2x - 4$, find *x*. **20**
2. If $BE = 6y + 2$ and $CE = 4y + 6$, find *y*. **2**
3. If $BC = 24$ and $AD = 5y - 1$, find *y*. **5**
4. If $m\angle BEA = 62$, find $m\angle BAC$. **59**
5. If $m\angle AED = 12x$ and $m\angle BEC = 10x + 20$, find $m\angle AED$. **120**
6. If $BD = 8y - 4$ and $AC = 7y + 3$, find *BD*. **52**
7. If $m\angle DBC = 10x$ and $m\angle ACB = 4x^2 - 6$, find $m\angle ACB$. **30**
8. If $AB = 6y$ and $BC = 8y$, find *BD* in terms of *y*. **10*y***
9. In rectangle *MNOP*, $m\angle 1 = 40$. Find the measure of each numbered angle.
   m∠2 = 40; m∠3 = 50; m∠4 = 50; m∠5 = 80; m∠6 = 100



Lesson 8-4

NAME ______________________ DATE __________ PERIOD _____

8-4 Study Guide and Intervention *(continued)*

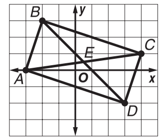
Rectangles

Prove that Parallelograms Are Rectangles The diagonals of a rectangle are congruent, and the converse is also true.

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

In the coordinate plane you can use the Distance Formula, the Slope Formula, and properties of diagonals to show that a figure is a rectangle.

Example **Determine whether $A(-3, 0)$, $B(-2, 3)$, $C(4, 1)$, and $D(3, -2)$ are the vertices of a rectangle.**



Method 1: Use the Slope Formula.

slope of $\overline{AB} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1}$ or 3 slope of $\overline{AD} = \frac{-2 - 0}{3 - (-3)} = \frac{-2}{6}$ or $-\frac{1}{3}$

slope of $\overline{CD} = \frac{-2 - 1}{3 - 4} = \frac{-3}{-1}$ or 3 slope of $\overline{BC} = \frac{1 - 3}{4 - (-2)} = \frac{-2}{6}$ or $-\frac{1}{3}$

Opposite sides are parallel, so the figure is a parallelogram. Consecutive sides are perpendicular, so *ABCD* is a rectangle.

Method 2: Use the Midpoint and Distance Formulas.

The midpoint of $\overline{AC}$ is $\left(\frac{-3 + 4}{2}, \frac{0 + 1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and the midpoint of $\overline{BD}$ is $\left(\frac{-2 + 3}{2}, \frac{3 - 2}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$. The diagonals have the same midpoint so they bisect each other. Thus, *ABCD* is a parallelogram.

$AC = \sqrt{(-3 - 4)^2 + (0 - 1)^2} = \sqrt{49 + 1}$ or $\sqrt{50}$

$BD = \sqrt{(-2 - 3)^2 + (3 - (-2))^2} = \sqrt{25 + 25}$ or $\sqrt{50}$

The diagonals are congruent. *ABCD* is a parallelogram with diagonals that bisect each other, so it is a rectangle.

Exercises

Determine whether *ABCD* is a rectangle given each set of vertices. Justify your answer. **Sample justifications are given.**

1. $A(-3, 1)$, $B(-3, 3)$, $C(3, 3)$, $D(3, 1)$
   Yes; consecutive sides are ⊥.
2. $A(-3, 0)$, $B(-2, 3)$, $C(4, 5)$, $D(3, 2)$
   No; consecutive sides are not ⊥.
3. $A(-3, 0)$, $B(-2, 2)$, $C(3, 0)$, $D(2, -2)$
   No; consecutive sides are not ⊥.
4. $A(-1, 0)$, $B(0, 2)$, $C(4, 0)$, $D(3, -2)$
   Yes; consecutive sides are ⊥.
5. $A(-1, -5)$, $B(-3, 0)$, $C(2, 2)$, $D(4, -3)$
   Yes; the diagonals are congruent and bisect each other.
6. $A(-1, -1)$, $B(0, 2)$, $C(4, 3)$, $D(3, 0)$
   No; the diagonals are not ≅.
7. A parallelogram has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(5, -2)$. Find the coordinates of *U* so that *RSTU* is a rectangle. **(3, −5)**

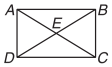
Answers

NAME ______________________ DATE __________ PERIOD _____

8-4 Skills Practice

Rectangles

ALGEBRA ***ABCD* is a rectangle.**



1. If $AC = 2x + 13$ and $DB = 4x - 1$, find x. **7**

2. If $AC = x + 3$ and $DB = 3x - 19$, find AC. **14**

3. If $AE = 3x + 3$ and $EC = 5x - 15$, find AC. **60**

4. If $DE = 6x - 7$ and $AE = 4x + 9$, find DB. **82**

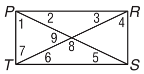
5. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find x. **17**

6. If $m\angle BDC = 7x + 1$ and $m\angle ADB = 9x - 7$, find $m\angle BDC$. **43**

7. If $m\angle ABD = x^2 - 7$ and $m\angle CDB = 4x + 5$, find x. **6**

8. If $m\angle BAC = x^2 + 3$ and $m\angle CAD = x + 15$, find $m\angle BAC$. **67 or 84**

***PRST* is a rectangle. Find each measure if $m\angle 1 = 50$.**



9. $m\angle 2$ **40**

10. $m\angle 3$ **40**

11. $m\angle 4$ **50**

12. $m\angle 5$ **40**

13. $m\angle 6$ **40**

14. $m\angle 7$ **50**

15. $m\angle 8$ **100**

16. $m\angle 9$ **80**

COORDINATE GEOMETRY **Determine whether *TUXY* is a rectangle given each set of vertices. Justify your answer.**

17. $T(-3, -2)$, $U(-4, 2)$, $X(2, 4)$, $Y(3, 0)$

No; sample answer: Angles are not right angles.

18. $T(-6, 3)$, $U(0, 6)$, $X(2, 2)$, $Y(-4, -1)$

Yes; sample answer: Opposite sides are congruent and diagonals are congruent.

19. $T(4, 1)$, $U(3, -1)$, $X(-3, 2)$, $Y(-2, 4)$

Yes; sample answer: Opposite sides are parallel and consecutive sides are perpendicular.

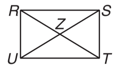
Lesson 8-4

NAME ______________________ DATE __________ PERIOD _____

8-4 Practice (Average)

Rectangles

ALGEBRA ***RSTU* is a rectangle.**



1. If $UZ = x + 21$ and $ZS = 3x - 15$, find US. **78**

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find UZ. **44**

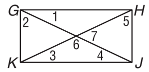
3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find ZT. **9**

4. If $m\angle SUT = 3x + 6$ and $m\angle RUS = 5x - 4$, find $m\angle SUT$. **39**

5. If $m\angle SRT = x^2 + 9$ and $m\angle UTR = 2x + 44$, find x. **−5 or 7**

6. If $m\angle RSU = x^2 - 1$ and $m\angle TUS = 3x + 9$, find $m\angle RSU$. **24 or 3**

***GHJK* is a rectangle. Find each measure if $m\angle 1 = 37$.**



7. $m\angle 2$ **53**

8. $m\angle 3$ **37**

9. $m\angle 4$ **37**

10. $m\angle 5$ **53**

11. $m\angle 6$ **106**

12. $m\angle 7$ **74**

COORDINATE GEOMETRY **Determine whether *BGHL* is a rectangle given each set of vertices. Justify your answer.**

13. $B(-4, 3)$, $G(-2, 4)$, $H(1, -2)$, $L(-1, -3)$

Yes; sample answer: Opposite sides are parallel and consecutive sides are perpendicular.

