

**GLENCOE  
MATHEMATICS**

# Geometry

## **Chapter 6 Resource Masters**

**Mc  
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## Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

<i>Study Guide and Intervention Workbook</i>	0-07-860191-6
<i>Skills Practice Workbook</i>	0-07-860192-4
<i>Practice Workbook</i>	0-07-860193-2
<i>Reading to Learn Mathematics Workbook</i>	0-07-861061-3

**ANSWERS FOR WORKBOOKS** The answers for Chapter 6 of these workbooks can be found in the back of this Chapter Resource Masters booklet.



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*Geometry*  
*Chapter 6 Resource Masters*

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# Teacher's Guide to Using the Chapter 6 Resource Masters

The **Fast File** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 6 Resource Masters* includes the core materials needed for Chapter 6. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

**Vocabulary Builder** Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

**WHEN TO USE** Give these pages to students before beginning Lesson 6-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

**Vocabulary Builder** Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

**WHEN TO USE** Give these pages to students before beginning Lesson 6-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

## Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

**WHEN TO USE** Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** There is one master for each lesson. These provide computational practice at a basic level.

**WHEN TO USE** These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

**Practice** There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

**WHEN TO USE** These provide additional practice options or may be used as homework for second day teaching of the lesson.

## Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

**WHEN TO USE** This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

**Enrichment** There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

**WHEN TO USE** These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

## Assessment Options

The assessment masters in the *Chapter 6 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

## Chapter Assessment

### CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

All of the above tests include a free-response Bonus question.

- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

## Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

## Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

## Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 338–339. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.



**6**

# Reading to Learn Mathematics

## Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 6. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
cross products		
extremes		
fractal		
iteration <small>                     {                      ID·uh·RAY·shuhn                 </small>		
means		

*(continued on the next page)*

## 6

**Reading to Learn Mathematics****Vocabulary Builder** *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
midsegment		
proportion		
ratio		
scale factor		
self-similar		
similar polygons		



## 6

**Learning to Read Mathematics*****Proof Builder***

This is a list of key theorems and postulates you will learn in Chapter 6. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 6.1 <i>Side-Side-Side (SSS) Similarity</i>		
Theorem 6.2 <i>Side-Angle-Side (SAS) Similarity</i>		
Theorem 6.3		
Theorem 6.4 <i>Triangle Proportionality Theorem</i>		
Theorem 6.5 <i>Converse of the Triangle Proportionality Theorem</i>		
Theorem 6.6 <i>Triangle Midsegment Theorem</i>		

(continued on the next page)

## 6

**Learning to Read Mathematics*****Proof Builder*** (continued)

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 6.7 <i>Proportional Perimeters Theorem</i>		
Theorem 6.8		
Theorem 6.9		
Theorem 6.10		
Theorem 6.11 <i>Angle Bisector Theorem</i>		
Postulate 6.1 <i>Angle-Angle (AA) Similarity</i>		

**6-1 Study Guide and Intervention****Proportions**

**Write Ratios** A **ratio** is a comparison of two quantities. The ratio  $a$  to  $b$ , where  $b$  is not zero, can be written as  $\frac{a}{b}$  or  $a:b$ . The ratio of two quantities is sometimes called a **scale factor**. For a scale factor, the units for each quantity are the same.

**Example 1** In 2002, the Chicago Cubs baseball team won 67 games out of 162. Write a ratio for the number of games won to the total number of games played. To find the ratio, divide the number of games won by the total number of games played. The result is  $\frac{67}{162}$ , which is about 0.41. The Chicago Cubs won about 41% of their games in 2002.

**Example 2** A doll house that is 15 inches tall is a scale model of a real house with a height of 20 feet. What is the ratio of the height of the doll house to the height of the real house?

To start, convert the height of the real house to inches.

$$20 \text{ feet} \times 12 \text{ inches per foot} = 240 \text{ inches}$$

To find the ratio or scale factor of the heights, divide the height of the doll house by the height of the real house. The ratio is 15 inches:240 inches or 1:16. The height of the doll house is  $\frac{1}{16}$  the height of the real house.

**Exercises**

1. In the 2002 Major League baseball season, Sammy Sosa hit 49 home runs and was at bat 556 times. Find the ratio of home runs to the number of times he was at bat.
2. There are 182 girls in the sophomore class of 305 students. Find the ratio of girls to total students.
3. The length of a rectangle is 8 inches and its width is 5 inches. Find the ratio of length to width.
4. The sides of a triangle are 3 inches, 4 inches, and 5 inches. Find the scale factor between the longest and the shortest sides.
5. The length of a model train is 18 inches. It is a scale model of a train that is 48 feet long. Find the scale factor.

**6-1 Study Guide and Intervention** *(continued)***Proportions**

**Use Properties of Proportions** A statement that two ratios are equal is called a **proportion**. In the proportion  $\frac{a}{b} = \frac{c}{d}$ , where  $b$  and  $d$  are not zero, the values  $a$  and  $d$  are the **extremes** and the values  $b$  and  $c$  are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so  $ad = bc$ .

$$\frac{a}{b} = \frac{c}{d}$$

$$a \cdot d = b \cdot c$$

$\uparrow$                      $\uparrow$   
 extremes        means

**Example 1** Solve  $\frac{9}{16} = \frac{27}{x}$ .

$$\frac{9}{16} = \frac{27}{x}$$

$$9 \cdot x = 16 \cdot 27 \quad \text{Cross products}$$

$$9x = 432 \quad \text{Multiply.}$$

$$x = 48 \quad \text{Divide each side by 9.}$$

**Example 2** A room is 49 centimeters by 28 centimeters on a scale drawing of a house. For the actual room, the larger dimension is 14 feet. Find the shorter dimension of the actual room.

If  $x$  is the room's shorter dimension, then

$$\frac{28}{49} = \frac{x}{14} \quad \begin{array}{l} \text{shorter dimension} \\ \text{longer dimension} \end{array}$$

$$49x = 392 \quad \text{Cross products}$$

$$x = 8 \quad \text{Divide each side by 49.}$$

The shorter side of the room is 8 feet.

**Exercises**

**Solve each proportion.**

1.  $\frac{1}{2} = \frac{28}{x}$

2.  $\frac{3}{8} = \frac{y}{24}$

3.  $\frac{x + 22}{x + 2} = \frac{30}{10}$

4.  $\frac{3}{18.2} = \frac{9}{y}$

5.  $\frac{2x + 3}{8} = \frac{5}{4}$

6.  $\frac{x + 1}{x - 1} = \frac{3}{4}$

**Use a proportion to solve each problem.**

7. If 3 cassettes cost \$44.85, find the cost of one cassette.
8. The ratio of the sides of a triangle are 8:15:17. If the perimeter of the triangle is 480 inches, find the length of each side of the triangle.
9. The scale on a map indicates that one inch equals 4 miles. If two towns are 3.5 inches apart on the map, what is the actual distance between the towns?

**6-1 Skills Practice****Proportions**

- 1. FOOTBALL** A tight end scored 6 touchdowns in 14 games. Find the ratio of touchdowns per game.
- 2. EDUCATION** In a schedule of 6 classes, Marta has 2 elective classes. What is the ratio of elective to non-elective classes in Marta's schedule?
- 3. BIOLOGY** Out of 274 listed species of birds in the United States, 78 species made the endangered list. Find the ratio of endangered species of birds to listed species in the United States.
- 4. ART** An artist in Portland, Oregon, makes bronze sculptures of dogs. The ratio of the height of a sculpture to the actual height of the dog is 2:3. If the height of the sculpture is 14 inches, find the height of the dog.
- 5. SCHOOL** The ratio of male students to female students in the drama club at Campbell High School is 3:4. If the number of male students in the club is 18, what is the number of female students?

**Solve each proportion.**

6.  $\frac{2}{5} = \frac{x}{40}$

7.  $\frac{7}{10} = \frac{21}{x}$

8.  $\frac{20}{5} = \frac{4x}{6}$

9.  $\frac{5x}{4} = \frac{35}{8}$

10.  $\frac{x+1}{3} = \frac{7}{2}$

11.  $\frac{15}{3} = \frac{x-3}{5}$

**Find the measures of the sides of each triangle.**

- 12.** The ratio of the measures of the sides of a triangle is 3:5:7, and its perimeter is 450 centimeters.
- 13.** The ratio of the measures of the sides of a triangle is 5:6:9, and its perimeter is 220 meters.
- 14.** The ratio of the measures of the sides of a triangle is 4:6:8, and its perimeter is 126 feet.
- 15.** The ratio of the measures of the sides of a triangle is 5:7:8, and its perimeter is 40 inches.

# 6-1 Practice

## Proportions

- 1. NUTRITION** One ounce of cheddar cheese contains 9 grams of fat. Six of the grams of fat are saturated fats. Find the ratio of saturated fats to total fat in an ounce of cheese.
- 2. FARMING** The ratio of goats to sheep at a university research farm is 4:7. The number of sheep at the farm is 28. What is the number of goats?
- 3. ART** Edward Hopper's oil on canvas painting *Nighthawks* has a length of 60 inches and a width of 30 inches. A print of the original has a length of 2.5 inches. What is the width of the print?

**Solve each proportion.**

4.  $\frac{5}{8} = \frac{x}{12}$

5.  $\frac{x}{1.12} = \frac{1}{5}$

6.  $\frac{6x}{27} = \frac{4}{3}$

7.  $\frac{x+2}{3} = \frac{8}{9}$

8.  $\frac{3x-5}{4} = \frac{-5}{7}$

9.  $\frac{x-2}{4} = \frac{x+4}{2}$

**Find the measures of the sides of each triangle.**

- 10.** The ratio of the measures of the sides of a triangle is 3:4:6, and its perimeter is 104 feet.
- 11.** The ratio of the measures of the sides of a triangle is 7:9:12, and its perimeter is 84 inches.
- 12.** The ratio of the measures of the sides of a triangle is 6:7:9, and its perimeter is 77 centimeters.

**Find the measures of the angles in each triangle.**

- 13.** The ratio of the measures of the angles is 4:5:6.
- 14.** The ratio of the measures of the angles is 5:7:8.
- 15. BRIDGES** The span of the Benjamin Franklin suspension bridge in Philadelphia, Pennsylvania, is 1750 feet. A model of the bridge has a span of 42 inches. What is the ratio of the span of the model to the span of the actual Benjamin Franklin Bridge?

## 6-1

## Reading to Learn Mathematics

## Proportions

## Pre-Activity How do artists use ratios?

Read the introduction to Lesson 6-1 at the top of page 282 in your textbook.

Estimate the ratio of length to width for the background rectangles in Tiffany's Clematis Skylight.

## Reading the Lesson

1. Match each description in the first column with a word or phrase from the second column.

- |  |                    |
|--|--------------------|
| a. The ratio of two corresponding quantities                 | i. proportion      |
| b. $r$ and $u$ in the equation $\frac{r}{s} = \frac{t}{u}$   | ii. cross products |
| c. a comparison of two quantities                            | iii. means         |
| d. $ru$ and $st$ in the equation $\frac{r}{s} = \frac{t}{u}$ | iv. scale factor   |
| e. an equation stating that two ratios are equal             | v. extremes        |
| f. $s$ and $t$ in the equation $\frac{r}{s} = \frac{t}{u}$   | vi. ratio          |

2. If  $m$ ,  $n$ ,  $p$ , and  $q$  are nonzero numbers such that  $\frac{m}{n} = \frac{p}{q}$ , which of the following statements could be *false*?

- |                                |                                |
|--------------------------------|--------------------------------|
| A. $np = mq$                   | B. $\frac{p}{n} = \frac{q}{m}$ |
| C. $mp = nq$                   | D. $qm = pn$                   |
| E. $\frac{m}{n} = \frac{q}{p}$ | F. $\frac{q}{p} = \frac{n}{m}$ |
| G. $m:p = n:q$                 | H. $m:n = p:q$                 |

3. Write two proportions that match each description.

- Means are 5 and 8; extremes are 4 and 10.
- Means are 5 and 4; extremes are positive integers that are different from means.

## Helping You Remember

4. Sometimes it is easier to remember a mathematical idea if you put it into words without using any mathematical symbols. How can you use this approach to remember the concept of equality of cross products?

# 6-1 Enrichment

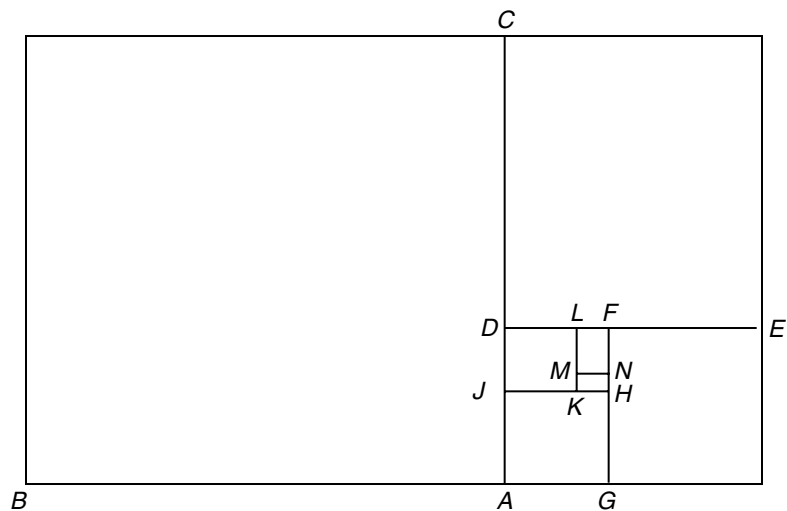
## Golden Rectangles

Use a straightedge, compass, and the instructions below to construct a golden rectangle.

1. Construct square  $ABCD$  with sides of 2 cm.
2. Construct the midpoint of  $\overline{AB}$ . Call the midpoint  $M$ .
3. Draw  $\overline{AB}$ . Set your compass at an opening equal to  $MC$ . Use  $M$  as the center to draw an arc that intersects  $\overline{AB}$ . Call the point of intersection  $P$ .
4. Construct a line through  $P$  that is perpendicular to  $\overline{AB}$ .
5. Draw  $\overline{DC}$  so that it intersects the perpendicular line in step 4. Call the intersection point  $Q$ .  $APQD$  is a **golden rectangle** because the ratio of its length to its width is 1.618. Check this conclusion by finding the value of  $\frac{QP}{AP}$ . Rectangles whose sides have this ratio are, it is said, the most pleasing to the human eye.

A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.

6. Using  $A$  as a center, draw an arc that passes through  $B$  and  $C$ .
7. Using  $D$  as a center, draw an arc that passes through  $C$  and  $E$ .
8. Using  $F$  as a center, draw an arc that passes through  $E$  and  $G$ .
9. Using  $H$  as a center, draw an arc that passes through  $G$  and  $J$ .
10. Using  $K$  as a center, draw an arc that passes through  $J$  and  $L$ .
11. Using  $M$  as a center, draw an arc that passes through  $L$  and  $N$ .





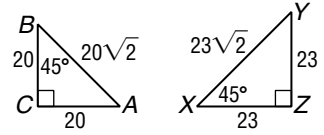
# 6-2 Study Guide and Intervention

## Similar Polygons

### Identify Similar Figures

**Example 1** Determine whether the triangles are similar.

Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.



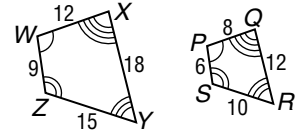
$\angle C \cong \angle Z$  because they are right angles, and  $\angle B \cong \angle X$ .  
By the Third Angle Theorem,  $\angle A \cong \angle Y$ .

For the sides,  $\frac{BC}{XZ} = \frac{20}{23}$ ,  $\frac{BA}{XY} = \frac{20\sqrt{2}}{23\sqrt{2}} = \frac{20}{23}$ , and  $\frac{AC}{YZ} = \frac{20}{23}$ .

The side lengths are proportional. So  $\triangle BCA \sim \triangle XZY$ .

**Example 2** Is polygon  $WXYZ \sim$  polygon  $PQRS$ ?

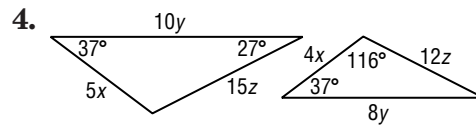
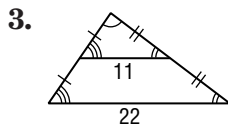
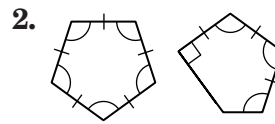
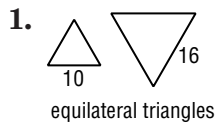
For the sides,  $\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}$ ,  $\frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}$ ,  $\frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2}$ ,  
and  $\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$ . So corresponding sides are proportional.



Also,  $\angle W \cong \angle P$ ,  $\angle X \cong \angle Q$ ,  $\angle Y \cong \angle R$ , and  $\angle Z \cong \angle S$ , so corresponding angles are congruent. We can conclude that polygon  $WXYZ \sim$  polygon  $PQRS$ .

### Exercises

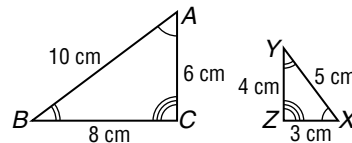
Determine whether each pair of figures is similar. If they are similar, give the ratio of corresponding sides.



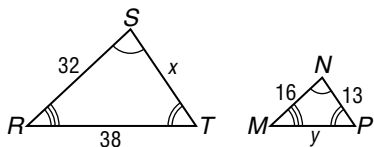
# 6-2 Study Guide and Intervention *(continued)*

## Similar Polygons

**Scale Factors** When two polygons are similar, the ratio of the lengths of corresponding sides is called the **scale factor**. At the right,  $\triangle ABC \sim \triangle XYZ$ . The scale factor of  $\triangle ABC$  to  $\triangle XYZ$  is 2 and the scale factor of  $\triangle XYZ$  to  $\triangle ABC$  is  $\frac{1}{2}$ .



**Example 1** The two polygons are similar. Find  $x$  and  $y$ .



Use the congruent angles to write the corresponding vertices in order.

$$\triangle RST \sim \triangle MNP$$

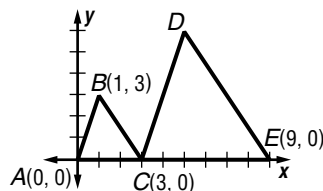
Write proportions to find  $x$  and  $y$ .

$$\frac{32}{16} = \frac{x}{13} \quad \frac{38}{y} = \frac{32}{16}$$

$$16x = 32(13) \quad 32y = 38(16)$$

$$x = 26 \quad y = 19$$

**Example 2**  $\triangle ABC \sim \triangle CDE$ . Find the scale factor and find the lengths of  $\overline{CD}$  and  $\overline{DE}$ .

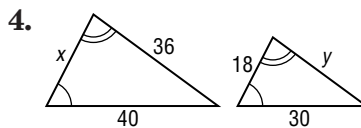
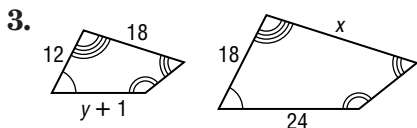
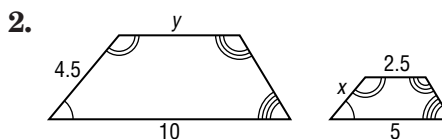
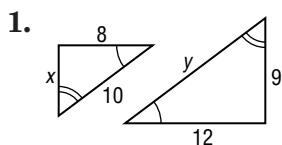


$AC = 3 - 0 = 3$  and  $CE = 9 - 3 = 6$ . The scale factor of  $\triangle CDE$  to  $\triangle ABC$  is  $6:3$  or  $2:1$ .

Using the Distance Formula,  
 $AB = \sqrt{1 + 9} = \sqrt{10}$  and  
 $BC = \sqrt{4 + 9} = \sqrt{13}$ . The lengths of the sides of  $\triangle CDE$  are twice those of  $\triangle ABC$ , so  $DC = 2(BA)$  or  $2\sqrt{10}$  and  $DE = 2(BC)$  or  $2\sqrt{13}$ .

### Exercises

Each pair of polygons is similar. Find  $x$  and  $y$ .



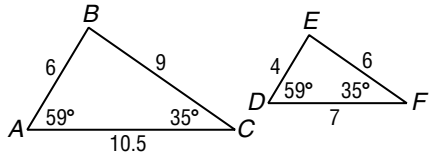
5. In Example 2 above, point  $D$  has coordinates  $(5, 6)$ . Use the Distance Formula to verify the lengths of  $\overline{CD}$  and  $\overline{DE}$ .

# 6-2 Skills Practice

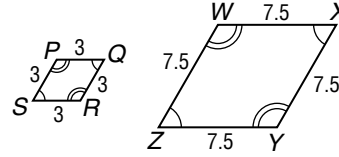
## Similar Polygons

Determine whether each pair of figures is similar. Justify your answer.

1.

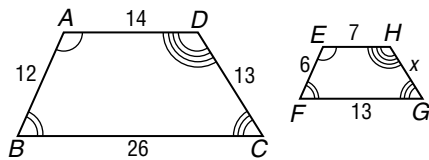


2.

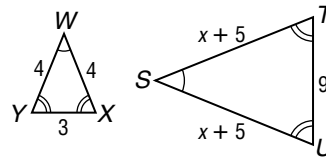


Each pair of polygons is similar. Write a similarity statement, and find  $x$ , the measure(s) of the indicated side(s), and the scale factor.

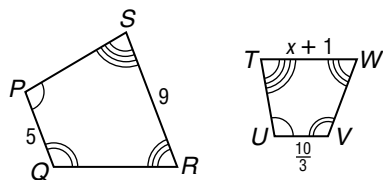
3.  $\overline{GH}$



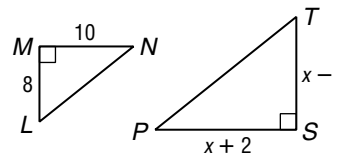
4.  $\overline{ST}$  and  $\overline{SU}$



5.  $\overline{WT}$



6.  $\overline{TS}$  and  $\overline{SP}$

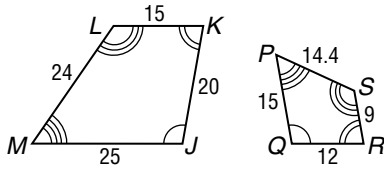


# 6-2 Practice

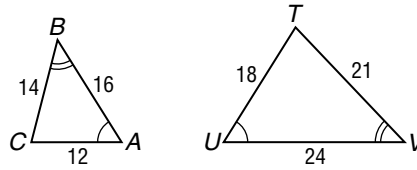
## Similar Polygons

Determine whether each pair of figures is similar. Justify your answer.

1.

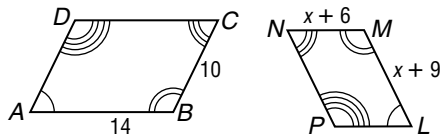


2.

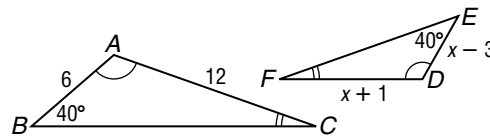


Each pair of polygons is similar. Write a similarity statement, and find  $x$ , the measure(s) of the indicated side(s), and the scale factor.

3.  $\overline{LM}$  and  $\overline{MN}$

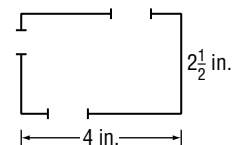


4.  $\overline{DE}$  and  $\overline{DF}$



5. **COORDINATE GEOMETRY** Triangle  $ABC$  has vertices  $A(0, 0)$ ,  $B(-4, 0)$ , and  $C(-2, 4)$ . The coordinates of each vertex are multiplied by 3 to create  $\triangle AEF$ . Show that  $\triangle AEF$  is similar to  $\triangle ABC$ .

6. **INTERIOR DESIGN** Graham used the scale drawing of his living room to decide where to place furniture. Find the dimensions of the living room if the scale in the drawing is 1 inch = 4.5 feet.



## 6-2

## Reading to Learn Mathematics

*Similar Polygons***Pre-Activity** How do artists use geometric patterns?

Read the introduction to Lesson 6-2 at the top of page 289 in your textbook.

- Describe the figures that have similar shapes.
  
- What happens to the figures as your eyes move from the center to the outer edge?

**Reading the Lesson**

1. Complete each sentence.
  - a. Two polygons that have exactly the same shape, but not necessarily the same size, are \_\_\_\_\_.
  - b. Two polygons are congruent if they have exactly the same shape and the same \_\_\_\_\_.
  - c. Two polygons are similar if their corresponding angles are \_\_\_\_\_ and their corresponding sides are \_\_\_\_\_.
  - d. Two polygons are congruent if their corresponding angles are \_\_\_\_\_ and their corresponding sides are \_\_\_\_\_.
  - e. The ratio of the lengths of corresponding sides of two similar figures is called the \_\_\_\_\_.
  - f. Multiplying the coordinates of all points of a figure in the coordinate plane by a scale factor to get a similar figure is called a \_\_\_\_\_.
  - g. If two polygons are similar with a scale factor of 1, then the polygons are \_\_\_\_\_.
  
2. Determine whether each statement is *always*, *sometimes*, or *never* true.
  - a. Two similar triangles are congruent.
  - b. Two equilateral triangles are congruent.
  - c. An equilateral triangle is similar to a scalene triangle.
  - d. Two rectangles are similar.
  - e. Two isosceles right triangles are congruent.
  - f. Two isosceles right triangles are similar.
  - g. A square is similar to an equilateral triangle.
  - h. Two acute triangles are similar.
  - i. Two rectangles in which the length is twice the width are similar.
  - j. Two congruent polygons are similar.

**Helping You Remember**

3. A good way to remember a new mathematical vocabulary term is to relate it to words used in everyday life. The word *scale* has many meanings in English. Give three phrases that include the word *scale* in a way that is related to proportions.

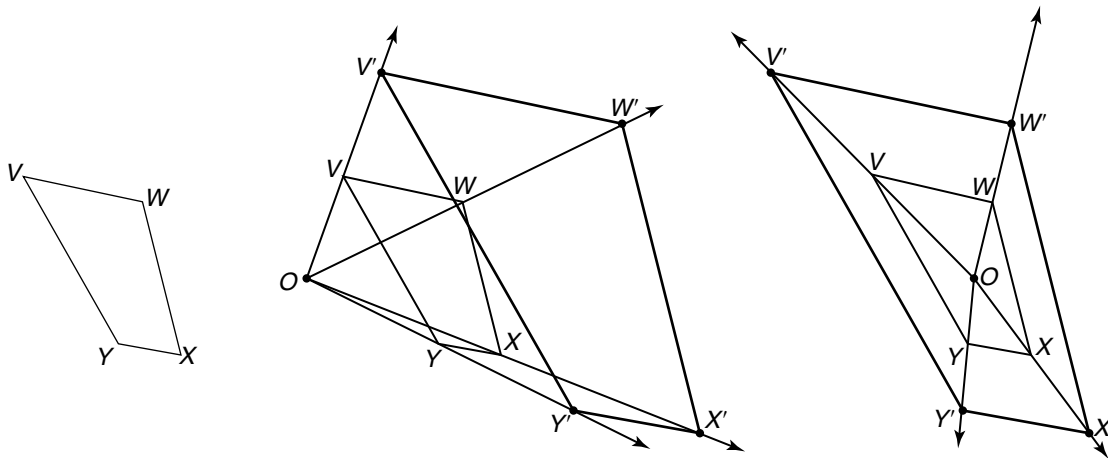
# 6-2 Enrichment

## Constructing Similar Polygons

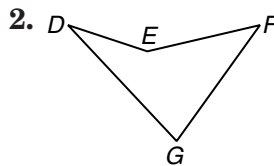
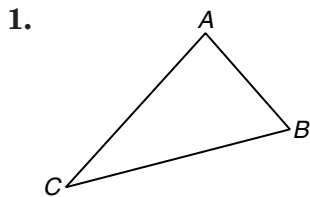
Here are four steps for constructing a polygon that is similar to and with sides twice as long as those of an existing polygon.

- Step 1** Choose any point either inside or outside the polygon and label it  $O$ .
- Step 2** Draw rays from  $O$  through each vertex of the polygon.
- Step 3** For vertex  $V$ , set the compass to length  $OV$ . Then locate a new point  $V'$  on ray  $OV$  such that  $VV' = OV$ . Thus,  $OV' = 2(OV)$ .
- Step 4** Repeat Step 3 for each vertex. Connect points  $V', W', X'$  and  $Y'$  to form the new polygon.

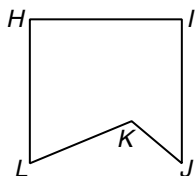
Two constructions of polygons similar to and with sides twice those of  $VWXY$  are shown below. Notice that the placement of point  $O$  does not affect the size or shape of  $V'W'X'Y'$ , only its location.



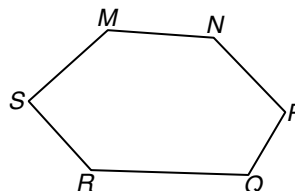
Trace each polygon. Then construct a similar polygon with sides twice as long as those of the given polygon.



3. Explain how to construct a similar polygon with sides three times the length of those of polygon  $HJKLM$ . Then do the construction.



4. Explain how to construct a similar polygon  $1\frac{1}{2}$  times the length of those of polygon  $MNPQRS$ . Then do the construction.



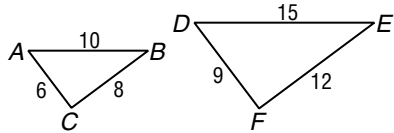
# 6-3 Study Guide and Intervention

## Similar Triangles

**Identify Similar Triangles** Here are three ways to show that two triangles are similar.

<b>AA Similarity</b>	Two angles of one triangle are congruent to two angles of another triangle.
<b>SSS Similarity</b>	The measures of the corresponding sides of two triangles are proportional.
<b>SAS Similarity</b>	The measures of two sides of one triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.

**Example 1** Determine whether the triangles are similar.



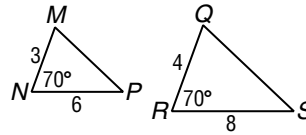
$$\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$$

$\triangle ABC \sim \triangle DEF$  by SSS Similarity.

**Example 2** Determine whether the triangles are similar.



$$\frac{3}{4} = \frac{6}{8}, \text{ so } \frac{MN}{RQ} = \frac{NP}{QS}$$

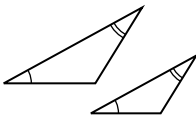
$m\angle N = m\angle R$ , so  $\angle N \cong \angle R$ .

$\triangle MNP \sim \triangle RQS$  by SAS Similarity.

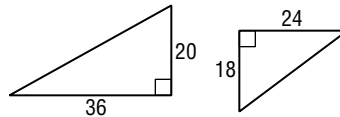
### Exercises

Determine whether each pair of triangles is similar. Justify your answer.

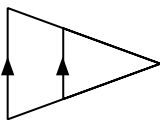
1.



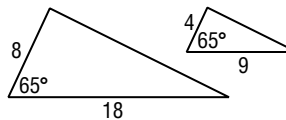
2.



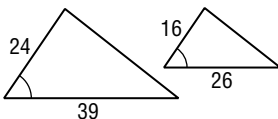
3.



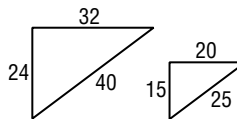
4.



5.



6.



# 6-3 Study Guide and Intervention *(continued)*

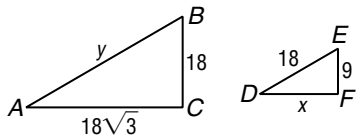
## Similar Triangles

**Use Similar Triangles** Similar triangles can be used to find measurements.

### Example 1

$$\triangle ABC \sim \triangle DEF.$$

Find  $x$  and  $y$ .



$$\frac{AC}{DF} = \frac{BC}{EF} \qquad \frac{AB}{DE} = \frac{BC}{EF}$$

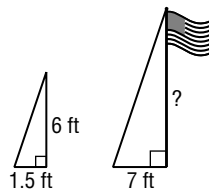
$$\frac{18\sqrt{3}}{x} = \frac{18}{9} \qquad \frac{y}{18} = \frac{18}{9}$$

$$18x = 9(18\sqrt{3}) \qquad 9y = 324$$

$$x = 9\sqrt{3} \qquad y = 36$$

### Example 2

A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?

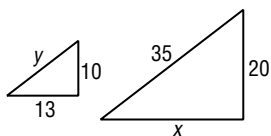


The sun's rays form similar triangles. Using  $x$  for the height of the pole,  $\frac{6}{x} = \frac{1.5}{7}$ , so  $1.5x = 42$  and  $x = 28$ . The flagpole is 28 feet tall.

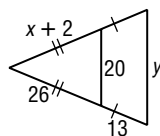
### Exercises

Each pair of triangles is similar. Find  $x$  and  $y$ .

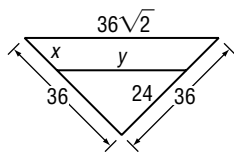
1.



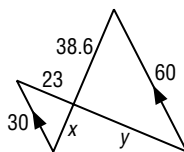
2.



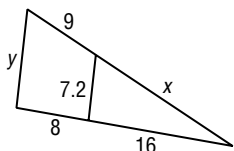
3.



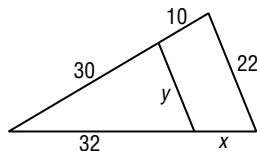
4.



5.



6.



7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post's shadow to the nearest hundredth?

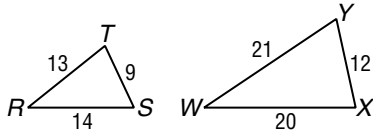


# 6-3 Skills Practice

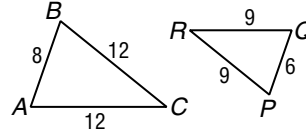
## Similar Triangles

Determine whether each pair of triangles is similar. Justify your answer.

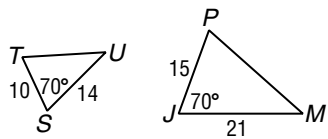
1.



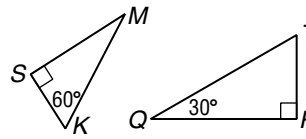
2.