14. $B(-4, 5)$, $G(6, 0)$, $H(3, -6)$, $L(-7, -1)$

Yes; sample answer: Opposite sides are congruent and diagonals are congruent.

15. $B(0, 5)$, $G(4, 7)$, $H(5, 4)$, $L(1, 2)$

No; sample answer: Diagonals are not congruent.

16. LANDSCAPING Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

No; if you only know that opposite sides are congruent and parallel, the most you can conclude is that the plot is a parallelogram.

NAME ______________________ DATE __________ PERIOD _____

8-4 Reading to Learn Mathematics

Rectangles

Pre-Activity **How are rectangles used in tennis?**

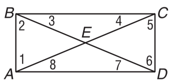
Read the introduction to Lesson 8-4 at the top of page 424 in your textbook.

Are the singles court and doubles court similar rectangles? Explain your answer. **No; sample answer: All of their angles are congruent, but their sides are not proportional. The doubles court is wider in relation to its length.**

Reading the Lesson

1. Determine whether each sentence is *always*, *sometimes*, or *never* true.
   - **a.** If a quadrilateral has four congruent angles, it is a rectangle. **always**
   - **b.** If consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a rectangle. **sometimes**
   - **c.** The diagonals of a rectangle bisect each other. **always**
   - **d.** If the diagonals of a quadrilateral bisect each other, the quadrilateral is a rectangle. **sometimes**
   - **e.** Consecutive angles of a rectangle are complementary. **never**
   - **f.** Consecutive angles of a rectangle are congruent. **always**
   - **g.** If the diagonals of a quadrilateral are congruent, the quadrilateral is a rectangle. **sometimes**
   - **h.** A diagonal of a rectangle bisects two of its angles. **sometimes**
   - **i.** A diagonal of a rectangle divides the rectangle into two congruent right triangles. **always**
   - **j.** If the diagonals of a quadrilateral bisect each other and are congruent, the quadrilateral is a rectangle. **always**
   - **k.** If a parallelogram has one right angle, it is a rectangle. **always**
   - **l.** If a parallelogram has four congruent sides, it is a rectangle. **sometimes**

2. *ABCD* is a rectangle with $AD > AB$. Name each of the following in this figure.



   - **a.** all segments that are congruent to $\overline{BE}$ **$\overline{AE}$, $\overline{CE}$, $\overline{DE}$**
   - **b.** all angles congruent to $\angle 1$ **$\angle 2$, $\angle 5$, $\angle 6$**
   - **c.** all angles congruent to $\angle 7$ **$\angle 3$, $\angle 4$, $\angle 8$**
   - **d.** two pairs of congruent triangles **$\triangle AED \cong \triangle BEC$, $\triangle AEB \cong \triangle DEC$, $\triangle BCD \cong \triangle DAB$, $\triangle ABC \cong \triangle CDA$**

Helping You Remember

3. It is easier to remember a large number of geometric relationships and theorems if you are able to combine some of them. How can you combine the two theorems about diagonals that you studied in this lesson? **Sample answer: A parallelogram is a rectangle if and only if its diagonals are congruent.**

Lesson 8-4

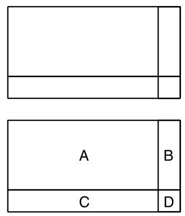
NAME ______________________ DATE __________ PERIOD _____

8-4 Enrichment

Counting Squares and Rectangles

Each puzzle below contains many squares and/or rectangles. Count them carefully. You may want to label each region so you can list all possibilities.

Example **How many rectangles are in the figure at the right?**

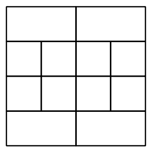


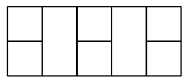
Label each small region with a letter. Then list each rectangle by writing the letters of regions it contains.

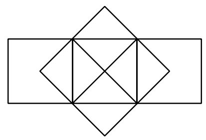
A, B, C, D, AB, CD, AC, BD, ABCD

There are 9 rectangles.

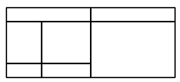
How many squares are in each figure?

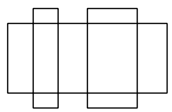
1.  **16**

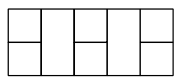
2.  **10**

3.  **8**

How many rectangles are in each figure?

1.  **19**

2.  **25**

3.  **21**

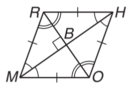
Answers

NAME ______________________ DATE __________ PERIOD _____

8-5 Study Guide and Intervention

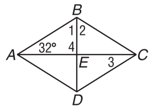
Rhombi and Squares

Properties of Rhombi A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.



The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.
If the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.	If *RHOM* is a parallelogram and $\overline{RO} \perp \overline{MH}$, then *RHOM* is a rhombus.

Example **In rhombus *ABCD*, $m\angle BAC = 32$. Find the measure of each numbered angle.**

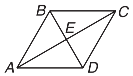


ABCD is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

***ABCD* is a rhombus.**



1. If $m\angle ABD = 60$, find $m\angle BDC$. **60**

2. If $AE = 8$, find AC. **16**

3. If $AB = 26$ and $BD = 20$, find AE. **24**

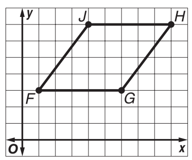
4. Find $m\angle CEB$. **90**

5. If $m\angle CBD = 58$, find $m\angle ACB$. **32**

6. If $AE = 3x - 1$ and $AC = 16$, find x. **3**

7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y. **10**

8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x. **4**



9. a. What is the midpoint of $\overline{FH}$? **(5, 5)**

b. What is the midpoint of $\overline{GJ}$? **(5, 5)**

c. What kind of figure is *FGHJ*? Explain.
***FGHJ* is a parallelogram because the diagonals bisect each other.**

d. What is the slope of $\overline{FH}$? $\frac{1}{2}$

e. What is the slope of $\overline{GJ}$? **−2**

f. Based on parts **c**, **d**, and **e**, what kind of figure is *FGHJ*? Explain.
***FGHJ* is a parallelogram with perpendicular diagonals, so it is a rhombus.**

Lesson 8-5

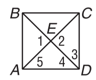
NAME ______________________ DATE __________ PERIOD _____

8-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Properties of Squares A square has all the properties of a rhombus and all the properties of a rectangle.

Example **Find the measure of each numbered angle of square *ABCD*.**



Using properties of rhombi and rectangles, the diagonals are perpendicular and congruent. $\triangle ABE$ is a right triangle, so $m\angle 1 = m\angle 2 = 90$.

Each vertex angle is a right angle and the diagonals bisect the vertex angles, so $m\angle 3 = m\angle 4 = m\angle 5 = 45$.

Exercises

Determine whether the given vertices represent a *parallelogram*, *rectangle*, *rhombus*, or *square*. Explain your reasoning.

1. $A(0, 2), B(2, 4), C(4, 2), D(2, 0)$
Square; the four sides are ≅ and consecutive sides are ⊥.

2. $D(-2, 1), E(-1, 3), F(3, 1), G(2, -1)$
Rectangle; both pairs of opposite sides are ∥ and consecutive sides are ⊥.

3. $A(-2, -1), B(0, 2), C(2, -1), D(0, -4)$
Rhombus; the four sides are ≅ and consecutive sides are not ⊥.

4. $A(-3, 0), B(-1, 3), C(5, -1), D(3, -4)$
Rectangle; both pairs of opposite sides are ∥ and consecutive sides are ⊥.

5. $S(-1, 4), T(3, 2), U(1, -2), V(-3, 0)$
Square; the four sides are ≅ and consecutive sides are ⊥.

6. $F(-1, 0), G(1, 3), H(4, 1), I(2, -2)$
Square; the four sides are ≅ and consecutive sides are ⊥.

7. Square *RSTU* has vertices $R(-3, -1)$, $S(-1, 2)$, and $T(2, 0)$. Find the coordinates of vertex *U*. **(0, −3)**

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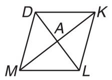
8-5 Skills Practice

Rhombi and Squares

Use rhombus *DKLM* with $AM = 4x$, $AK = 5x - 3$, and $DL = 10$.



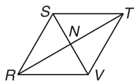
1. Find *x*.
3

2. Find *AL*.
5

3. Find $m\angle KAL$.
90

4. Find *DM*.
13

Use rhombus *RSTV* with $RS = 5y + 2$, $ST = 3y + 6$, and $NV = 6$.



5. Find *y*.
2

6. Find *TV*.
12

7. Find $m\angle NTV$.
30

8. Find $m\angle SVT$.
60

9. Find $m\angle RST$.
120

10. Find $m\angle SRV$.
60

COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*QRST* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.

11. $Q(3, 5)$, $R(3, 1)$, $S(-1, 1)$, $T(-1, 5)$
Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

12. $Q(-5, 12)$, $R(5, 12)$, $S(-1, 4)$, $T(-11, 4)$
Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

13. $Q(-6, -1)$, $R(4, -6)$, $S(2, 5)$, $T(-8, 10)$
Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

14. $Q(2, -4)$, $R(-6, -8)$, $S(-10, 2)$, $T(-2, 6)$
None; opposite sides are congruent, but the diagonals are neither congruent nor perpendicular.

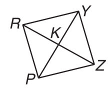
Lesson 8-5

NAME ______________________ DATE ___________ PERIOD _____

8-5 Practice (Average)

Rhombi and Squares

Use rhombus *PRYZ* with $RK = 4y + 1$, $ZK = 7y - 14$, $PK = 3x - 1$, and $YK = 2x + 6$.



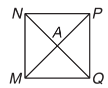
1. Find *PY*.
40

2. Find *RZ*.
42

3. Find *RY*.
29

4. Find $m\angle YKZ$.
90

Use rhombus *MNPQ* with $PQ = 3\sqrt{2}$, $PA = 4x - 1$, and $AM = 9x - 6$.



5. Find *AQ*.
3

6. Find $m\angle APQ$.
45

7. Find $m\angle MNP$.
90

8. Find *PM*.
6

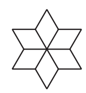
COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*BEFG* is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain your reasoning.

9. $B(-9, 1)$, $E(2, 3)$, $F(12, -2)$, $G(1, -4)$
Rhombus; all sides are congruent and the diagonals are perpendicular, but not congruent.

10. $B(1, 3)$, $E(7, -3)$, $F(1, -9)$, $G(-5, -3)$
Rhombus, rectangle, square; all sides are congruent and the diagonals are perpendicular and congruent.

11. $B(-4, -5)$, $E(1, -5)$, $F(-7, -1)$, $G(-2, -1)$
None; two of the opposite sides are not congruent.

12. TESSELATIONS The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.
The figure consists of 6 congruent rhombi.



Answers

NAME ______________________ DATE ________ PERIOD _____

8-5 Reading to Learn Mathematics

Rhombi and Squares

Pre-Activity **How can you ride a bicycle with square wheels?**

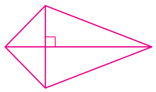
Read the introduction to Lesson 8-5 at the top of page 431 in your textbook.

If you draw a diagonal on the surface of one of the square wheels shown in the picture in your textbook, how can you describe the two triangles that are formed? **Sample answer: two congruent isosceles right triangles**

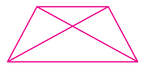
Reading the Lesson

1. Sketch each of the following. **Sample answers are given.**

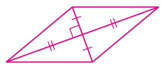
a. a quadrilateral with perpendicular diagonals that is not a rhombus



b. a quadrilateral with congruent diagonals that is not a rectangle



c. a quadrilateral whose diagonals are perpendicular and bisect each other, but are not congruent



2. List all of the following special quadrilaterals that have each listed property: *parallelogram*, *rectangle*, *rhombus*, *square*.

a. The diagonals are congruent. **rectangle, square**

b. Opposite sides are congruent. **parallelogram, rectangle, rhombus, square**

c. The diagonals are perpendicular. **rhombus, square**

d. Consecutive angles are supplementary. **parallelogram, rectangle, rhombus, square**

e. The quadrilateral is equilateral. **rhombus, square**

f. The quadrilateral is equiangular. **rectangle, square**

g. The diagonals are perpendicular and congruent. **square**

h. A pair of opposite sides is both parallel and congruent. **parallelogram, rectangle, rhombus, square**

3. What is the common name for a regular quadrilateral? Explain your answer. **Square; sample answer: All four sides of a square are congruent and all four angles are congruent.**

Helping You Remember

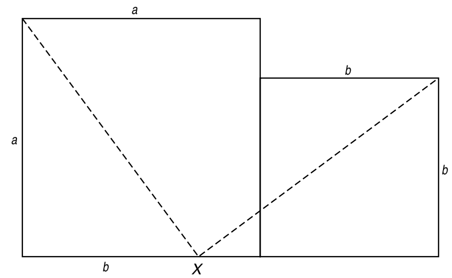
4. A good way to remember something is to explain it to someone else. Suppose that your classmate Luis is having trouble remembering which of the properties he has learned in this chapter apply to squares. How can you help him? **Sample answer: A square is a parallelogram that is both a rectangle and a rhombus, so all properties of parallelograms, rectangles, and rhombi apply to squares.**

NAME ______________________ DATE ________ PERIOD _____

8-5 Enrichment

Creating Pythagorean Puzzles

By drawing two squares and cutting them in a certain way, you can make a puzzle that demonstrates the Pythagorean Theorem. A sample puzzle is shown. You can create your own puzzle by following the instructions below.

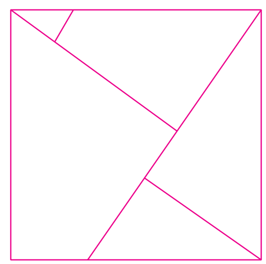


See students' work. A sample answer is shown.

1. Carefully construct a square and label the length of a side as a. Then construct a smaller square to the right of it and label the length of a side as b, as shown in the figure above. The bases should be adjacent and collinear.

2. Mark a point X that is b units from the left edge of the larger square. Then draw the segments from the upper left corner of the larger square to point X, and from point X to the upper right corner of the smaller square.