3.

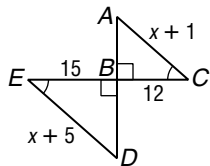


4.

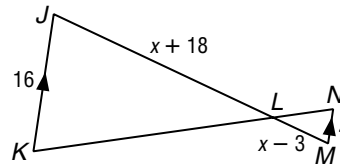


**ALGEBRA** Identify the similar triangles, and find  $x$  and the measures of the indicated sides.

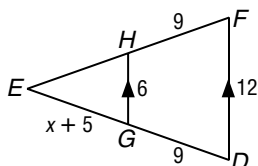
5.  $\overline{AC}$  and  $\overline{ED}$



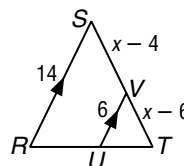
6.  $\overline{JL}$  and  $\overline{LM}$



7.  $\overline{EH}$  and  $\overline{EF}$



8.  $\overline{UT}$  and  $\overline{RT}$

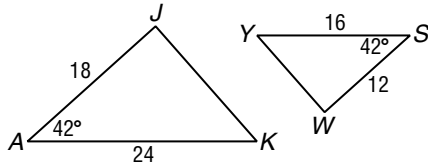


# 6-3 Practice

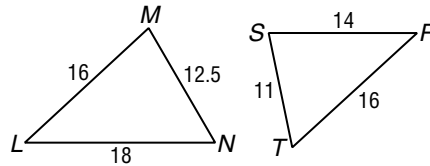
## Similar Triangles

Determine whether each pair of triangles is similar. Justify your answer.

1.

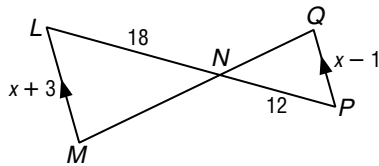


2.

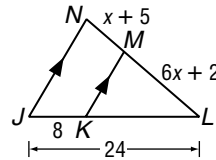


**ALGEBRA** Identify the similar triangles, and find  $x$  and the measures of the indicated sides.

3.  $\overline{LM}$  and  $\overline{QP}$

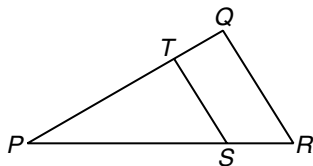


4.  $\overline{NL}$  and  $\overline{ML}$

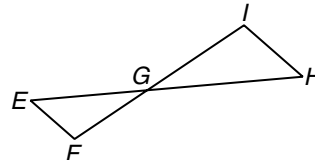


Use the given information to find each measure.

5. If  $\overline{TS} \parallel \overline{QR}$ ,  $TS = 6$ ,  $PS = x + 7$ ,  $QR = 8$ , and  $SR = x - 1$ , find  $PS$  and  $PR$ .



6. If  $\overline{EF} \parallel \overline{HI}$ ,  $EF = 3$ ,  $EG = x + 1$ ,  $HI = 4$ , and  $HG = x + 3$ , find  $EG$  and  $HG$ .



**INDIRECT MEASUREMENT** For Exercises 7 and 8, use the following information.

A lighthouse casts a 128-foot shadow. A nearby lamppost that measures 5 feet 3 inches casts an 8-foot shadow.

7. Write a proportion that can be used to determine the height of the lighthouse.

8. What is the height of the lighthouse?

## 6-3

## Reading to Learn Mathematics

*Similar Triangles***Pre-Activity** How do engineers use geometry?

Read the introduction to Lesson 6-3 at the top of page 298 in your textbook.

- What does it mean to say that triangular shapes result in rigid construction?
  
- What would happen if the shapes used in the construction were quadrilaterals?

**Reading the Lesson**

1. State whether each condition guarantees that two triangles are *congruent* or *similar*. If the condition guarantees that the triangles are both similar and congruent, write *congruent*. If there is not enough information to guarantee that the triangles will be congruent or similar, write *neither*.
  - a. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
  - b. The measures of all three pairs of corresponding sides are proportional.
  - c. Two angles of one triangle are congruent to two angles of the other triangle.
  - d. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
  - e. The measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.
  - f. The three sides of one triangle are congruent to the three sides of the other triangle.
  - g. The three angles of one triangle are congruent to the three angles of the other triangle.
  - h. One acute angle of a right triangle is congruent to one acute angle of another right triangle.
  - i. The measures of two sides of a triangle are proportional to the measures of two sides of another triangle.
2. Identify each of the following as an example of a *reflexive*, *symmetric*, or *transitive* property.
  - a. If  $\triangle RST \sim \triangle UVW$ , then  $\triangle UVW \sim \triangle RST$ .
  - b. If  $\triangle RST \sim \triangle UVW$  and  $\triangle UVW \sim \triangle OPQ$ , then  $\triangle RST \sim \triangle OPQ$ .
  - c.  $\triangle RST \sim \triangle RST$

**Helping You Remember**

3. A good way to remember something is to explain it to someone else. Suppose one of your classmates is having trouble understanding the difference between SAS for congruent triangles and SAS for similar triangles. How can you explain the difference to him?

## 6-3 Enrichment

### Ratio Puzzles with Triangles

If you know the perimeter of a triangle and the ratios of the sides, you can find the lengths of the sides.

#### Example

The perimeter of a triangle is 84 units. The sides have lengths  $r$ ,  $s$ , and  $t$ . The ratio of  $s$  to  $r$  is 5:3, and the ratio of  $t$  to  $r$  is 2:1. Find the length of each side.

Since both ratios contain  $r$ , rewrite one or both ratios to make  $r$  the same. You can write the ratio of  $t$  to  $r$  as 6:3. Now you can write a three-part ratio.

$$r:s:t = 3:5:6$$

There is a number  $x$  such that  $r = 3x$ ,  $s = 5x$ , and  $t = 6x$ . Since you know the perimeter, 84, you can use algebra to find the lengths of the sides.

$$r + s + t = 84$$

$$3x + 5x + 6x = 84$$

$$14x = 84$$

$$x = 6$$

$$3x = 18, 5x = 30, 6x = 36$$

So  $r = 18$ ,  $s = 30$ , and  $t = 36$ .

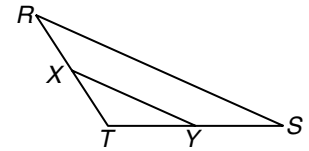
#### Find the lengths of the sides of each triangle.

- The perimeter of a triangle is 75 units. The sides have lengths  $a$ ,  $b$ , and  $c$ . The ratio of  $b$  to  $a$  is 3:5, and the ratio of  $c$  to  $a$  is 7:5. Find the length of each side.
- The perimeter of a triangle is 88 units. The sides have lengths  $d$ ,  $e$ , and  $f$ . The ratio of  $e$  to  $d$  is 3:1, and the ratio of  $f$  to  $e$  is 10:9. Find the length of each side.
- The perimeter of a triangle is 91 units. The sides have lengths  $p$ ,  $q$ , and  $r$ . The ratio of  $p$  to  $r$  is 3:1, and the ratio of  $q$  to  $r$  is 5:2. Find the length of each side.
- The perimeter of a triangle is 68 units. The sides have lengths  $g$ ,  $h$ , and  $j$ . The ratio of  $j$  to  $g$  is 2:1, and the ratio of  $h$  to  $g$  is 5:4. Find the length of each side.
- Write a problem similar to those above involving ratios in triangles.

# 6-4 Study Guide and Intervention

## Parallel Lines and Proportional Parts

**Proportional Parts of Triangles** In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. The converse is also true.



If  $X$  and  $Y$  are the midpoints of  $\overline{RT}$  and  $\overline{TS}$ , then  $\overline{XY}$  is a **midsegment** of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

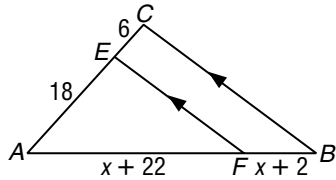
If  $\overline{XY} \parallel \overline{RS}$ , then  $\frac{RX}{XT} = \frac{SY}{YT}$ .

If  $\frac{RX}{XT} = \frac{SY}{YT}$ , then  $\overline{XY} \parallel \overline{RS}$ .

If  $\overline{XY}$  is a midsegment, then  $\overline{XY} \parallel \overline{RS}$  and  $XY = \frac{1}{2}RS$ .

### Example 1

In  $\triangle ABC$ ,  $\overline{EF} \parallel \overline{CB}$ . Find  $x$ .



Since  $\overline{EF} \parallel \overline{CB}$ ,  $\frac{AF}{FB} = \frac{AE}{EC}$ .

$$\frac{x + 22}{x + 2} = \frac{18}{6}$$

$$6x + 132 = 18x + 36$$

$$96 = 12x$$

$$8 = x$$

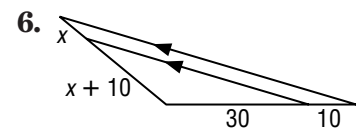
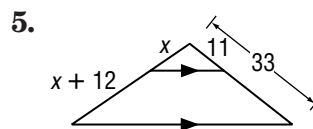
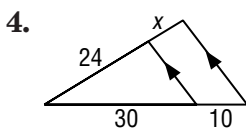
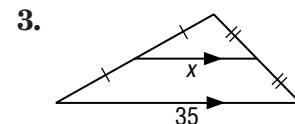
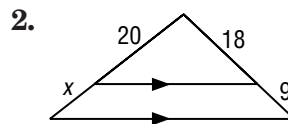
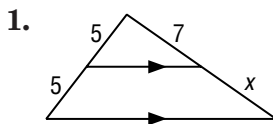
### Example 2

A triangle has vertices  $D(3, 6)$ ,  $E(-3, -2)$ , and  $F(7, -2)$ . Midsegment  $\overline{GH}$  is parallel to  $\overline{EF}$ . Find the length of  $\overline{GH}$ .

$\overline{GH}$  is a midsegment, so its length is one-half that of  $\overline{EF}$ . Points  $E$  and  $F$  have the same  $y$ -coordinate, so  $EF = 7 - (-3) = 10$ . The length of midsegment  $\overline{GH}$  is 5.

### Exercises

Find  $x$ .

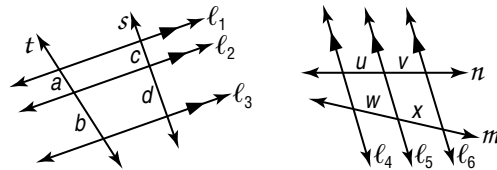


7. In Example 2, find the slope of  $\overline{EF}$  and show that  $\overline{EF} \parallel \overline{GH}$ .

# 6-4 Study Guide and Intervention *(continued)*

## Parallel Lines and Proportional Parts

**Divide Segments Proportionally** When three or more parallel lines cut two transversals, they separate the transversals into proportional parts. If the ratio of the parts is 1, then the parallel lines separate the transversals into congruent parts.

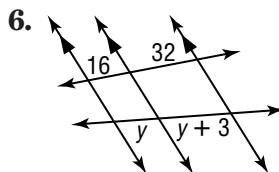
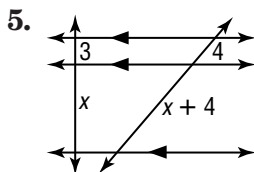
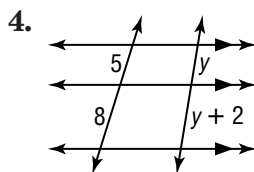
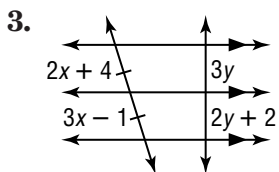
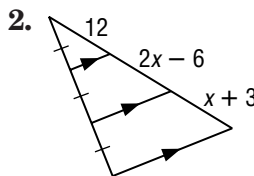
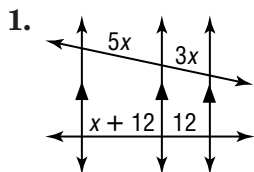


If  $l_1 \parallel l_2 \parallel l_3$ , then  $\frac{a}{b} = \frac{c}{d}$ .  
 If  $l_4 \parallel l_5 \parallel l_6$  and  $\frac{u}{v} = 1$ , then  $\frac{w}{x} = 1$ .

**Example** Refer to lines  $l_1, l_2,$  and  $l_3$  above. If  $a = 3, b = 8,$  and  $c = 5,$  find  $d$ .  
 $l_1 \parallel l_2 \parallel l_3$  so  $\frac{3}{8} = \frac{5}{d}$ . Then  $3d = 40$  and  $d = 13\frac{1}{3}$ .

### Exercises

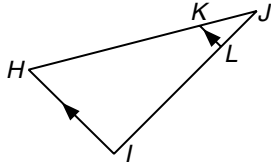
Find  $x$  and  $y$ .



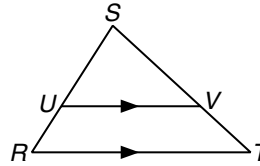
# 6-4 Skills Practice

## Parallel Lines and Proportional Parts

1. If  $JK = 7$ ,  $KH = 21$ , and  $JL = 6$ , find  $LI$ .

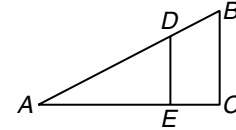


2. Find  $x$  and  $TV$  if  $RU = 8$ ,  $US = 14$ ,  $TV = x - 1$  and  $VS = 17.5$ .



Determine whether  $\overline{BC} \parallel \overline{DE}$ .

3.  $AD = 15$ ,  $DB = 12$ ,  $AE = 10$ , and  $EC = 8$

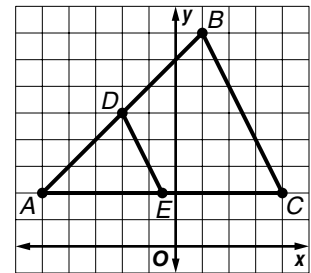


4.  $BD = 9$ ,  $BA = 27$ , and  $CE$  is one third of  $EA$

5.  $AE = 30$ ,  $AC = 45$ , and  $AD$  is twice  $DB$

**COORDINATE GEOMETRY** For Exercises 6–8, use the following information.

Triangle  $ABC$  has vertices  $A(-5, 2)$ ,  $B(1, 8)$ , and  $C(4, 2)$ . Point  $D$  is the midpoint of  $\overline{AB}$  and  $E$  is the midpoint of  $\overline{AC}$ .

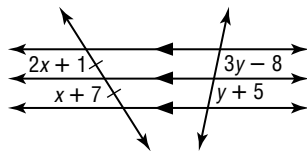


6. Identify the coordinates of  $D$  and  $E$ .

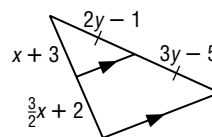
7. Show that  $\overline{BC}$  is parallel to  $\overline{DE}$ .

8. Show that  $DE = \frac{1}{2}BC$ .

9. Find  $x$  and  $y$ .



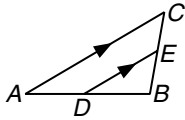
10. Find  $x$  and  $y$ .



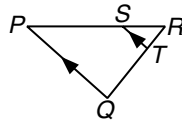
# 6-4 Practice

## Parallel Lines and Proportional Parts

1. If  $AD = 24$ ,  $DB = 27$ , and  $EB = 18$ , find  $CE$ .

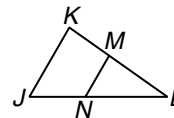


2. Find  $x$ ,  $QT$ , and  $TR$  if  $QT = x + 6$ ,  $SR = 12$ ,  $PS = 27$ , and  $TR = x - 4$ .



Determine whether  $\overline{JK} \parallel \overline{NM}$ .

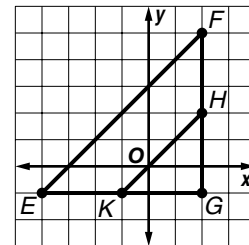
3.  $JN = 18$ ,  $JL = 30$ ,  $KM = 21$ , and  $ML = 35$



4.  $KM = 24$ ,  $KL = 44$ , and  $NL = \frac{5}{6}JN$

**COORDINATE GEOMETRY** For Exercises 5 and 6, use the following information.

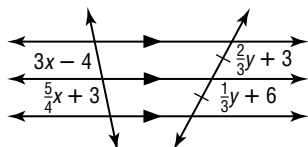
Triangle  $EFG$  has vertices  $E(-4, -1)$ ,  $F(2, 5)$ , and  $G(2, -1)$ . Point  $K$  is the midpoint of  $\overline{EG}$  and  $H$  is the midpoint of  $\overline{FG}$ .



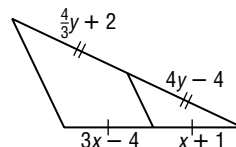
5. Show that  $\overline{EF}$  is parallel to  $\overline{KH}$ .

6. Show that  $KH = \frac{1}{2}EF$ .

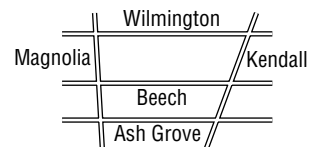
7. Find  $x$  and  $y$ .



8. Find  $x$  and  $y$ .



9. **MAPS** The distance from Wilmington to Ash Grove along Kendall is 820 feet and along Magnolia, 660 feet. If the distance between Beech and Ash Grove along Magnolia is 280 feet, what is the distance between the two streets along Kendall?