3. Cut out and rearrange your five pieces to form a larger square. Draw a diagram to show your answer.



4. Verify that the length of each side is equal to $\sqrt{a^2 + b^2}$.

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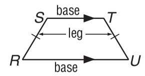
8-6 Study Guide and Intervention

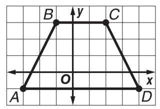
Trapezoids

Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases** and the nonparallel sides are called **legs**. If the legs are congruent, the trapezoid is an **isosceles trapezoid**. In an isosceles trapezoid both pairs of **base angles** are congruent.



STUR is an isosceles trapezoid. $\overline{SR} \cong \overline{TU}$; $\angle R \cong \angle U$, $\angle S \cong \angle T$

Example **The vertices of *ABCD* are $A(-3, -1)$, $B(-1, 3)$, $C(2, 3)$, and $D(4, -1)$. Verify that *ABCD* is a trapezoid.**



slope of $\overline{AB} = \frac{3 - (-1)}{-1 - (-3)} = \frac{4}{2} = 2$

slope of $\overline{AD} = \frac{-1 - (-1)}{4 - (-3)} = \frac{0}{7} = 0$

slope of $\overline{BC} = \frac{3 - 3}{2 - (-1)} = \frac{0}{3} = 0$

slope of $\overline{CD} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$

$AB = \sqrt{(-3 - (-1))^2 + (-1 - 3)^2}$
$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$CD = \sqrt{(2 - 4)^2 + (3 - (-1))^2}$
$= \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

Exactly two sides are parallel, $\overline{AD}$ and $\overline{BC}$, so *ABCD* is a trapezoid. $AB = CD$, so *ABCD* is an isosceles trapezoid.

Exercises

In Exercises 1–3, determine whether *ABCD* is a trapezoid. If so, determine whether it is an isosceles trapezoid. Explain.

1. $A(-1, 1)$, $B(2, 1)$, $C(3, -2)$, and $D(2, -2)$

Slope of $\overline{AB} = 0$, slope of $\overline{DC} = 0$, slope of $\overline{AD} = -1$, slope of $\overline{BC} = -3$. Exactly two sides are parallel, so *ABCD* is a trapezoid. $AD = 3\sqrt{2}$ and $BC = \sqrt{10}$; $AD \neq BC$, so *ABCD* is not isosceles.

2. $A(3, -3)$, $B(-3, -3)$, $C(-2, 3)$, and $D(2, 3)$

Slope of $\overline{AB} = 0$, slope of $\overline{DC} = 0$, slope of $\overline{BC} = 6$, slope of $\overline{AD} = -6$. Exactly 2 sides are ∥, so *ABCD* is a trapezoid. $BC = \sqrt{37}$ and $AD = \sqrt{37}$; $BC = AD$, so *ABCD* is isosceles.

3. $A(1, -4)$, $B(-3, -3)$, $C(-2, 3)$, and $D(2, 2)$

Slope of $\overline{AB} = -\frac{1}{4}$, slope of $\overline{DC} = -\frac{1}{4}$, slope of $\overline{BC} = 6$, slope of $\overline{AD} = 6$. Both pairs of opposite sides are parallel, so *ABCD* is not a trapezoid.

4. The vertices of an isosceles trapezoid are $R(-2, 2)$, $S(2, 2)$, $T(4, -1)$, and $U(-4, -1)$. Verify that the diagonals are congruent.

$RT = \sqrt{((4 - (-2))^2 + (-1 - 2)^2)} = \sqrt{45}$

$SU = \sqrt{((-4 - (2))^2 + (-1 - 2)^2)} = \sqrt{45}$

Lesson 8-6

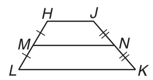
NAME ______________________ DATE __________ PERIOD _____

8-6 Study Guide and Intervention *(continued)*

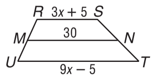
Trapezoids

Medians of Trapezoids The **median** of a trapezoid is the segment that joins the midpoints of the legs. It is parallel to the bases, and its length is one-half the sum of the lengths of the bases.

In trapezoid *HJKL*, $MN = \frac{1}{2}(HJ + LK)$.



Example **$\overline{MN}$ is the median of trapezoid *RSTU*. Find x.**



$MN = \frac{1}{2}(RS + UT)$

$30 = \frac{1}{2}(3x + 5 + 9x - 5)$

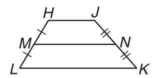
$30 = \frac{1}{2}(12x)$

$30 = 6x$

$5 = x$

Exercises

$\overline{MN}$ is the median of trapezoid *HJKL*. Find each indicated value.



1. Find MN if $HJ = 32$ and $LK = 60$.
46

2. Find LK if $HJ = 18$ and $MN = 28$.
38

3. Find MN if $HJ + LK = 42$.
21

4. Find $m\angle LMN$ if $m\angle LHJ = 116$.
116

5. Find $m\angle JKL$ if *HJKL* is isosceles and $m\angle HLK = 62$.
62

6. Find HJ if $MN = 5x + 6$, $HJ = 3x + 6$, and $LK = 8x$.
24

7. Find the length of the median of a trapezoid with vertices $A(-2, 2)$, $B(3, 3)$, $C(7, 0)$, and $D(-3, -2)$.
$\frac{3}{2}\sqrt{26}$

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8-6 Skills Practice

Trapezoids

COORDINATE GEOMETRY ***ABCD*** **is a quadrilateral with vertices** ***A*(−4, −3), *B*(3, −3), *C*(6, 4), *D*(−7, 4).**

1. Verify that *ABCD* is a trapezoid.

$\overline{AB} \parallel \overline{CD}$

2. Determine whether *ABCD* is an isosceles trapezoid. Explain.

isosceles; $AD = \sqrt{58}$ and $BC = \sqrt{58}$

COORDINATE GEOMETRY ***EFGH*** **is a quadrilateral with vertices** ***E*(1, 3), *F*(5, 0), *G*(8, −5), *H*(−4, 4).**

3. Verify that *EFGH* is a trapezoid.

$\overline{EF} \parallel \overline{GH}$

4. Determine whether *EFGH* is an isosceles trapezoid. Explain.

not isosceles; $EH = \sqrt{26}$ and $FG = \sqrt{34}$

COORDINATE GEOMETRY ***LMNP*** **is a quadrilateral with vertices** ***L*(−1, 3), *M*(−4, 1), *N*(−6, 3), *P*(0, 7).**

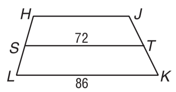
5. Verify that *LMNP* is a trapezoid. $\overline{LM} \parallel \overline{NP}$

6. Determine whether *LMNP* is an isosceles trapezoid. Explain.

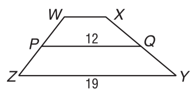
not isosceles; $LP = \sqrt{17}$ and $MN = \sqrt{8}$

ALGEBRA Find the missing measure(s) for the given trapezoid.

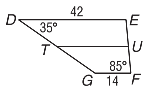
7. For trapezoid *HJKL*, *S* and *T* are midpoints of the legs. Find *HJ*. **58**



8. For trapezoid *WXYZ*, *P* and *Q* are midpoints of the legs. Find *WX*. **5**

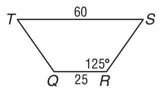


9. For trapezoid *DEFG*, *T* and *U* are midpoints of the legs. Find *TU*, $m\angle E$, and $m\angle G$.



28, 95, 145

10. For isosceles trapezoid *QRST*, find the length of the median, $m\angle Q$, and $m\angle S$.



42.5, 125, 55

Lesson 8-6

NAME ______________________ DATE ________ PERIOD _____

8-6 Practice (Average)

Trapezoids

COORDINATE GEOMETRY ***RSTU*** **is a quadrilateral with vertices** ***R*(−3, −3), *S*(5, 1), *T*(10, −2), *U*(−4, −9).**

1. Verify that *RSTU* is a trapezoid.

$\overline{RS} \parallel \overline{TU}$

2. Determine whether *RSTU* is an isosceles trapezoid. Explain.

not isosceles; $RU = \sqrt{37}$ and $ST = \sqrt{34}$

COORDINATE GEOMETRY ***BGHJ*** **is a quadrilateral with vertices** ***B*(−9, 1), *G*(2, 3), *H*(12, −2), *J*(−10, −6).**

3. Verify that *BGHJ* is a trapezoid.

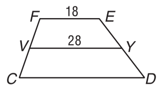
$\overline{BG} \parallel \overline{HJ}$

4. Determine whether *BGHJ* is an isosceles trapezoid. Explain.

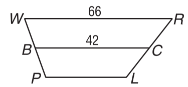
not isosceles; $BJ = \sqrt{50}$ and $GH = \sqrt{125}$

ALGEBRA Find the missing measure(s) for the given trapezoid.

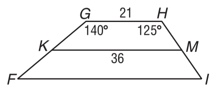
5. For trapezoid *CDEF*, *V* and *Y* are midpoints of the legs. Find *CD*. **38**



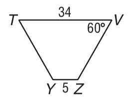
6. For trapezoid *WRLP*, *B* and *C* are midpoints of the legs. Find *LP*. **18**



7. For trapezoid *FGHI*, *K* and *M* are midpoints of the legs. Find *FI*, $m\angle F$, and $m\angle I$. **51, 40, 55**



8. For isosceles trapezoid *TVZY*, find the length of the median, $m\angle T$, and $m\angle Z$. **19.5, 60, 120**



9. CONSTRUCTION A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14, find the width of the stairs halfway to the top. **17.5 ft**

10. DESK TOPS A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

Sample answer: the measures of the base angles

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8-6 Reading to Learn Mathematics

Trapezoids

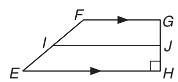
Pre-Activity **How are trapezoids used in architecture?**

Read the introduction to Lesson 8-6 at the top of page 439 in your textbook.

How might trapezoids be used in the interior design of a home?
Sample answer: floor tiles for a kitchen or bathroom

Reading the Lesson

1. In the figure at the right, *EFGH* is a trapezoid, *I* is the midpoint of $\overline{FE}$, and *J* is the midpoint of $\overline{GH}$. Identify each of the following segments or angles in the figure.



a. the bases of trapezoid *EFGH* **$\overline{FG}$, $\overline{EH}$**

b. the two pairs of base angles of trapezoid *EFGH* **$\angle E$ and $\angle H$; $\angle F$ and $\angle G$**

c. the legs of trapezoid *EFGH* **$\overline{FE}$, $\overline{GH}$**

d. the median of trapezoid *EFGH* **$\overline{IJ}$**

2. Determine whether each statement is *true* or *false*. If the statement is false, explain why.

a. A trapezoid is a special kind of parallelogram. **False; sample answer: A parallelogram has two pairs of parallel sides, but a trapezoid has only one pair of parallel sides.**

b. The diagonals of a trapezoid are congruent. **False; sample answer: This is only true for isosceles trapezoids.**

c. The median of a trapezoid is parallel to the legs. **False; sample answer: The median is parallel to the bases.**

d. The length of the median of a trapezoid is the average of the length of the bases. **true**

e. A trapezoid has three medians. **False; sample answer: A trapezoid has only one median.**

f. The bases of an isosceles trapezoid are congruent. **False: sample answer: The legs are congruent.**

g. An isosceles trapezoid has two pairs of congruent angles. **true**

h. The median of an isosceles trapezoid divides the trapezoid into two smaller isosceles trapezoids. **true**

Helping You Remember

3. A good way to remember a new geometric theorem is to relate it to one you already know. Name and state in words a theorem about triangles that is similar to the theorem in this lesson about the median of a trapezoid. **Triangle Midsegment Theorem; a midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.**

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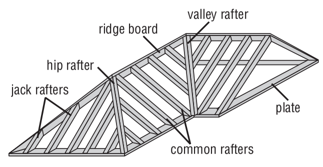
8-6 Enrichment

Quadrilaterals in Construction

Quadrilaterals are often used in construction work.

1. The diagram at the right represents a roof frame and shows many quadrilaterals. Find the following shapes in the diagram and shade in their edges. **See students' work.**



Roof Frame

a. isosceles triangle

b. scalene triangle

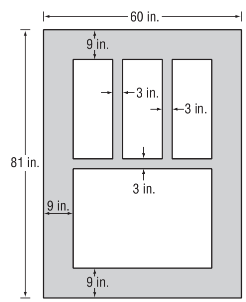
c. rectangle

d. rhombus

e. trapezoid (not isosceles)

f. isosceles trapezoid

2. The figure at the right represents a window. The wooden part between the panes of glass is 3 inches wide. The frame around the outer edge is 9 inches wide. The outside measurements of the frame are 60 inches by 81 inches. The height of the top and bottom panes is the same. The top three panes are the same size.

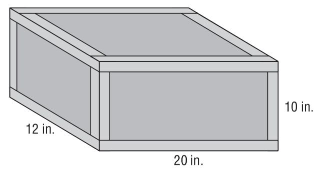


a. How wide is the bottom pane of glass? **42 in.**

b. How wide is each top pane of glass? **12 in.**

c. How high is each pane of glass? **30 in.**

3. Each edge of this box has been reinforced with a piece of tape. The box is 10 inches high, 20 inches wide, and 12 inches deep. What is the length of the tape that has been used? **168 in.**



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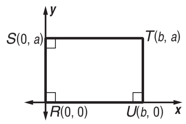
8-7 Study Guide and Intervention

Coordinate Proof with Quadrilaterals

Position Figures Coordinate proofs use properties of lines and segments to prove geometric properties. The first step in writing a coordinate proof is to place the figure on the coordinate plane in a convenient way. Use the following guidelines for placing a figure on the coordinate plane.

1.	Use the origin as a vertex, so one set of coordinates is (0, 0), or use the origin as the center of the figure.
2.	Place at least one side of the quadrilateral on an axis so you will have some zero coordinates.
3.	Try to keep the quadrilateral in the first quadrant so you will have positive coordinates.
4.	Use coordinates that make the computations as easy as possible. For example, use even numbers if you are going to be finding midpoints.

Example **Position and label a rectangle with sides a and b units long on the coordinate plane.**

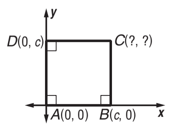


- Place one vertex at the origin for R, so one vertex is $R(0, 0)$.
- Place side $\overline{RU}$ along the x-axis and side $\overline{RS}$ along the y-axis, with the rectangle in the first quadrant.
- The sides are a and b units, so label two vertices $S(0, a)$ and $U(b, 0)$.
- Vertex T is b units right and a units up, so the fourth vertex is $T(b, a)$.

Exercises

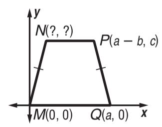
Name the missing coordinates for each quadrilateral.

1.



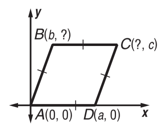
***C*(*c*, *c*)**

2.