## 6-4

## Reading to Learn Mathematics

*Parallel Lines and Proportional Parts***Pre-Activity** How do city planners use geometry?

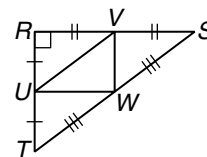
Read the introduction to Lesson 6-4 at the top of page 307 in your textbook.

Use a geometric idea to explain why the distance between Chicago Avenue and Ontario Street is shorter along Michigan Avenue than along Lake Shore Drive.

**Reading the Lesson**

- Provide the missing words to complete the statement of each theorem. Then state the name of the theorem.
  - If a line intersects two sides of a triangle and separates the sides into corresponding segments of \_\_\_\_\_ lengths, then the line is \_\_\_\_\_ to the third side.
  - A midsegment of a triangle is \_\_\_\_\_ to one side of the triangle and its length is \_\_\_\_\_ the length of that side.
  - If a line is \_\_\_\_\_ to one side of a triangle and intersects the other two sides in \_\_\_\_\_ distinct points, then it separates these sides into \_\_\_\_\_ of proportional length.

- Refer to the figure at the right.



- Name the three midsegments of  $\triangle RST$ .
- If  $RS = 8$ ,  $RU = 3$ , and  $TW = 5$ , find the length of each of the midsegments.
- What is the perimeter of  $\triangle RST$ ?
- What is the perimeter of  $\triangle UVW$ ?
- What are the perimeters of  $\triangle RUV$ ,  $\triangle SVW$ , and  $\triangle TUW$ ?
- How are the perimeters of each of the four small triangles related to the perimeter of the large triangle?
- Would the relationship that you found in part f apply to any triangle in which the midpoints of the three sides are connected?

**Helping You Remember**

- A good way to remember a new mathematical term is to relate it to other mathematical vocabulary that you already know. What is an easy way to remember the definition of *midsegment* using other geometric terms?

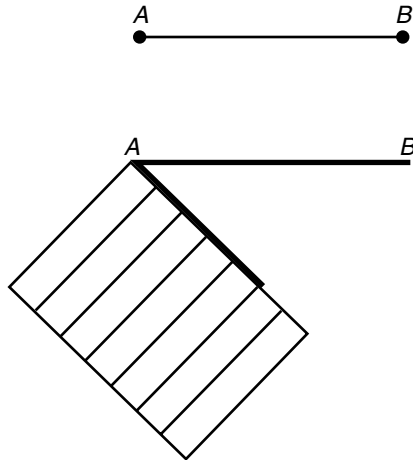
# 6-4 Enrichment

## Parallel Lines and Congruent Parts

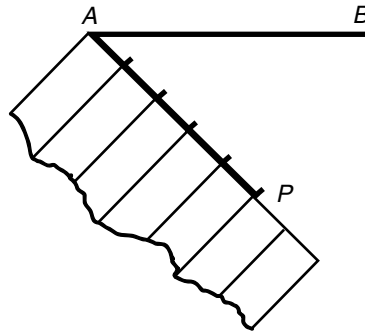
There is a theorem stating that if three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal. This can be shown for any number of parallel lines. The following drafting technique uses this fact to divide a segment into congruent parts.

$\overline{AB}$  is to be separated into five congruent parts. This can be done very accurately without using a ruler. All that is needed is a compass and a piece of notebook paper.

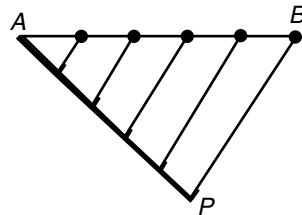
**Step 1** Hold the corner of a piece of notebook paper at point  $A$ .



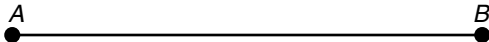
**Step 2** From point  $A$ , draw a segment along the paper that is five spaces long. Mark where the lines of the notebook paper meet the segment. Label the fifth point,  $P$ .



**Step 3** Draw  $\overline{PB}$ . Through each of the other marks on  $\overline{AP}$ , construct a line parallel to  $\overline{BP}$ . The points where these lines intersect  $\overline{AB}$  will divide  $\overline{AB}$  into five congruent segments.



**Use a compass and a piece of notebook paper to divide each segment into the given number of congruent parts.**

1. six congruent parts 

2. seven congruent parts 

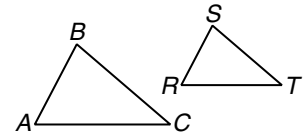
# 6-5 Study Guide and Intervention

## Parts of Similar Triangles

**Perimeters** If two triangles are similar, their perimeters have the same proportion as the corresponding sides.

If  $\triangle ABC \sim \triangle RST$ , then

$$\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$$



**Example** Use the diagram above with  $\triangle ABC \sim \triangle RST$ . If  $AB = 24$  and  $RS = 15$ , find the ratio of their perimeters.

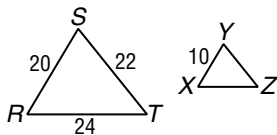
Since  $\triangle ABC \sim \triangle RST$ , the ratio of the perimeters of  $\triangle ABC$  and  $\triangle RST$  is the same as the ratio of corresponding sides.

Therefore  $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle RST} = \frac{24}{15} = \frac{8}{5}$

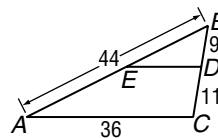
### Exercises

Each pair of triangles is similar. Find the perimeter of the indicated triangle.

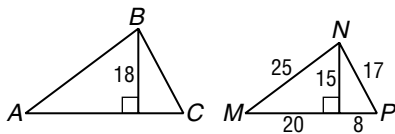
1.  $\triangle XYZ$



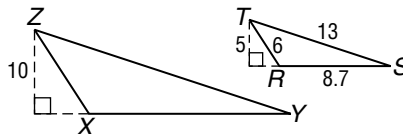
2.  $\triangle BDE$



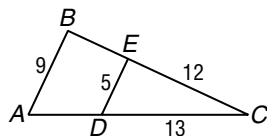
3.  $\triangle ABC$



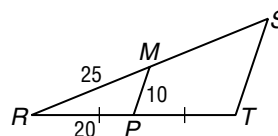
4.  $\triangle XYZ$



5.  $\triangle ABC$



6.  $\triangle RST$



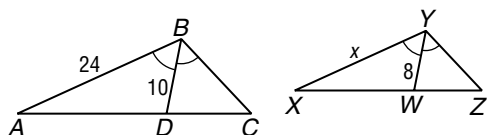
# 6-5 Study Guide and Intervention *(continued)*

## Parts of Similar Triangles

**Special Segments of Similar Triangles** When two triangles are similar, corresponding altitudes, angle bisectors, and medians are proportional to the corresponding sides. Also, in any triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides of the triangle.

**Example 1** In the figure,

$\triangle ABC \sim \triangle XYZ$ , with angle bisectors as shown. Find  $x$ .



Since  $\triangle ABC \sim \triangle XYZ$ , the measures of the angle bisectors are proportional to the measures of a pair of corresponding sides.

$$\frac{AB}{XY} = \frac{BD}{YW}$$

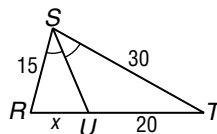
$$\frac{24}{x} = \frac{10}{8}$$

$$10x = 24(8)$$

$$10x = 192$$

$$x = 19.2$$

**Example 2**  $\overline{SU}$  bisects  $\angle RST$ . Find  $x$ .



Since  $\overline{SU}$  is an angle bisector,  $\frac{RU}{TU} = \frac{RS}{TS}$ .

$$\frac{x}{20} = \frac{15}{30}$$

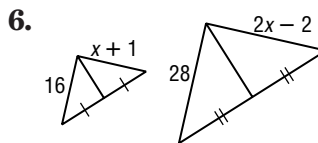
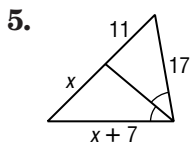
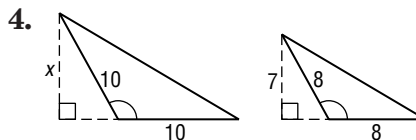
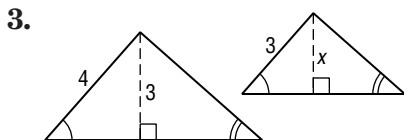
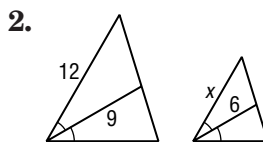
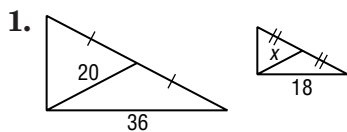
$$30x = 20(15)$$

$$30x = 300$$

$$x = 10$$

### Exercises

Find  $x$  for each pair of similar triangles.

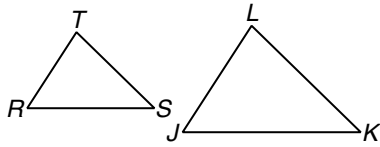


# 6-5 Skills Practice

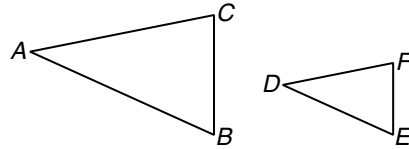
## Parts of Similar Triangles

Find the perimeter of the given triangle.

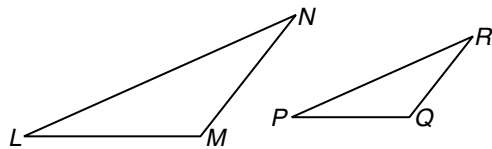
1.  $\triangle JKL$ , if  $\triangle JKL \sim \triangle RST$ ,  $RS = 14$ ,  $ST = 12$ ,  $TR = 10$ , and  $LJ = 14$



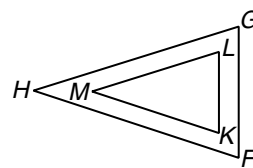
2.  $\triangle DEF$ , if  $\triangle DEF \sim \triangle ABC$ ,  $AB = 27$ ,  $BC = 16$ ,  $CA = 25$ , and  $FD = 15$



3.  $\triangle PQR$ , if  $\triangle PQR \sim \triangle LMN$ ,  $LM = 16$ ,  $MN = 14$ ,  $NL = 27$ , and  $RP = 18$

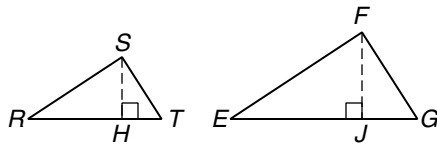


4.  $\triangle KLM$ , if  $\triangle KLM \sim \triangle FGH$ ,  $FG = 30$ ,  $GH = 38$ ,  $HF = 38$ , and  $KL = 24$

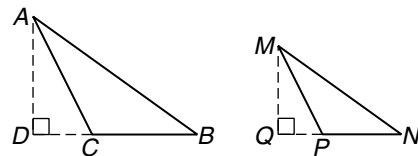


Use the given information to find each measure.

5. Find  $FG$  if  $\triangle RST \sim \triangle EFG$ ,  $\overline{SH}$  is an altitude of  $\triangle RST$ ,  $\overline{FJ}$  is an altitude of  $\triangle EFG$ ,  $ST = 6$ ,  $SH = 5$ , and  $FJ = 7$ .

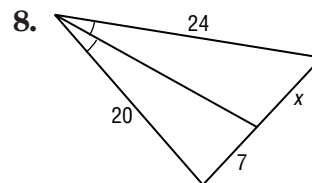
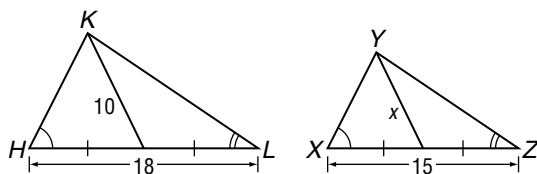


6. Find  $MN$  if  $\triangle ABC \sim \triangle MNP$ ,  $\overline{AD}$  is an altitude of  $\triangle ABC$ ,  $\overline{MQ}$  is an altitude of  $\triangle MNP$ ,  $AB = 24$ ,  $AD = 14$ , and  $MQ = 10.5$ .



Find  $x$ .

7.  $\triangle HKL \sim \triangle XYZ$

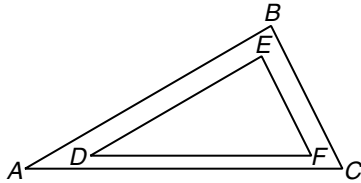


# 6-5 Practice

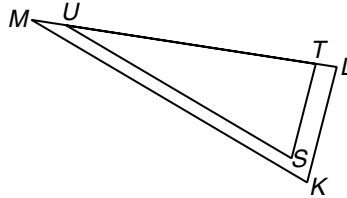
## Parts of Similar Triangles

Find the perimeter of the given triangle.

1.  $\triangle DEF$ , if  $\triangle ABC \sim \triangle DEF$ ,  $AB = 36$ ,  $BC = 20$ ,  $CA = 40$ , and  $DE = 35$

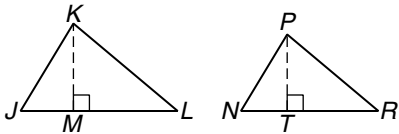


2.  $\triangle STU$ , if  $\triangle STU \sim \triangle KLM$ ,  $KL = 12$ ,  $LM = 31$ ,  $MK = 32$ , and  $US = 28$

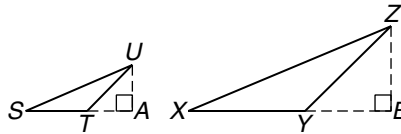


Use the given information to find each measure.

3. Find  $PR$  if  $\triangle JKL \sim \triangle NPR$ ,  $\overline{KM}$  is an altitude of  $\triangle JKL$ ,  $\overline{PT}$  is an altitude of  $\triangle NPR$ ,  $KL = 28$ ,  $KM = 18$ , and  $PT = 15.75$ .



4. Find  $ZY$  if  $\triangle STU \sim \triangle XYZ$ ,  $\overline{UA}$  is an altitude of  $\triangle STU$ ,  $\overline{ZB}$  is an altitude of  $\triangle XYZ$ ,  $UT = 8.5$ ,  $UA = 6$ , and  $ZB = 11.4$ .



Find  $x$ .

- 5.

- 6.

**PHOTOGRAPHY** For Exercises 7 and 8, use the following information.

Francine has a camera in which the distance from the lens to the film is 24 millimeters.

7. If Francine takes a full-length photograph of her friend from a distance of 3 meters and the height of her friend is 140 centimeters, what will be the height of the image on the film? (*Hint*: Convert to the same unit of measure.)
8. Suppose the height of the image on the film of her friend is 15 millimeters. If Francine took a full-length shot, what was the distance between the camera and her friend?

# 6-5 Reading to Learn Mathematics

## Parts of Similar Triangles

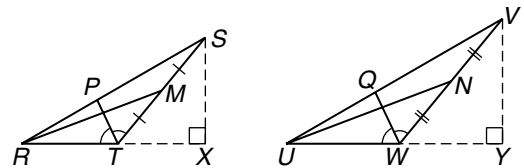
### Pre-Activity How is geometry related to photography?

Read the introduction to Lesson 6-5 at the top of page 316 in your textbook.

- How is similarity involved in the process of making a photographic print from a negative?
  
- Why do photographers place their cameras on tripods?

### Reading the Lesson

1. In the figure,  $\triangle RST \sim \triangle UVW$ . Complete each proportion involving the lengths of segments in this figure by replacing the question mark. Then identify the definition or theorem from the list below that the completed proportion illustrates.



- |  |                                    |
|--|------------------------------------|
| i. Definition of congruent polygons  | ii. Definition of similar polygons |
| iii. Proportional Perimeters Theorem   | iv. Angle Bisectors Theorem        |
| v. Similar triangles have corresponding altitudes proportional to corresponding sides.         |                                    |
| vi. Similar triangles have corresponding medians proportional to corresponding sides.          |                                    |
| vii. Similar triangles have corresponding angle bisectors proportional to corresponding sides. |                                    |

a.  $\frac{RS + ST + TR}{?} = \frac{RS}{UV}$

b.  $\frac{RT}{UW} = \frac{SX}{?}$

c.  $\frac{RM}{UN} = \frac{?}{VW}$

d.  $\frac{RS}{UV} = \frac{ST}{?}$

e.  $\frac{RP}{PS} = \frac{?}{ST}$

f.  $\frac{UN}{?} = \frac{UW}{RT}$

g.  $\frac{TP}{WQ} = \frac{RT}{?}$

h.  $\frac{UW}{VW} = \frac{?}{QV}$

### Helping You Remember

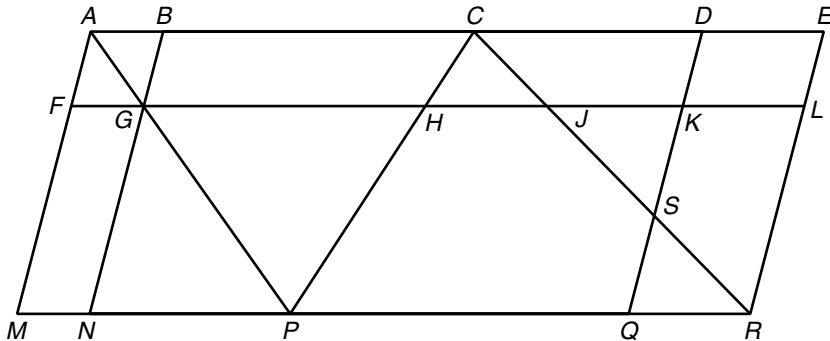
2. A good way to remember a large amount of information is to remember key words. What key words will help you remember the features of similar triangles that are proportional to the lengths of the corresponding sides?

# 6-5 Enrichment

## Proportions for Similar Triangles

Recall that if a line crosses two sides of a triangle and is parallel to the third side, then the line separates the two sides that it crosses into segments of proportional lengths.

You can write many proportions by identifying similar triangles in the following diagram. In the diagram,  $\overline{AM} \parallel \overline{BN}$ ,  $\overline{AE} \parallel \overline{FL} \parallel \overline{MR}$ , and  $\overline{DQ} \parallel \overline{ER}$ .



Answer each question. Use the diagram above.

1. Name a triangle similar to  $\triangle GNP$ .
2. Name a triangle similar to  $\triangle CJH$ .
3. Name two triangles similar to  $\triangle JKS$ .
4. Name a triangle similar to  $\triangle ACP$ .

Complete each proportion.

5.  $\frac{AG}{AP} = \frac{AF}{?}$
6.  $\frac{CP}{CH} = \frac{CR}{?}$
7.  $\frac{JS}{JR} = \frac{?}{JL}$
8.  $\frac{PH}{PC} = \frac{PG}{?}$
9.  $\frac{ER}{LR} = \frac{?}{JR}$
10.  $\frac{MN}{MP} = \frac{?}{AP}$

Solve.

11. If  $CJ = 16$ ,  $JR = 48$ , and  $LR = 30$ , find  $EL$ .
12. If  $DK = 5$ ,  $KS = 7$ , and  $CJ = 8$ , find  $JS$ .
13. If  $MN = 12$ ,  $NP = 32$ , and  $AP = 48$ , find  $AG$ . Round to the nearest tenth.
14. If  $CH = 18$ ,  $HP = 82$ , and  $CR = 130$ , find  $CJ$ .
15. Write three more problems that can be solved using the diagram above.

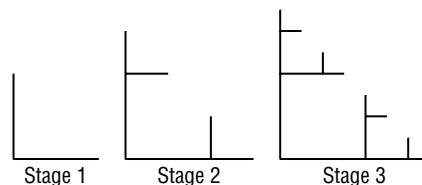


# 6-6 Study Guide and Intervention

## Fractals and Self-Similarity

**Characteristics of Fractals** The act of repeating a process over and over, such as finding a third of a segment, then a third of the new segment, and so on, is called **iteration**. When the process of iteration is applied to some geometric figures, the results are called **fractals**. For objects such as fractals, when a portion of the object has the same shape or characteristics as the entire object, the object can be called **self-similar**.

**Example** In the diagram at the right, notice that the details at each stage are similar to the details at Stage 1.

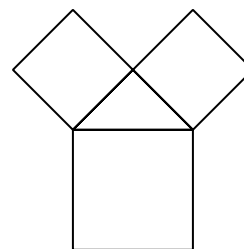


### Exercises

1. Follow the iteration process below to produce a fractal.

#### Stage 1

- Draw a square.
- Draw an isosceles right triangle on the top side of the square. Use the side of the square as the hypotenuse of the triangle.
- Draw a square on each leg of the right triangle.



#### Stage 2

Repeat the steps in Stage 1, drawing an isosceles triangle and two small squares for each of the small squares from Stage 1.

#### Stage 3

Repeat the steps in Stage 1 for each of the smallest squares in Stage 2.

2. Is the figure produced in Stage 3 self-similar?

**6-6 Study Guide and Intervention** *(continued)***Fractals and Self-Similarity**

**Nongeometric Iteration** An iterative process can be applied to an algebraic expression or equation. The result is called a **recursive formula**.

**Example**

Find the value of  $x^3$ , where the initial value of  $x$  is 2. Repeat the process three times and describe the pattern.

Initial value: 2

First time:  $2^3 = 8$

Second time:  $8^3 = 512$

Third time:  $512^3 = 134,217,728$

The result of each step of the iteration is used for the next step. For this example, the  $x$  values are greater with each iteration. There is no maximum value, so the values are described as *approaching infinity*.

**Exercises**

For Exercises 1–5, find the value of each expression. Then use that value as the next  $x$  in the expression. Repeat the process three times, and describe your observations.

1.  $\sqrt{x}$ , where  $x$  initially equals 5

2.  $\frac{1}{x}$ , where  $x$  initially equals 2

3.  $3^x$ , where  $x$  initially equals 1

4.  $x - 5$  where  $x$  initially equals 10

5.  $x^2 - 4$ , where  $x$  initially equals 16

6. Harpesh paid \$1000 for a savings certificate. It earns interest at an annual rate of 2.8%, and interest is added to the certificate each year. What will the certificate be worth after four years?

## 6-6

**Skills Practice*****Fractals and Self-Similarity***

Stages 1 and 2 of a fractal known as the Cantor set are shown. To get Stage 2, the segment in Stage 1 is trisected, and the interior of the middle segment is removed. (The interior of a segment is the segment with its endpoints removed.) The process is repeated for subsequent stages.

Stage 1 \_\_\_\_\_

Stage 2 \_\_\_\_\_

1. Draw stages 3 and 4 of the Cantor set.

Stage 3

Stage 4

2. How many segments are there in Stage 3? Stage 4?

3. What happens to the length of the line segments in each stage?

4. The Cantor set is the set of points after infinitely many iterations. Is the Cantor set self-similar?

**Find the value of each expression. Then, use that value as the next  $x$  in the expression. Repeat the process three times, and describe your observations.**

5.  $2x - 3$ , where  $x$  initially equals 5

6.  $\frac{x}{2}$ , where  $x$  initially equals 1

**Find the first three iterates of each expression.**

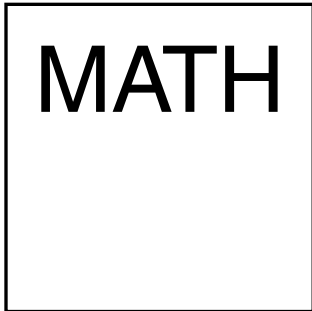
7.  $x^2 - 1$ , where  $x$  initially equals 2

8.  $\frac{1}{x}$ , where  $x$  initially equals 8

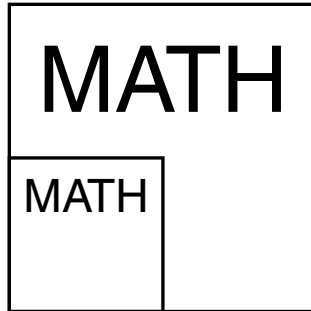
**6-6 Practice****Fractals and Self-Similarity**

An artist is designing a book cover and wants to show a copy of the book cover in the lower left corner of the cover. After Stages 1 and 2 of the design, he realizes that the design is developing into a fractal!

Stage 1



Stage 2



1. Draw Stages 3 and 4 of the book cover fractal.

Stage 3

Stage 4

2. After infinitely many iterations, will the result be a self similar fractal?
3. On what part of the book cover should you focus your attention to be sure you can find a copy of the entire figure?

**Find the value of each expression. Then, use that value as the next  $x$  in the expression. Repeat the process three times and describe your observations.**

4.  $2(x + 3)$ , where  $x$  initially equals 0      5.  $\frac{x}{2} - 3$ , where  $x$  initially equals 10

**Find the first three iterates of each expression.**

6.  $\frac{x}{2} + 1$ , where  $x$  initially equals 1      7.  $2x^2$ , where  $x$  initially equals 2

8. **HOUSING** The Andrews purchased a house for \$96,000. The real estate agent who sold the house said that comparable houses in the area appreciate at a rate of 4.5% per year. If this pattern continues, what will be the value of the house in three years? Round to the nearest whole number.

**6-6**

# Reading to Learn Mathematics

## Fractals and Self-Similarity

### Pre-Activity How is mathematics found in nature?

Read the introduction to Lesson 6-6 at the top of page 325 in your textbook.

Name two objects from nature other than broccoli in which a small piece resembles the whole.

### Reading the Lesson

1. Match each definition from the first column with a term from the second column. (Some words or phrases in the second column may be used more than once or not at all.)

**Phrase**

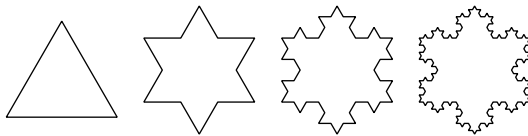
- a. a geometric figure that is created using iteration
- b. a pattern in which smaller and smaller details of a shape have the same geometric characteristics as the original shape
- c. the result of translating an iterative process into a formula or algebraic equation
- d. the process of repeating the same procedure over and over again

**Term**

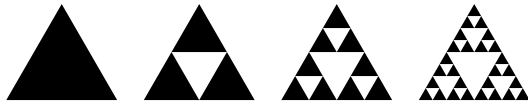
- i. similar
- ii. iteration
- iii. fractal
- iv. self-similar
- v. recursion formula
- vi. congruent

2. Refer to the three patterns below.

i.



ii.



iii.



- a. Give the special name for each of the fractals you obtain by continuing the pattern without end.
- b. Which of these fractals are self-similar?
- c. Which of these fractals are strictly self-similar?

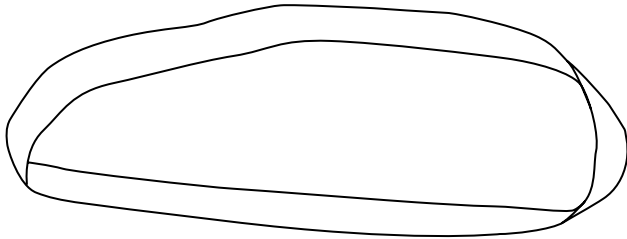
### Helping You Remember

3. A good way to remember a new mathematical term is to relate it to everyday English words. The word *fractal* is related to *fraction* and *fragment*. Use your dictionary to find at least one definition for each of these words that you think is related to the meaning of the word *fractal* and explain the connection.

## 6-6 Enrichment

### *The Möbius Strip*

A Möbius strip is a special surface with only one side. It was discovered by August Ferdinand Möbius, a German astronomer and mathematician.



- To make a Möbius strip, cut a strip of paper about 16 inches long and 1 inch wide. Mark the ends with the letters *A*, *B*, *C*, and *D* as shown below.



Twist the paper once, connecting *A* to *D* and *B* to *C*. Tape the ends together on both sides.

- Use a crayon or pencil to shade one side of the paper. Shade around the strip until you get back to where you started. What happens?
- What do you think will happen if you cut the Möbius strip down the middle? Try it.
- Make another Möbius strip. Starting a third of the way in from one edge, cut around the strip, staying always the same distance in from the edge. What happens?
- Start with another long strip of paper. Twist the paper twice and connect the ends. What happens when you cut down the center of this strip?
- Start with another long strip of paper. Twist the paper three times and connect the ends. What happens when you cut down the center of this strip?

# 6 Chapter 6 Test, Form 1

Write the letter for the correct answer in the blank at the right of each question.

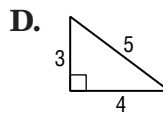
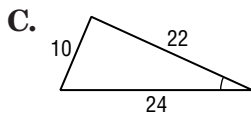
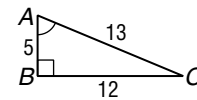
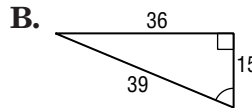
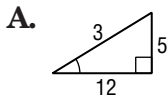
1. There are 15 plums and 9 apples in a fruit bowl. Find the ratio of apples to plums. 1. \_\_\_\_\_  
**A.** 3:5                      **B.** 3:8                      **C.** 5:3                      **D.** 8:3

2. The scale drawing of a porch is 8 inches wide by 12 inches long. If the actual porch is 12 feet wide, find the length of the porch. 2. \_\_\_\_\_  
**A.** 8 ft                      **B.** 10 ft                      **C.** 16 ft                      **D.** 18 ft

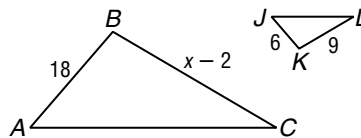
3. Solve  $\frac{5}{6} = \frac{4}{x}$ . 3. \_\_\_\_\_  
**A.** 4.6                      **B.** 4.8                      **C.** 5                      **D.** 7

4. A quality control technician checked a sample of 30 bulbs. Two of the bulbs were defective. If the sample was representative, find the number of bulbs expected to be defective in a case of 450. 4. \_\_\_\_\_  
**A.** 24                      **B.** 30                      **C.** 36                      **D.** 45

5. Find the triangle similar to  $\triangle ABC$  at the right. 5. \_\_\_\_\_

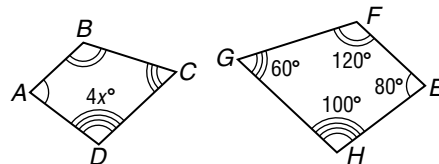


6. Find  $x$  if  $\triangle ABC \sim \triangle JKL$ . 6. \_\_\_\_\_  
**A.** 10                      **B.** 14  
**C.** 25                      **D.** 29

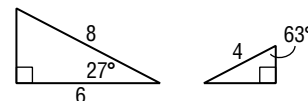


7. Quadrilateral  $ABCD \sim$  quadrilateral  $PQRS$ . If  $AB = 10$ ,  $BC = 6$ ,  $PS = 12$ , and  $QR = 4$ , find the scale factor of  $ABCD$  to  $PQRS$ . 7. \_\_\_\_\_  
**A.**  $\frac{1}{2}$                       **B.**  $\frac{3}{2}$                       **C.**  $\frac{5}{3}$                       **D.**  $\frac{5}{6}$

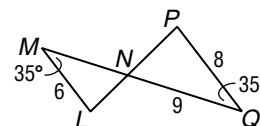
8. If quadrilateral  $ABCD \sim$  quadrilateral  $EFGH$ , find  $x$ . 8. \_\_\_\_\_  
**A.** 15                      **B.** 20  
**C.** 25                      **D.** 30



9. Which theorem or postulate can be used to prove that these two triangles are similar? 9. \_\_\_\_\_  
**A.** AA                      **B.** SAS                      **C.** SSA                      **D.** SSS



10. Find  $MN$ . 10. \_\_\_\_\_  
**A.**  $5\frac{1}{3}$                       **B.**  $6\frac{3}{4}$                       **C.** 7                      **D.** 12



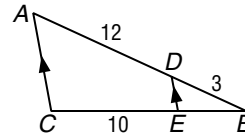
# 6 Chapter 6 Test, Form 1 *(continued)*

11. A 5-foot tall student cast a 4-foot shadow. If the tree next to her cast a 44-foot shadow, what is the height of the tree? 11. \_\_\_\_\_

- A.  $35\frac{1}{5}$  ft      B. 45 ft      C.  $51\frac{1}{2}$  ft      D. 55 ft

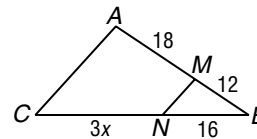
12. In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{AC}$ . If  $AD = 12$ ,  $BD = 3$ , and  $CE = 10$ , find  $BE$ . 12. \_\_\_\_\_

- A. 1      B.  $1\frac{1}{2}$   
C. 2      D.  $2\frac{1}{2}$



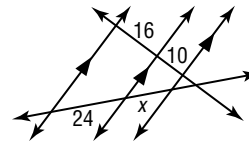
13. Find  $x$  so that  $\overline{AC} \parallel \overline{MN}$  in  $\triangle ABC$ . 13. \_\_\_\_\_

- A. 8      B. 10  
C. 25      D. 29



14. Find  $x$ . 14. \_\_\_\_\_

- A. 14      B. 15  
C. 16      D. 18

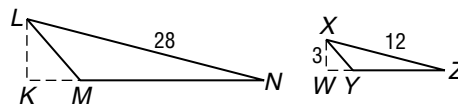


15. If  $\triangle FGH \sim \triangle PQR$ ,  $FG = 6$ ,  $PQ = 10$ , and the perimeter of  $\triangle PQR$  is 35, find the perimeter of  $\triangle FGH$ . 15. \_\_\_\_\_

- A. 21      B. 27      C. 31      D.  $58\frac{1}{3}$

16.  $\triangle LMN \sim \triangle XYZ$  with altitudes  $\overline{KL}$  and  $\overline{WX}$ . Find  $KL$ . 16. \_\_\_\_\_