***N*(*b*, *c*)**

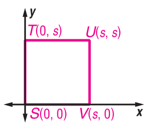
3.



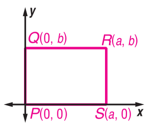
***B*(*b*, *c*); *C*(*a* + *b*, *c*)**

Position and label each quadrilateral on the coordinate plane. **Sample answers are given.**

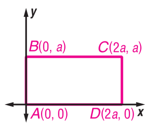
4. square $STUV$ with side s units



5. parallelogram $PQRS$ with congruent diagonals



6. rectangle $ABCD$ with length twice the width



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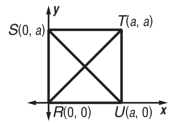
8-7 Study Guide and Intervention *(continued)*

Coordinate Proof With Quadrilaterals

Prove Theorems After a figure has been placed on the coordinate plane and labeled, a coordinate proof can be used to prove a theorem or verify a property. The Distance Formula, the Slope Formula, and the Midpoint Theorem are often used in a coordinate proof.

Example **Write a coordinate proof to show that the diagonals of a square are perpendicular.**



The first step is to position and label a square on the coordinate plane. Place it in the first quadrant, with one side on each axis. Label the vertices and draw the diagonals.

Given: square $RSTU$

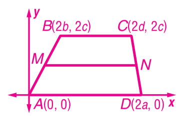
Prove: $\overline{SU} \perp \overline{RT}$

Proof: The slope of $\overline{SU}$ is $\frac{0 - a}{a - 0} = -1$, and the slope of $\overline{RT}$ is $\frac{a - 0}{a - 0} = 1$.

The product of the two slopes is -1, so $\overline{SU} \perp \overline{RT}$.

Exercise

Write a coordinate proof to show that the length of the median of a trapezoid is half the sum of the lengths of the bases.



Position $\overline{AD}$ on the x-axis and $\overline{BC}$ parallel to $\overline{AD}$. Label two vertices $A(0, 0)$ and $D(2a, 0)$. Label another vertex $B(2b, 2c)$. For vertex C, the y value is the same as for vertex B, so the last vertex is $C(2d, 2c)$.

The coordinates of midpoint M are

$\left(\frac{2b + 0}{2}, \frac{0 + 2c}{2}\right) = (b, c)$; the coordinates of midpoint N are

$\left(\frac{2a + 2d}{2}, \frac{2c + 0}{2}\right) = (a + d, c)$. $\overline{BC}$, $\overline{MN}$, and $\overline{AD}$ are horizontal segments, so find their lengths by subtracting the x-coordinates.

$AD = 2a - 0 = 2a$, $BC = 2d - 2b$, $MN = a + d - b$

Then $\frac{1}{2}(AD + BC) = \frac{1}{2}(2a + 2d - 2b) = a + d - b = MN$.

Therefore, the length of the median of a trapezoid is half the sum of the lengths of the bases.

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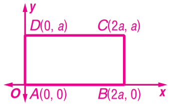
8-7 Skills Practice

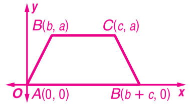
Coordinate Proof with Quadrilaterals

Position and label each quadrilateral on the coordinate plane.

1. rectangle with length $2a$ units and height a units

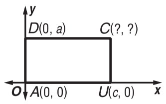


2. isosceles trapezoid with height a units, bases $c - b$ units and $b + c$ units

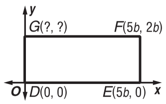


Name the missing coordinates for each quadrilateral.

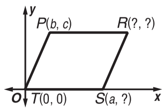
3. rectangle **$C(c, a)$**



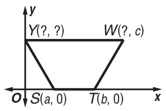
4. rectangle **$G(0, 2b)$**



5. parallelogram **$R(a + b, c)$, $S(a, 0)$**

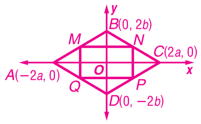


6. isosceles trapezoid **$W(a + b, c)$, $Y(0, c)$**



Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

7. The segments joining the midpoints of the sides of a rhombus form a rectangle.



Given: $ABCD$ is a rhombus.
M, N, P, and Q are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively.
Prove: $MNPQ$ is a rectangle.
Proof:
$M = (-a, b)$, $N = (a, b)$, $P = (a, -b)$, and $Q = (-a, -b)$.

slope of $\overline{MN} = \frac{b - b}{a - (-a)}$ or 0   slope of $\overline{QP} = \frac{-b - (-b)}{-a - a}$ or 0

slope of $\overline{MQ} = \frac{-b - b}{-a - (-a)}$ or undefined   slope of $\overline{NP} = \frac{b - (-b)}{a - a}$ or undefined

$\overline{MN}$ and $\overline{QP}$ have the same slope. $\overline{MQ}$ and $\overline{NP}$ also have the same slope. $\overline{MN}$ is perpendicular to $\overline{MQ}$. Therefore, both pairs of opposite sides are parallel and consecutive sides are perpendicular. This means that $MNPQ$ is a rectangle.

Lesson 8-7

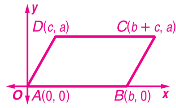
NAME ______________________ DATE __________ PERIOD _____

8-7 Practice (Average)

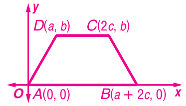
Coordinate Proof with Quadrilaterals

Position and label each quadrilateral on the coordinate plane.

1. parallelogram with side length b units and height a units

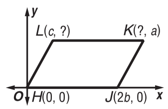


2. isosceles trapezoid with height b units, bases $2c - a$ units and $2c + a$ units



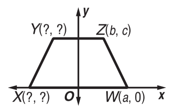
Name the missing coordinates for each quadrilateral.

3. parallelogram



$K(2b + c, a)$, $L(c, a)$

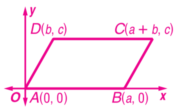
4. isosceles trapezoid



$X(-a, 0)$, $Y(-b, c)$

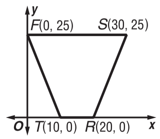
Position and label the figure on the coordinate plane. Then write a coordinate proof for the following.

5. The opposite sides of a parallelogram are congruent.



Given: $ABCD$ is a parallelogram.
Prove: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$
Proof:
$AB = \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2}$ or a
$CD = \sqrt{[(a + b) - b]^2 + (c - c)^2} = \sqrt{a^2}$ or a
$AD = \sqrt{(b - 0)^2 + (c - 0)^2} = \sqrt{b^2 + c^2}$
$AB = \sqrt{[(a + b) - a]^2 + (c - 0)^2} = \sqrt{b^2 + c^2}$
$AB = CD$ and $AD = BC$, so $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$

6. THEATER A stage is in the shape of a trapezoid. Write a coordinate proof to prove that $\overline{TR}$ and $\overline{SF}$ are parallel.



Given: $T(10, 0)$, $R(20, 0)$, $S(30, 25)$, $F(0, 25)$
Prove: $\overline{TR} \parallel \overline{SF}$
Proof: The slope of $\overline{TR} = \frac{0 - 0}{20 - 10} = \frac{0}{10}$ and the slope of $\overline{SF} = \frac{25 - 25}{30 - 0} = \frac{0}{30}$. Since $\overline{TR}$ and $\overline{SF}$ both have a slope of 0, $\overline{TR} \parallel \overline{SF}$.