- A. 6      B. 7  
C. 9      D. 19



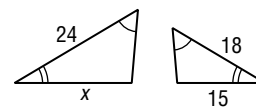
17. Find  $x$ . 17. \_\_\_\_\_

- A. 5      B. 6  
C.  $6\frac{1}{2}$       D.  $7\frac{1}{2}$



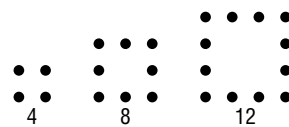
18. Find  $x$ . 18. \_\_\_\_\_

- A. 16      B. 18  
C. 20      D. 21

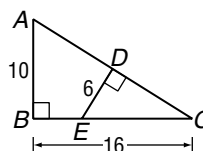


19. Each arrangement of dots forms a *box number*. Find the number of dots in the eighth box number. 19. \_\_\_\_\_

- A. 32      B. 48  
C. 64      D. 512



**Bonus** In  $\triangle ABC$ ,  $AB = 10$ ,  $BC = 16$ ,  $\overline{DE} \perp \overline{AC}$ , and  $DE = 6$ . Find  $CD$ . B: \_\_\_\_\_





# 6 Chapter 6 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. Of the 240 students eating lunch, 96 purchased their lunch and the rest brought a bag lunch. Find the ratio of students purchasing lunch to students bringing a bag lunch. 1. \_\_\_\_\_

A. 2:3                      B. 2:5                      C. 3:2                      D. 5:2

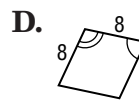
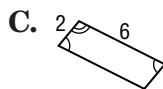
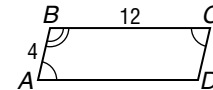
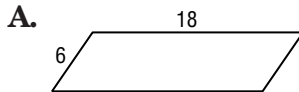
2. In a rectangle, the ratio of the width to the length is 4:5. If the rectangle is 40 centimeters long, find its width. 2. \_\_\_\_\_

A. 32 cm                      B. 36 cm                      C. 44 cm                      D. 50 cm

3. A postage stamp 25 millimeters wide and 40 millimeter tall is enlarged to make a poster. The poster is 4 feet wide. Find the height of the poster. 3. \_\_\_\_\_

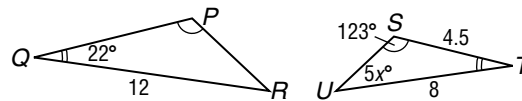
A. 2.5 ft                      B. 5.25 ft                      C. 5.8 ft                      D. 6.4 ft

4. Find the polygon that is similar to  $ABCD$ . 4. \_\_\_\_\_



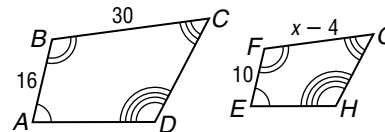
5. If  $\triangle PQR \sim \triangle STU$ , find  $x$ . 5. \_\_\_\_\_

A. 4.4                      B. 7  
C. 24.6                      D. 35



6. If  $ABCD \sim EFGH$ , find  $x$ . 6. \_\_\_\_\_

A. 18.75                      B. 20  
C. 22.75                      D. 28

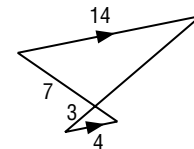


7. If  $\triangle ABC \sim \triangle LMN$ ,  $AB = 18$ ,  $BC = 12$ ,  $LN = 9$ , and  $LM = 6$ , find the scale factor of  $\triangle ABC$  to  $\triangle LMN$ . 7. \_\_\_\_\_

A.  $\frac{9}{2}$                       B.  $\frac{3}{2}$                       C.  $\frac{3}{1}$                       D.  $\frac{2}{1}$

8. Name the theorem or postulate that can be used to prove that these triangles are similar. 8. \_\_\_\_\_

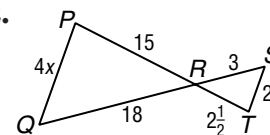
A. AA                      B. SSS  
C. SAS                      D. SSA



For Questions 9 and 10, refer to the figure at the right.

9. Identify the true statement. 9. \_\_\_\_\_

A.  $\triangle PQR \sim \triangle RST$                       B.  $\triangle PQR \sim \triangle STR$   
C.  $\triangle PQR \sim \triangle TSR$                       D.  $\triangle PQR \sim \triangle TRS$



10. Find  $x$ . 10. \_\_\_\_\_

A.  $2\frac{1}{2}$                       B. 3                      C.  $3\frac{1}{2}$                       D. 4

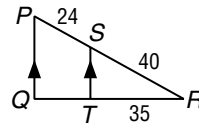
# 6 Chapter 6 Test, Form 2A *(continued)*

11. A 24-foot flagpole cast a 20-foot shadow. The building next to it cast an 85-foot shadow. Find the height of the building. 11. \_\_\_\_\_

- A.  $70\frac{5}{6}$  ft      B. 89 ft      C.  $96\frac{1}{6}$  ft      D. 102 ft

12. Find  $QT$ .

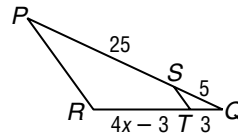
- A. 15      B. 17  
C. 19      D. 21



12. \_\_\_\_\_

13. Find  $x$  so that  $\overline{ST} \parallel \overline{PR}$ .

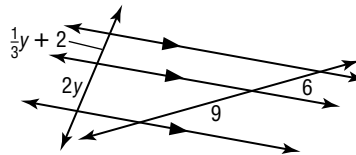
- A. 4      B.  $4\frac{1}{2}$   
C. 6      D.  $6\frac{1}{2}$



13. \_\_\_\_\_

14. Find  $y$ .

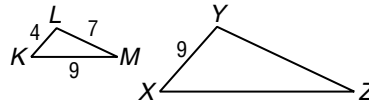
- A.  $\frac{4}{3}$       B. 2  
C.  $\frac{7}{3}$       D. 3



14. \_\_\_\_\_

15. If  $\triangle KLM \sim \triangle XYZ$ , find the perimeter of  $\triangle XYZ$ .

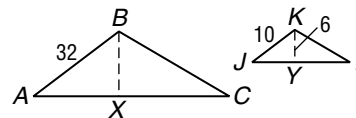
- A. 40      B. 42  
C. 45      D. 48



15. \_\_\_\_\_

16.  $\triangle ABC \sim \triangle JKL$  with altitudes  $\overline{BX}$  and  $\overline{KY}$ . Find  $BX$ .

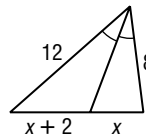
- A. 19.2      B. 21  
C. 24.6      D. 28



16. \_\_\_\_\_

17. Find  $x$ .

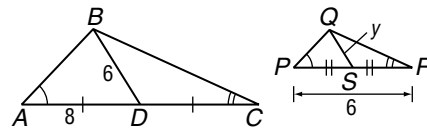
- A. 4      B. 5  
C. 6      D. 8



17. \_\_\_\_\_

18. Find  $y$ .

- A.  $2\frac{1}{4}$       B.  $2\frac{3}{4}$   
C.  $3\frac{1}{2}$       D.  $4\frac{1}{2}$

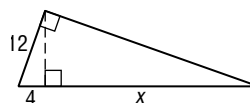


18. \_\_\_\_\_

19. Find the third iterate of the expression  $2(3x + 1)$ , where  $x$  initially equals 1. 19. \_\_\_\_\_

- A. 24      B. 64      C. 302      D. 474

**Bonus** Find  $x$ .



**B:** \_\_\_\_\_

# 6 Chapter 6 Test, Form 2B

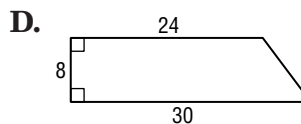
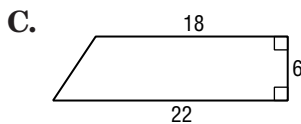
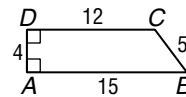
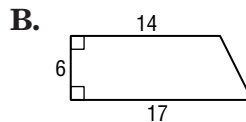
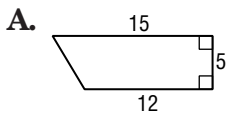
Write the letter for the correct answer in the blank at the right of each question.

1. Given the choice between doing an oral and a written report, 18 of the 28 students chose to do an oral report. Find the ratio of written to oral reports. 1. \_\_\_\_\_  
**A.** 5:9                      **B.** 9:5                      **C.** 9:14                      **D.** 14:9

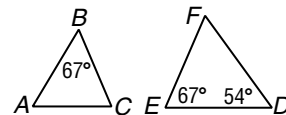
2. A model of a lighthouse has diameter 8 inches and height 18 inches. If the actual diameter of the lighthouse is 20 feet, find its actual height. 2. \_\_\_\_\_  
**A.** 30 ft                      **B.** 35 ft                      **C.** 45 ft                      **D.** 50 ft

3. The three sides of a triangle are in the ratio 2:4:5. If the shortest side of the triangle is 4 meters long, find the perimeter. 3. \_\_\_\_\_  
**A.** 17 m                      **B.** 22 m                      **C.** 32 m                      **D.** 40 m

4. Find the polygon that may be similar to  $ABCD$ . 4. \_\_\_\_\_



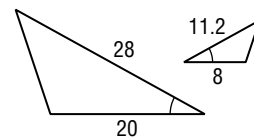
5. If  $\triangle ABC \sim \triangle DEF$ , find  $m\angle C$ . 5. \_\_\_\_\_  
**A.**  $54^\circ$                       **B.**  $59^\circ$   
**C.**  $67^\circ$                       **D.**  $69^\circ$



6. If quadrilateral  $JKLM \sim$  quadrilateral  $WXYZ$ ,  $JK = 15$ ,  $LM = 10$ ,  $XY = 6$ , and  $WX = 9$ , find  $KL$ . 6. \_\_\_\_\_  
**A.** 8                      **B.** 10                      **C.** 11                      **D.** 12

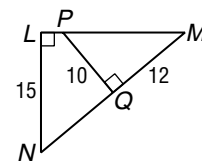
7. If  $\triangle LMN \sim \triangle RST$ ,  $LN = 21$ ,  $MN = 28$ , and the scale factor of  $\triangle RST$  to  $\triangle LMN$  is  $\frac{4}{3}$ , find  $ST$ . 7. \_\_\_\_\_  
**A.**  $15\frac{3}{4}$                       **B.** 21                      **C.** 28                      **D.**  $37\frac{1}{3}$

8. Name the theorem or postulate that can be used to prove that these triangles are similar. 8. \_\_\_\_\_  
**A.** AA                      **B.** SAS  
**C.** SSA                      **D.** SSS



For Questions 9 and 10, refer to the figure at the right.

9. Identify the similar triangles. 9. \_\_\_\_\_  
**A.**  $\triangle LMN \sim \triangle MPQ$                       **B.**  $\triangle LMN \sim \triangle QMP$   
**C.**  $\triangle LMN \sim \triangle QPM$                       **D.**  $\triangle LMN \sim \triangle PQM$



10. Find  $LM$ . 10. \_\_\_\_\_  
**A.** 16                      **B.** 17                      **C.** 18                      **D.** 20

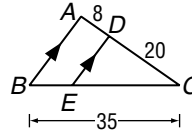
# 6 Chapter 6 Test, Form 2B *(continued)*

11. A 6-foot tall fence post cast a  $2\frac{1}{2}$ -foot shadow. A nearby clock tower cast a 35-foot shadow. Find the height of the tower. 11. \_\_\_\_\_

- A.  $37\frac{1}{2}$  ft      B. 71 ft      C. 78 ft      D. 84 ft

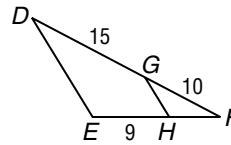
12. Find  $CE$ . 12. \_\_\_\_\_

- A. 25      B. 26  
C. 27      D. 28



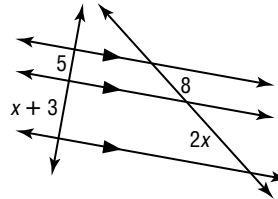
13. Find  $FH$  so that  $\overline{GH} \parallel \overline{DE}$ . 13. \_\_\_\_\_

- A. 4      B. 5  
C. 6      D. 7



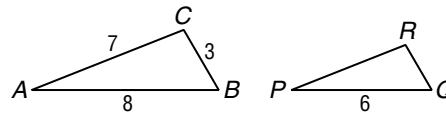
14. Find  $x$ . 14. \_\_\_\_\_

- A. 4      B. 6  
C. 9      D. 12



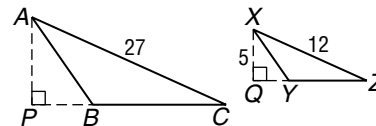
15. If  $\triangle ABC \sim \triangle PQR$ , find the perimeter of  $\triangle PQR$ . 15. \_\_\_\_\_

- A. 12      B.  $13\frac{1}{2}$   
C.  $14\frac{1}{2}$       D. 16



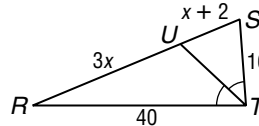
16.  $\triangle ABC \sim \triangle XYZ$  with altitudes  $\overline{AP}$  and  $\overline{XQ}$ . Find  $AP$ . 16. \_\_\_\_\_

- A.  $11\frac{1}{4}$       B. 14  
C.  $15\frac{1}{4}$       D. 20



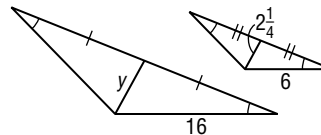
17. Find  $x$ . 17. \_\_\_\_\_

- A. 7      B. 8  
C. 9      D. 10



18. Find  $y$ . 18. \_\_\_\_\_

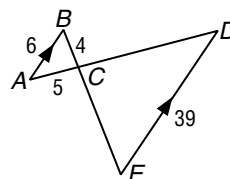
- A.  $3\frac{3}{4}$       B. 4  
C. 6      D.  $7\frac{1}{2}$



19. Find the third iterate of the expression  $3(2x - 1)$ , where  $x$  initially equals 2. 19. \_\_\_\_\_

- A. 27      B. 63      C. 174      D. 303

**Bonus** Find  $CE$ .



**B:** \_\_\_\_\_

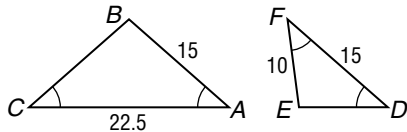
# 6 Chapter 6 Test, Form 2C

1. Of the 300 television sets sold at an electronics store last month, 90 were flat-screen TVs. Find the ratio of flat-screen TVs to other TVs sold last month.

1. \_\_\_\_\_

2. Determine whether  $\triangle ABC \sim \triangle DEF$ . Justify your answer.

2. \_\_\_\_\_



3. When a 5-foot vertical pole casts a 3-foot, 4-inch shadow, an oak tree casts a 20-foot shadow. Find the height of the tree.

3. \_\_\_\_\_

4. If quadrilateral  $ABCD \sim$  quadrilateral  $WXYZ$ ,  $AB = 15$ ,  $BC = 27$ , and the scale factor of  $WXYZ$  to  $ABCD$  is  $\frac{2}{3}$ , find  $XY$ .

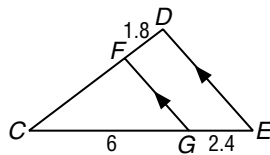
4. \_\_\_\_\_

5. The blueprint for a swimming pool is 8 inches by  $2\frac{1}{2}$  inches. The actual pool is 136 feet long. Find the width of the pool.

5. \_\_\_\_\_

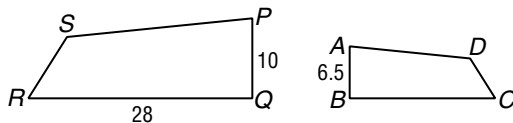
6. Find  $CD$ .

6. \_\_\_\_\_



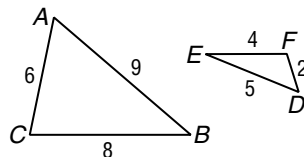
7. If quadrilateral  $ABCD \sim$  quadrilateral  $PQRS$ , find  $BC$ .

7. \_\_\_\_\_



8. Determine whether  $\triangle ABC \sim \triangle DEF$ . Justify your answer.

8. \_\_\_\_\_



9.  $\triangle ABC \sim \triangle XYZ$ ,  $AB = 12$ ,  $AC = 16$ ,  $BC = 20$ , and  $XZ = 24$ . Find the perimeter of  $\triangle XYZ$ .

9. \_\_\_\_\_

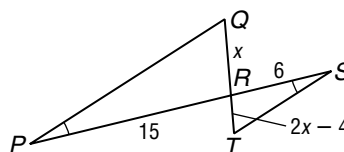
**For Questions 10 and 11, use the figure.**

10. Identify the similar triangles.

10. \_\_\_\_\_

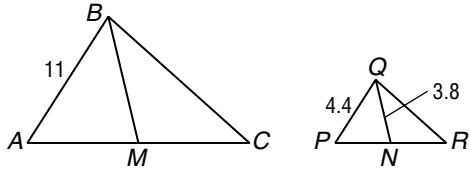
11. Find  $x$ .

11. \_\_\_\_\_



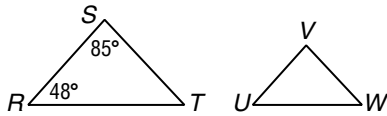
# 6 Chapter 6 Test, Form 2C *(continued)*

12. If  $\triangle ABC \sim \triangle PQR$  and  $\overline{BM}$  and  $\overline{QN}$  are medians, find  $BM$ . 12. \_\_\_\_\_

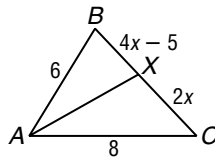


13. The ratio of the measures of the three sides of a triangle is 3:4:6. If the perimeter is 91, find the measure of the longest side. 13. \_\_\_\_\_

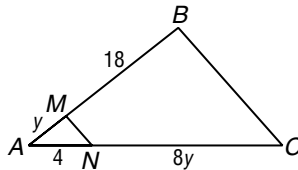
14. If  $\triangle RST \sim \triangle UVW$ , find  $m\angle W$ . 14. \_\_\_\_\_



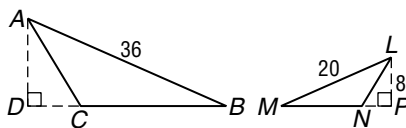
15. In  $\triangle ABC$ ,  $\overline{AX}$  bisects  $\angle BAC$ . Find  $x$ . 15. \_\_\_\_\_



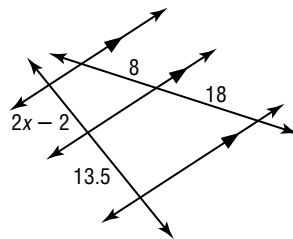
16. Find  $y$  so that  $\overline{MN} \parallel \overline{BC}$ . 16. \_\_\_\_\_



17.  $\triangle ABC \sim \triangle LMN$ , and  $\overline{AD}$  and  $\overline{LP}$  are altitudes. Find  $AD$ . 17. \_\_\_\_\_

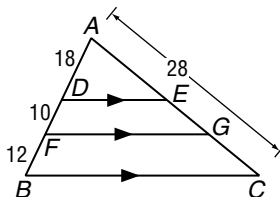


18. Find  $x$ . 18. \_\_\_\_\_



19. Find the third iterate of the expression  $x^2 + 2$ , where  $x$  initially equals 1. 19. \_\_\_\_\_

- Bonus** Find  $EG$ . B: \_\_\_\_\_



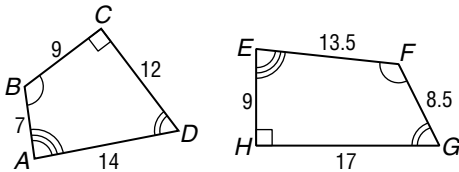
# 6 Chapter 6 Test, Form 2D

1. Of the 112 students in the marching band, 35 were in the drum section. Find the ratio of drummers to other musicians in the band.

1. \_\_\_\_\_

2. Determine whether quadrilateral  $ABCD \sim$  quadrilateral  $EFGH$ . Justify your answer.

2. \_\_\_\_\_



3. When a 9-foot tall garden shed cast a 5-foot, 3-inch shadow, a house cast a 28-foot shadow. Find the height of the house.

3. \_\_\_\_\_

4.  $\triangle ABC \sim \triangle FGH$ ,  $AB = 24$ ,  $AC = 16$ ,  $GH = 9$ , and  $FH = 12$ . Find the scale factor of  $\triangle ABC$  to  $\triangle FGH$ .

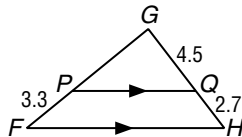
4. \_\_\_\_\_

5. The model of a suspension bridge is 18 inches long and 2 inches tall. If the length of the actual bridge is 1650 feet, find its height.

5. \_\_\_\_\_

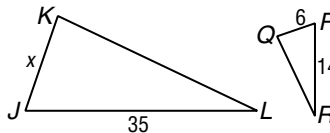
6. Find  $GP$ .

6. \_\_\_\_\_



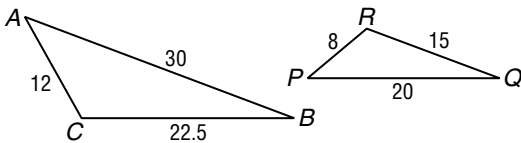
7. If  $\triangle JKL \sim \triangle PQR$ , find  $x$ .

7. \_\_\_\_\_



8. Determine whether  $\triangle ABC \sim \triangle PQR$ . Justify your answer.

8. \_\_\_\_\_



9.  $\triangle ABC \sim \triangle PQR$ ,  $AB = 18$ ,  $BC = 20$ ,  $AC = 22$ , and  $QR = 25$ . Find the perimeter of  $\triangle PQR$ .

9. \_\_\_\_\_

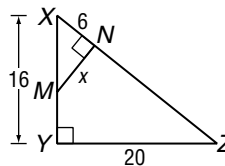
For Questions 10 and 11, use the figure.

10. Identify the similar triangles.

10. \_\_\_\_\_

11. Find  $MN$ .

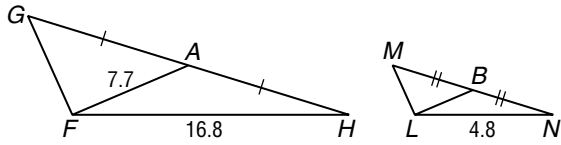
11. \_\_\_\_\_



# 6 Chapter 6 Test, Form 2D *(continued)*

12. If  $\triangle FGH \sim \triangle LMN$  and  $\overline{AF}$  and  $\overline{BL}$  are medians, find  $BL$ .

12. \_\_\_\_\_

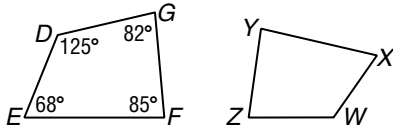


13. The ratio of the measures of the three angles of a triangle is 3:4:8. Find the measure of the largest angle.

13. \_\_\_\_\_

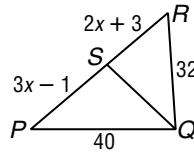
14. If quadrilateral  $DEFG \sim$  quadrilateral  $WXYZ$ , find  $m\angle Y$ .

14. \_\_\_\_\_



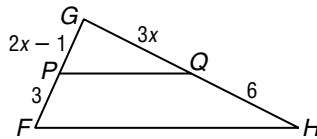
15. In  $\triangle PQR$ ,  $\overline{QS}$  bisects  $\angle PQR$ . Find  $x$ .

15. \_\_\_\_\_



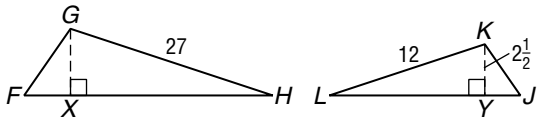
16. Find  $x$  so that  $\overline{PQ} \parallel \overline{FH}$ .

16. \_\_\_\_\_



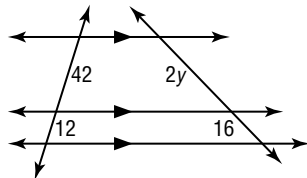
17. If  $\triangle FGH \sim \triangle JKL$ , find  $GX$ .

17. \_\_\_\_\_



18. Find  $y$ .

18. \_\_\_\_\_

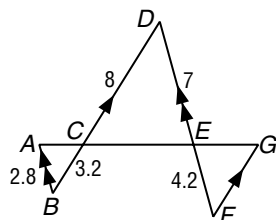


19. Find the third iterate of the expression  $4(x - 2)$ , where  $x$  initially equals 10.

19. \_\_\_\_\_

**Bonus** Find  $FG$ .

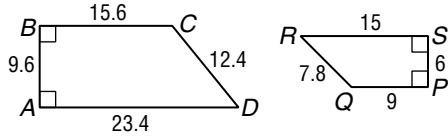
**B:** \_\_\_\_\_



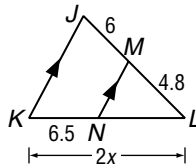


# 6 Chapter 6 Test, Form 3

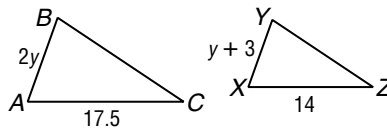
- In an orchard of apple and peach trees,  $\frac{3}{7}$  of the trees are peaches. Find the ratio of apple trees to peach trees.
- Determine whether trapezoid  $ABCD \sim$  trapezoid  $PQRS$ . Justify your answer.



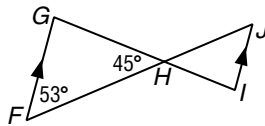
- In  $\triangle ABC$ ,  $m\angle A = 51$ ,  $AB = 14$ , and  $AC = 20$ . In  $\triangle DEF$ ,  $m\angle D = 51$ ,  $DE = 16.8$ , and  $DF = 24$ . Determine whether  $\triangle ABC \sim \triangle DEF$ . Justify your answer.
- A painting that is 48 inches by 12 inches is reduced to fit on an area that is 30 centimeters by 10 centimeters. Find the maximum dimensions of the reduced painting.
- Find  $x$ .



- If  $\triangle ABC \sim \triangle XYZ$ , find  $y$ .

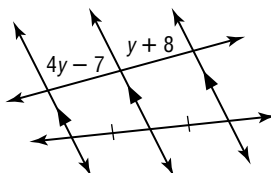


- The ratio of the measures of the sides of a triangle is 2:5:6. If the length of the longest side is 48 inches, find the perimeter.
- Find  $m\angle I$ .



- $\triangle ABC \sim \triangle DEF$ ,  $AB = 8$ ,  $BC = 13$ ,  $AC = 15$ , and  $DF = 20$ . Find the perimeter of  $\triangle DEF$ .
- $\triangle ABC \sim \triangle JKL$ ,  $AB = 12$ ,  $BC = 18.4$ ,  $KL = 6.9$ , and  $JL = 5.6$ . Find the scale factor of  $\triangle ABC$  to  $\triangle JKL$ .

- Find  $y$ .



1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

7. \_\_\_\_\_

8. \_\_\_\_\_

9. \_\_\_\_\_

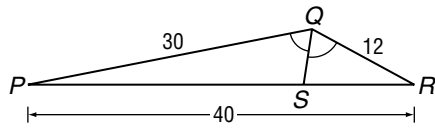
10. \_\_\_\_\_

11. \_\_\_\_\_

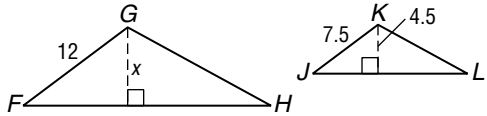
# 6 Chapter 6 Test, Form 3 *(continued)*

12. Find  $SR$ .

12. \_\_\_\_\_



For Questions 13 and 14,  $\triangle FGH \sim \triangle JKL$ .



13. Find  $x$ .

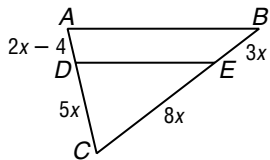
13. \_\_\_\_\_

14. Find the ratio of the perimeter of  $\triangle FGH$  to the perimeter of  $\triangle JKL$ .

14. \_\_\_\_\_

15. Find  $AD$  so that  $\overline{DE} \parallel \overline{AB}$ .

15. \_\_\_\_\_



16. Find the fourth iterate of the expression  $3x - 1$ , where  $x$  initially equals 0.8.

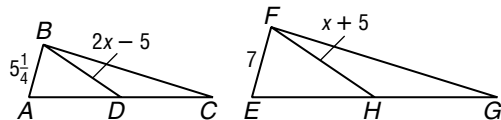
16. \_\_\_\_\_

17. When a 15-foot tall climbing wall cast a 20-foot shadow, a building cast a 32-foot shadow. Find the height of the building.

17. \_\_\_\_\_

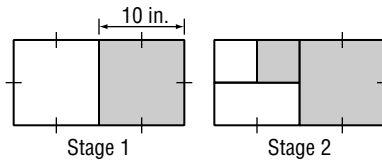
18. If  $\triangle ABC \sim \triangle EFG$  and  $\overline{BD}$  and  $\overline{FH}$  are medians, find  $BD$ .

18. \_\_\_\_\_



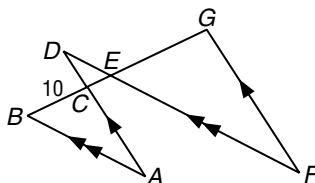
19. Stage 1 is repeated in the upper left half of the unshaded square of each stage. Find the perimeter of the smallest square in Stage 3.

19. \_\_\_\_\_



**Bonus** The ratio of the lengths of the sides of  $\triangle ABC$  is 4:5:2.5. The scale factor of  $\triangle ABC$  to  $\triangle DEC$  is 5:2, and the scale factor of  $\triangle DEC$  to  $\triangle FEG$  is 1:4.  $\overline{BC}$  is the shortest side and  $\overline{AB}$  is the longest side of  $\triangle ABC$ . Find  $FG$ .

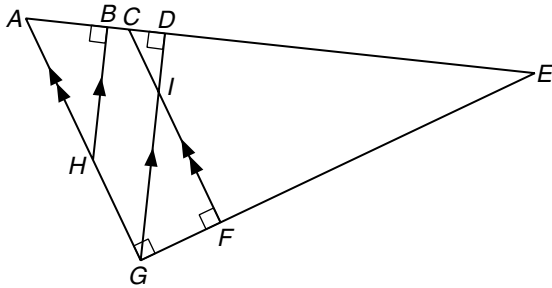
**B:** \_\_\_\_\_



# 6 Chapter 6 Open-Ended Assessment

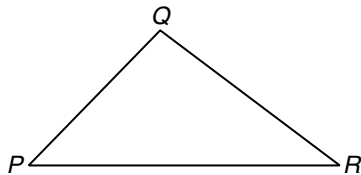
**Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.**

1. Write a possible proportion if the extremes are 3 and 10.
2.  $\triangle ABC$  and  $\triangle WXY$  are isosceles triangles.
  - a. Write a possible ratio for the sides of  $\triangle ABC$  if its perimeter is 42 inches.
  - b. Name possible measures for the sides of  $\triangle ABC$  using your answer to Part a.
  - c. If  $\triangle WXY$  has a perimeter of 28 and  $\triangle ABC$  has sides with the measures you gave in Part b, what must be the measure of the sides of  $\triangle WXY$  so that  $\triangle WXY \sim \triangle ABC$ ?
3. Write as many triangle similarity statements as possible for the figure below. How do you know that these triangles are similar?



4. Sketch two triangles that are *not* similar, but have one pair of corresponding angles congruent and two pairs of corresponding sides proportional. Label the corresponding angles and the proportional sides.

5.



- a. Draw  $\triangle XYZ$  inside  $\triangle PQR$  with half the perimeter of  $\triangle PQR$ . Explain your process and why it works.
- b. Draw the figure that results when your process in Part a is iterated 3 times.
- c. Explain why your figure in Part b is a fractal.
- d. The perimeter of  $\triangle PQR$  is  $\frac{1}{2}x$ . If  $x$  equals 96, what is the perimeter of the smallest triangle in your figure in Part b?

cross products  
extremes  
fractal

iteration  
means  
midsegment

proportion  
ratio  
scale factor

self-similar  
similar polygons

**Choose from the terms above to complete each sentence.**

1. If there are 15 girls and 9 boys in an art class, the \_\_\_\_\_ of girls to boys in the class is 5:3. 1. \_\_\_\_\_
2. If  $\triangle ABC \sim \triangle DEF$ ,  $AB = 10$ , and  $DE = 2.5$ , then the \_\_\_\_\_ of  $\triangle ABC$  to  $\triangle DEF$  is 4:1. 2. \_\_\_\_\_
3. In  $\triangle LMN$ ,  $P$  lies on  $\overline{LM}$  and  $Q$  on  $\overline{LN}$ . If  $PQ = \frac{1}{2}MN$ ,  $\overline{PQ}$  is called a(n) \_\_\_\_\_. 3. \_\_\_\_\_
4. The product of the \_\_\_\_\_ in the equation  $\frac{3}{x} = \frac{24}{30}$  is 90. 4. \_\_\_\_\_
5. The product of the \_\_\_\_\_ in the equation  $\frac{3}{x} = \frac{24}{30}$  is  $24x$ . 5. \_\_\_\_\_
6. A geometric figure created by using iteration is called a(n) \_\_\_\_\_. 6. \_\_\_\_\_
7. If quadrilaterals  $ABCD$  and  $WXYZ$  have corresponding angles congruent and corresponding sides proportional, they are called \_\_\_\_\_. 7. \_\_\_\_\_
8. In a recursive formula, an algebraic expression is evaluated repeatedly in a process called \_\_\_\_\_. 8. \_\_\_\_\_
9. The equation  $\frac{3}{x} = \frac{24}{30}$  is called a(n) \_\_\_\_\_. 9. \_\_\_\_\_
10. If some parts of a figure resemble the figure as a whole, the figure is called \_\_\_\_\_. 10. \_\_\_\_\_

**In your own words—**

11. Explain what is meant by *equality of cross products*. 11. \_\_\_\_\_
12. Describe a self-similar figure. 12. \_\_\_\_\_

# 6 Chapter 6 Quiz

(Lessons 6-1 and 6-2)

SCORE \_\_\_\_\_

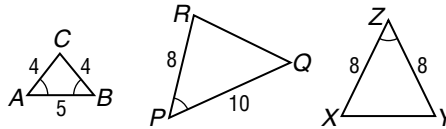
1. **GARDENS** The model of a circular garden is 8 inches in diameter. The actual garden will be 20 feet in diameter. What is the ratio of the diameter of the model to the diameter of the actual garden?

1. \_\_\_\_\_

2. **PHOTOS** A 4-inch by 6-inch photograph, set vertically, is enlarged to make a poster 22 inches wide. How tall is the poster?