Answers

NAME ______________________ DATE ________ PERIOD _____

8-7 Reading to Learn Mathematics

Coordinate Proof with Quadrilaterals

Pre-Activity **How can you use a coordinate plane to prove theorems about quadrilaterals?**

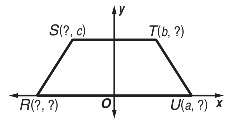
Read the introduction to Lesson 8-7 at the top of page 447 in your textbook.

What special kinds of quadrilaterals can be placed on a coordinate system so that two sides of the quadrilateral lie along the axes?
rectangles and squares

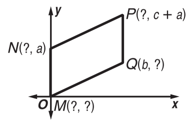
Reading the Lesson

1. Find the missing coordinates in each figure. Then write the coordinates of the four vertices of the quadrilateral.

a. isosceles trapezoid

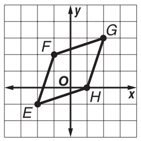


b. parallelogram



$R(-a, 0)$, $S(-b, c)$, $T(b, c)$, $U(a, 0)$ **$M(0, 0)$, $N(0, a)$, $P(b, c + a)$, $Q(b, c)$**

2. Refer to quadrilateral *EFGH*.



a. Find the slope of each side. **slope $\overline{EF} = 3$, slope $\overline{FG} = \frac{1}{3}$, slope $\overline{GH} = 3$, slope $\overline{HE} = \frac{1}{3}$**

b. Find the length of each side.
$EF = \sqrt{10}$, $FG = \sqrt{10}$, $GH = \sqrt{10}$, $HE = \sqrt{10}$

c. Find the slope of each diagonal. **slope $\overline{EG} = 1$, slope $\overline{FH} = -1$**

d. Find the length of each diagonal. **$EG = \sqrt{32}$, $FH = \sqrt{8}$**

e. What can you conclude about the sides of *EFGH*? **All four sides are congruent and both pairs of opposite sides are parallel.**

f. What can you conclude about the diagonals of *EFGH*? **The diagonals are perpendicular and they are not congruent.**

g. Classify *EFGH* as a *parallelogram*, a *rhombus*, or a *square*. Choose the most specific term. Explain how your results from parts a-f support your conclusion.
***EFGH* is a rhombus. All four sides are congruent and the diagonals are perpendicular. Since the diagonals are not congruent, *EFGH* is not a square.**

Helping You Remember

3. What is an easy way to remember how best to draw a diagram that will help you devise a coordinate proof? **Sample answer: A key point in the coordinate plane is the origin. The everyday meaning of *origin* is *place where something begins*. So look to see if there is a good way to begin by placing a vertex of the figure at the origin.**

Lesson 8-7

NAME ______________________ DATE ________ PERIOD _____

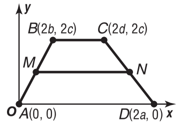
8-7 Enrichment

Coordinate Proofs

An important part of planning a coordinate proof is correctly placing and labeling the geometric figure on the coordinate plane.

Example **Draw a diagram you would use in a coordinate proof of the following theorem.**

The median of a trapezoid is parallel to the bases of the trapezoid.



In the diagram, note that one vertex, *A*, is placed at the origin. Also, the coordinates of *B*, *C*, and *D* use $2a$, $2b$, and $2c$ in order to avoid the use of fractions when finding the coordinates of the midpoints, *M* and *N*.

When doing coordinate proofs, the following strategies may be helpful.

1. If you are asked to prove that segments are parallel or perpendicular, use slopes.

2. If you are asked to prove that segments are congruent or have related measures, use the distance formula.

3. If you are asked to prove that a segment is bisected, use the midpoint formula.

For each of the following theorems, a diagram has been provided to be used in a formal proof. Name the missing coordinates in the diagram. Then, using the *Given* and the *Prove* statements, prove the theorem.

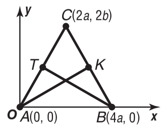
1. The median of a trapezoid is parallel to the bases of the trapezoid. (Use the diagram given in the example above.)

Coordinates of *M* and *N*: **$M(b, c)$; $N(d + a, c)$**

Given: Trapezoid *ABCD* has median $\overline{MN}$.
Prove: $\overline{BC} \parallel \overline{MN}$ and $\overline{AD} \parallel \overline{MN}$

Proof: **The slope of $\overline{AD}$ is $\frac{0 - 0}{2a - 0}$ or 0. The slope of $\overline{BC}$ is $\frac{2c - 2c}{2d - 2b}$ or 0. The slope of $\overline{MN}$ is $\frac{c - c}{(a + b) - b}$ or 0. Since all three slopes are equal, $\overline{BC} \parallel \overline{MN}$ and $\overline{AD} \parallel \overline{MN}$.**

2. The medians to the legs of an isosceles triangle are congruent.



Coordinates of *T* and *K*: **$T(a, b)$; $K(3a, b)$**

Given: $\triangle ABC$ is isosceles with medians $\overline{TB}$ and $\overline{KA}$.
Prove: $\overline{TB} \cong \overline{KA}$

Proof: **$TB = \sqrt{(4a - a)^2 + (0 - b)^2}$ or $\sqrt{9a^2 + b^2}$**
$KA = \sqrt{(3a - 0)^2 + (b - 0)^2}$ or $\sqrt{9a^2 + b^2}$
Since $TB = KA$, $\overline{TB} \cong \overline{KA}$.

Chapter 8 Assessment Answer Key

Form 1
Page 459

1. B
2. C
3. B
4. C
5. D
6. A
7. B
8. A
9. D
10. A
11. C

Page 460

12. C
13. B
14. D
15. A
16. B
17. A
18. D
19. B
20. B

B: $x = 7$, $m\angle WYZ = 41$

Form 2A
Page 461

1. B
2. B
3. D
4. A
5. C
6. A
7. A
8. C
9. B
10. A
11. D

(continued on the next page)

Answers

Chapter 8 Assessment Answer Key

Form 2A *(continued)*

Page 462

12. D
13. C
14. C
15. A
16. B
17. D
18. B
19. D
20. B

B: 7 or −4

Form 2B

Page 463

1. B
2. D
3. D
4. A
5. B
6. B
7. A
8. C
9. C
10. D
11. B

Page 464

12. B
13. D
14. B
15. D
16. C
17. C
18. A
19. A
20. C

B: 22

Chapter 8 Assessment Answer Key

Form 2C

Page 465

1. 10,440
2. 19
3. 40
4. 18
5. 8
6. 122
7. (6, 4)
8. Yes; $\overline{AB}$ and $\overline{CD}$ are ∥ and ≅.
9. No; the slopes are $\frac{9}{4}$, $\frac{1}{7}$, 1, and $\frac{2}{3}$. Thus, *ABCD* does not have ∥ sides.
10. $-\frac{2}{3}$
11. 22
12. Yes; if the diagonals of a ▱ are ≅, then the ▱ is a rectangle.
13. 67.5

Page 466

14. (4, 0)
15. 31
16. Yes; *ABCD* has only one pair of opposite sides ∥, $\overline{BC}$ and $\overline{AD}$.
17. 6
18. $A(a, 0)$, $\overline{AC} \perp \overline{BD}$
19. true
20. true
21. true
22. false
23. true
24. true
25. false

B: $x = 9$, $y = 2$

Answers

Chapter 8 Assessment Answer Key

Form 2D

Page 467

1. 6120
2. 38
3. 90
4. 50
5. 3.6
6. 117
7. $\left(\frac{1}{2}, 3\right)$
8. Yes; Both pairs of opp. sides are ≅.
9. *ABCD* has two pairs of ∥ sides, $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \parallel \overline{DA}$; it is a ▱.
10. −4
11. 8
12. One rt. ∠ means that the other ⦣s will be rt. ⦣s. If all 4 ⦣s are rt. ⦣s, the ▱ is a rectangle.

Page 468

13. 72
14. (−3, 1)
15. 16
16. Yes; *ABCD* has only one pair of opp. sides ∥, $\overline{AD}$ and $\overline{BC}$.
17. 8
18. *B*(*a*, *a*); The midpoint of both diagonals is at $\left(\frac{a}{2}, \frac{a}{2}\right)$.
19. false
20. true
21. false
22. false
23. true
24. false
25. false

B: 90

Chapter 8 Assessment Answer Key

Form 3

Page 469

1. 3960
2. 30; 30, 47, 120, 179, 174, and 170
3. $\frac{180}{x}$
4. 65
5. 8 or 32
6. Yes; the diagonals bisect each other.
7. slope of $\overline{CD} = \frac{2}{3}$; slope of $\overline{DA} = -2$.
8. 28
9. 72
10. $8\sqrt{2}$
11. 4
12. 9

Page 470

13. 13
14. Sample answer: $(b + d, c)$
15. Yes; $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, so all ∠s are rt. ∠s.
16. *ABCD* has 2 pairs of opp. sides ≅, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, so *ABCD* is a ▱.
17. $C(-2a, 0)$; the segment joining the midpoints of $\overline{BC}$ and $\overline{AB}$ is ∥ to $\overline{AC}$.
18. Opp. sides of a ▱ are ≅.
19. If both pairs of opp. sides of a quad. are ≅, then the quad. is a ▱.
20. 27

B: 54

Answers

Chapter 8 Assessment Answer Key

Page 471, Open-Ended Assessment
Scoring Rubric

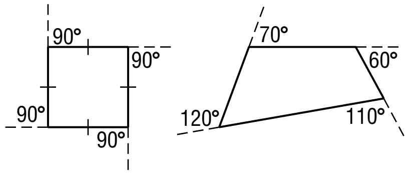
Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *angles of polygons*, *properties of parallelograms*, *rectangles*, *rhombi*, *squares*, *and trapezoids*, and *coordinate proofs*. • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs and figures are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *angles of polygons*, *properties of parallelograms*, *rectangles*, *rhombi*, *squares*, *and trapezoids*, and *coordinate proofs*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs and figures are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *angles of polygons*, *properties of parallelograms*, *rectangles*, *rhombi*, *squares*, *and trapezoids*, and *coordinate proofs*. • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs and figures are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work shown to substantiate the final computation. • Graphs and figures may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *angles of polygons*, *properties of parallelograms*, *rectangles*, *rhombi*, *squares*, *and trapezoids*, and *coordinate proofs*. • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs and figures are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 8 Assessment Answer Key

Page 471, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

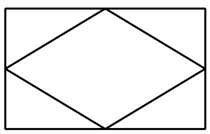
1. a. Any type of convex polygon can be drawn as long as one is regular and one is not regular and both have the same number of sides.



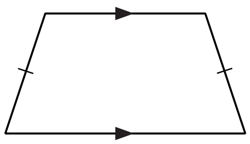
b. Check to be sure that the exterior angles have been properly drawn and accurately measured.

c. 4(90) = 360; 120 + 70 + 60 + 110 = 360; The sum of the exterior angles of each figure should be 360°. The sum of the exterior angles of both the regular convex polygon and the irregular convex polygon is 360°.

2. The student should draw a rectangle and join the midpoints of consecutive sides. The figure formed inside is a rhombus. Since all four small triangles can be proved to be congruent by SAS, the four sides of the interior quadrilateral are congruent by CPCTC, making it a rhombus.



3. The student should draw an isosceles trapezoid with one pair of opposite sides parallel and the other pair of opposite sides congruent, as in the figure below.



4. a. Possible properties:
A square has four congruent sides and a rectangle may not.
A square has perpendicular diagonals and a rectangle may not.
The diagonals of a square bisect the angles and those in a rectangle may not.

b. Possible properties:
A square has four right angles and a rhombus may not.
The diagonals of a square are congruent and those of a rhombus may not be.

c. Possible properties:
A rectangle has four right angles and a parallelogram may not.
The diagonals of a rectangle are congruent and those of a parallelogram may not be.

5. The slope of diagonal $\overline{AC}$ is $\frac{c}{a+b}$, while the slope of diagonal $\overline{BD}$ is $\frac{c}{b-a}$. These slopes are not necessarily opposite reciprocals of each other. Therefore John's figure could not be used to prove that the diagonals of a rhombus are perpendicular. The figure John drew on the coordinate plane has vertices that make it a parallelogram but not always a rhombus.

Chapter 8 Assessment Answer Key

Vocabulary Test/Review
Page 472

1. isosceles trapezoid
2. parallelogram
3. trapezoid
4. square
5. rhombus
6. rectangle
7. kite
8. diagonals
9. median
10. bases
11. angles formed by the base and one of the legs of a trapezoid
12. the nonparallel sides of a trapezoid

Quiz 1
Page 473

1. 12,240
2. 45
3. 20
4. (−1, 10)
5. A

Quiz 2
Page 473

1. No; none of the tests for ▱s are fulfilled.
2. false
3. false
4. true
5. $m\angle 1 = 68$, $m\angle 2 = 56$, $m\angle 3 = 34$

Quiz 3
Page 474

1. Rhombus; all sides are ≅.
2. 108
3. 50
4. true
5. 12

Quiz 4
Page 474

1. $C(-a, 0)$
2. $D(0, -a)$
3. $A(-d, c)$
4. $X(a + b, c)$, $Y(-e - d, c)$
5. $C(a + b, c)$, $AB = CD = \sqrt{b^2 + c^2}$, $BC = AD = a$

Chapter 8 Assessment Answer Key

Mid-Chapter Test
Page 475

Part I

1. B
2. D
3. A
4. B
5. C
6. 50

Part II

7. 42
8. yes
9. $m\angle 1 = 12$, $m\angle 2 = 78$, $m\angle 3 = 12$, $m\angle 4 = 156$
10. If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.

Cumulative Review
Page 476

1. $\overrightarrow{BR}$ and $\overrightarrow{BT}$
2. $y = -\frac{3}{2}x - 9$
3. 11, 2
4. Assume that neither bell cost more than $45.
5. 20.5 in., 33 in., 18.75 in., and 30 in.
6. $A(3, 0.5)$ and $B(-0.5, -1)$
7. ∠ of elevation: $\angle XYZ$; ∠ of depression: $\angle WXY$
8. 13
9. 55; 125; 5
10. 8.5
11. *LMNP* is a trapezoid since $\overline{PL} \parallel \overline{MN}$ and $\overline{PN} \nparallel \overline{LM}$.

Answers

Chapter 8 Assessment Answer Key

Standardized Test Practice

Page 477

1. **A** B C D

2. E **F** G H

3. A B C **D**

4. E F **G** H

5. A B **C** D

6. **E** F G H

7. **A** B C D

8. E **F** G H

Page 478

9. 1/7

11. 7

11. 11.2

12. 10

13. 900

14. hexagon; concave; irregular

15. 88.9 cm

16. $\overline{DL}$