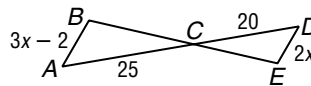
2. \_\_\_\_\_

3. Are any of the three triangles similar? If so, write the appropriate similarity statement.



3. \_\_\_\_\_

4. If  $\triangle ABC \sim \triangle DEC$ , find  $x$  and the scale factor of  $\triangle ABC$  to  $\triangle DEC$ .



4. \_\_\_\_\_

5. **STANDARDIZED TEST PRACTICE** The perimeter of a rectangle is 126 centimeters. The ratio of the length to the width is 5:2. Find the width of the rectangle.

5. \_\_\_\_\_

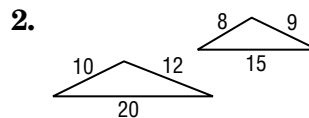
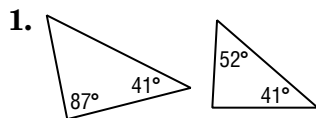
- A. 9 cm      B. 18 cm      C. 45 cm      D. 50.4 cm

# 6 Chapter 6 Quiz

(Lesson 6-3)

SCORE \_\_\_\_\_

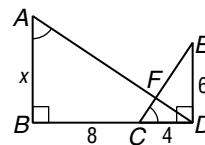
For Questions 1 and 2, determine whether each pair of triangles is similar. Justify your answer.



1. \_\_\_\_\_

2. \_\_\_\_\_

3. Identify the similar triangles in the figure, then find  $x$ .

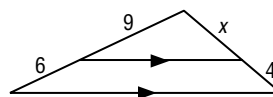


3. \_\_\_\_\_

4. **SHADOWS** A person who is 5 feet tall casts a shadow that is 4 feet long. At the same time, a flagpole casts a shadow that is 18 feet long. How tall is the flagpole?

4. \_\_\_\_\_

5. Find  $x$ .



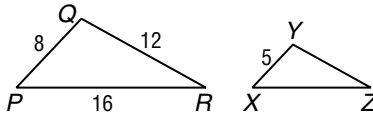
5. \_\_\_\_\_

# 6 Chapter 6 Quiz

(Lessons 6-4 and 6-5)

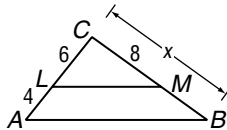
SCORE \_\_\_\_\_

1. If  $\triangle PQR \sim \triangle XYZ$ , find the perimeter of  $\triangle XYZ$ .



1. \_\_\_\_\_

2. Find  $x$  so that  $\overline{LM} \parallel \overline{AB}$ .

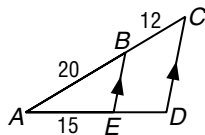


2. \_\_\_\_\_

3. In  $\triangle ABC$ ,  $\overline{DE}$  is parallel to  $\overline{AC}$  and  $DE = 10$ . What is the length of  $\overline{AC}$  if  $\overline{DE}$  is the midsegment of  $\triangle ABC$ ?

3. \_\_\_\_\_

4. Find  $DE$ .



4. \_\_\_\_\_

5. In  $\triangle RST$ ,  $\overline{TU}$  bisects  $\angle T$ . If  $U$  is a point on  $\overline{RS}$ ,  $RU = 6$ ,  $RT = 9$ , and  $ST = 12$ , find  $RS$ .

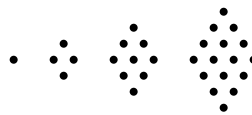
5. \_\_\_\_\_

# 6 Chapter 6 Quiz

(Lesson 6-6)

SCORE \_\_\_\_\_

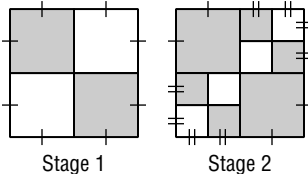
1. **DIAMOND NUMBERS** The dots in each arrangement form a *diamond number*. How many dots are in the ninth diamond number?



1. \_\_\_\_\_

2. In the fractal, Stage 1 is repeated in the upper right and lower left unshaded squares of each stage. Draw Stage 3.

2. \_\_\_\_\_



For Questions 3 and 4, find the first 3 iterates of the given expression.

3.  $4x - 1$ , where  $x$  initially equals 2

3. \_\_\_\_\_

4.  $x^2 - x$ , where  $x$  initially equals 3

4. \_\_\_\_\_

**Part I** Write the letter for the correct answer in the blank at the right of each question.

1. If quadrilateral  $ABCD \sim$  quadrilateral  $PQRS$ , which proportion must be true? 1. \_\_\_\_\_

- A.  $\frac{AC}{AD} = \frac{PQ}{PS}$       B.  $\frac{BC}{CD} = \frac{QR}{RS}$       C.  $\frac{AB}{BD} = \frac{PQ}{QR}$       D.  $\frac{CD}{AB} = \frac{PQ}{RS}$

2. This fall 126 students participated in the soccer program, while 54 played volleyball. What was the ratio of soccer players to volleyball players? 2. \_\_\_\_\_

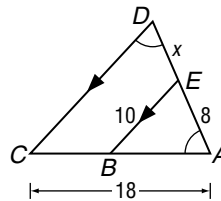
- A.  $\frac{3}{4}$       B.  $\frac{3}{7}$       C.  $\frac{4}{3}$       D.  $\frac{7}{3}$

3. The ratio of the measures of the angles of a triangle is 2:3:10. What is the least angle measure? 3. \_\_\_\_\_

- A. 12      B. 15      C. 24      D. 36

4. Find  $x$ . 4. \_\_\_\_\_

- A. 2      B. 4.8  
C. 6      D. 6.4

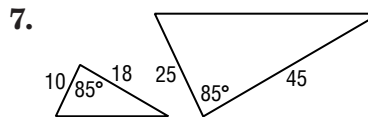
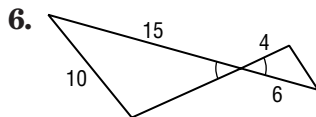


5. If rectangle  $ABCD \sim$  rectangle  $EFGH$ , the perimeter of  $ABCD$  is 54 centimeters, and the perimeter of  $EFGH$  is 36 centimeters, what is the scale factor of  $ABCD$  to  $EFGH$ ? 5. \_\_\_\_\_

- A.  $\frac{2}{3}$       B.  $\frac{3}{2}$       C.  $\frac{3}{5}$       D.  $\frac{5}{3}$

**Part II**

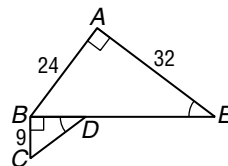
For Questions 6 and 7, determine whether each pair of triangles is similar. Justify your answer.



6. \_\_\_\_\_

7. \_\_\_\_\_

8. If  $\triangle ABE \sim \triangle BCD$ , find  $DE$  and the scale factor of  $\triangle ABE$  to  $\triangle BCD$ .



8. \_\_\_\_\_

9. Quadrilateral  $ABCD \sim$  quadrilateral  $RSUV$ ,  $m\angle ABC = 120$ , and the scale factor of  $ABCD$  to  $RSUV$  is  $\frac{8}{5}$ . What is  $m\angle RSU$ ? 9. \_\_\_\_\_

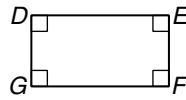
10. **MODELS** Sasha made a model of a clipper ship. If her model has a length of 18 inches, and the original ship had a length of 160 feet and a width of 32 feet, what should be the width of her model? 10. \_\_\_\_\_

# 6 Chapter 6 Cumulative Review

(Chapters 1–6)

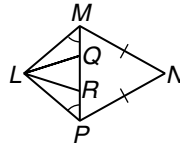
1. Find the coordinates of the midpoint of  $\overline{AB}$  for  $A(-24, 15)$  and  $B(13, -31)$ . (Lesson 1-3) 1. \_\_\_\_\_

For Questions 2–4, use the figure at the right.

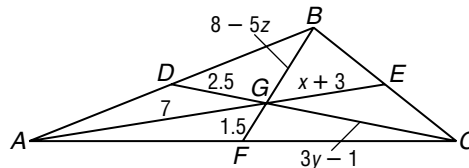


2. The perimeter of rectangle  $DEFG$  is 176,  $EF = h$ , and  $DE = 7h$ . Find  $h$ . (Lesson 1-6) 2. \_\_\_\_\_
3. If  $p$ :  $DEFG$  is a rectangle and  $q$ :  $DE = EF$ , find the truth value of  $p \vee q$ . (Lesson 2-2) 3. \_\_\_\_\_
4. How many line segments can be drawn connecting  $D, E, F$ , and  $G$ ? (Lesson 2-5) 4. \_\_\_\_\_
5. For  $S(-5, 7)$ ,  $T(1, 9)$ ,  $P(12, -1)$ , and  $R(3, 26)$ , determine whether  $\overline{ST}$  and  $\overline{PR}$  are *parallel*, *perpendicular*, or *neither*. (Lesson 3-3) 5. \_\_\_\_\_
6. Write an equation in slope-intercept form for the line  $y + 7 = 4(x - 10)$ . (Lesson 3-4) 6. \_\_\_\_\_
7. Given  $T(3, -1)$ ,  $U(1, -7)$ ,  $V(8, -5)$ ,  $W(2, 6)$ ,  $X(-4, 8)$ , and  $Y(-2, 1)$  determine whether  $\triangle TUV \cong \triangle WXY$ . Explain. (Lesson 4-4) 7. \_\_\_\_\_

8. Name a pair of congruent angles and a pair of congruent sides that are not marked in the figure. (Lesson 4-6)

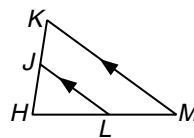


For Questions 9 and 10, use the figure at the right.

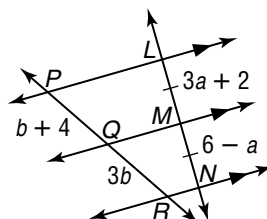


9. Points  $D, E$ , and  $F$  are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ , respectively. Find  $x, y$ , and  $z$ . (Lesson 5-1) 9. \_\_\_\_\_
10. Compare  $m\angle BEG$  to  $m\angle CEG$ . (Lesson 5-5) 10. \_\_\_\_\_

For Questions 11 and 12, use the figure at the right.



11. Determine whether  $\triangle HJL \sim \triangle HKM$ . Justify your answer. (Lesson 6-3) 11. \_\_\_\_\_
12. If  $JK = 7$ ,  $HJ = 8$ ,  $HL = 12$ , and  $LM = 2x - 7.5$ , find  $x$ . (Lesson 6-4) 12. \_\_\_\_\_
13. Find  $a$  and  $b$ . (Lesson 6-4) 13. \_\_\_\_\_



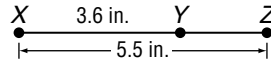


**Part 1: Multiple Choice**

**Instructions:** Fill in the appropriate oval for the best answer.

1. Find the length of  $\overline{YZ}$ . (Lesson 1-2)

- A. 1.9 in.      B. 5.3 in.  
C. 7.2 in.      D. 12.5 in.



1. (A) (B) (C) (D)

2. **Given:**  $3b + 4 < 16$

**Conjecture:**  $b > 0$

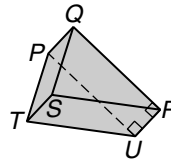
Which of the following would be a counterexample? (Lesson 2-1)

- E.  $b = -1$       F.  $b = 0$       G.  $b = 3.5$       H.  $b = 4$

2. (E) (F) (G) (H)

3. Find the plane that is parallel to plane  $PTU$ . (Lesson 3-1)

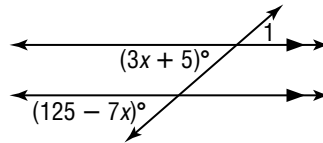
- A. plane  $QRU$       B. plane  $QRS$   
C. plane  $PQS$       D. plane  $SPU$



3. (A) (B) (C) (D)

4. Find  $m\angle 1$ . (Lesson 3-2)

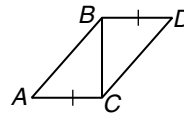
- E. 5      F. 12  
G. 41      H. 44



4. (E) (F) (G) (H)

5. Which statement *must* be true in order to prove  $\triangle ABC \cong \triangle DCB$  by SAS? (Lesson 4-4)

- A.  $\overline{CB}$  bisects  $\angle ABD$ .  
B.  $\angle BCA \cong \angle CBD$   
C.  $\angle BDC \cong \angle CAB$   
D.  $\overline{AB} \cong \overline{BC}$



5. (A) (B) (C) (D)

6. In an indirect proof, you assume that the conclusion is false and then find a(n) \_\_\_\_\_. (Lesson 5-3)

- E. assumption      F. contradiction  
G. truth value      H. conditional statement

6. (E) (F) (G) (H)

7. Demont and Tony are competing to see whose house is the tallest. Early in the afternoon, Tony, who is 4 feet tall, measured his shadow to be 9.6 inches and the shadow of his house to be 62.4 inches. Later in the day, Demont, who is 5 feet tall, measured his shadow to be 15.6 inches and the shadow of his house to be 62.4 inches. Who lives in the taller house? (Lesson 6-3)

- A. Demont      B. Both houses are the same height.  
C. Tony      D. There is not enough information.

7. (A) (B) (C) (D)

8. Which geometric figure is created using iteration? (Lesson 6-6)

- E. scalene triangle      F. square  
G. fractal      H. midsegment

8. (E) (F) (G) (H)

**6**

**Standardized Test Practice** *(continued)*

**Part 2: Grid In**

**Instructions:** Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

9. What is the distance between  $-76$  and  $43$  on a number line? (Lesson 1-3)

9.

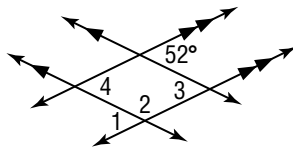
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10. If  $28 - 4x = -12$ , then  $5 + 2x = \underline{\quad? \quad}$ .  
(Lesson 2-6)

11. Find  $m\angle 2$ . (Lesson 3-2)



11.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.  $\triangle LMN$  is equilateral,  $LM$  is one more than three times a number,  $MN$  is nine less than five times the number, and  $NL$  is eleven more than the number. Find  $LM$ . (Lesson 4-1)

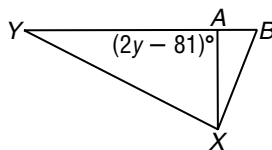
**Part 3: Short Response**

**Instructions:** Show your work or explain in words how you found your answer.

13. Solve  $\frac{-16}{40} = \frac{4x + 10}{5}$ . (Lesson 6-1)

13. \_\_\_\_\_

14.  $\overline{XA}$  is an altitude of  $\triangle XYB$ . Find  $y$ .  
(Lesson 5-1)

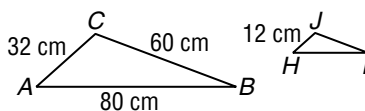


14. \_\_\_\_\_

15. Two sides of a triangle measure 21 inches and 32 inches, and the third side measures  $x$  inches. Find the range for  $x$ . (Lesson 5-4)

15. \_\_\_\_\_

16. If  $\triangle ABC \sim \triangle HIJ$ , find the perimeter of  $\triangle HIJ$ . (Lesson 6-5)



16. \_\_\_\_\_

**6**

# Standardized Test Practice

*Student Record Sheet (Use with pages 338–339 of the Student Edition.)*

## Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

4 (A) (B) (C) (D)

7 (A) (B) (C) (D)

2 (A) (B) (C) (D)

5 (A) (B) (C) (D)

8 (A) (B) (C) (D)

3 (A) (B) (C) (D)

6 (A) (B) (C) (D)

## Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 11 and 12, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

9 \_\_\_\_\_

10 \_\_\_\_\_

11 \_\_\_\_\_ (grid in)

12 \_\_\_\_\_ (grid in)

11

	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12

	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

## Part 3 Open-Ended

Record your answers for Questions 13–14 on the back of this paper.

## 6-1 Study Guide and Intervention (continued)

### Proportions

**Use Properties of Proportions** A statement that two ratios are equal is called a **proportion**. In the proportion  $\frac{a}{b} = \frac{c}{d}$ , where  $b$  and  $d$  are not zero, the values  $a$  and  $d$  are the **extremes** and the values  $b$  and  $c$  are the **means**. In a proportion, the product of the means is equal to the product of the extremes, so  $ad = bc$ .

$$\frac{a}{b} = \frac{c}{d}$$

↑ extremes means  
 $a \cdot d = b \cdot c$

**Example 1** Solve  $\frac{9}{16} = \frac{27}{x}$ .

$$\frac{9}{16} = \frac{27}{x}$$

$9 \cdot x = 16 \cdot 27$  Cross products  
 $9x = 432$  Multiply.  
 $x = 48$  Divide each side by 9.

**Example 2** A room is 49 centimeters by 28 centimeters on a scale drawing of a house. For the actual room, the larger dimension is 14 feet. Find the shorter dimension of the actual room.

If  $x$  is the room's shorter dimension, then

$$\frac{28}{49} = \frac{x}{14}$$

shorter dimension  
longer dimension  
 $49x = 392$  Cross products  
 $x = 8$  Divide each side by 49.

The shorter side of the room is 8 feet.

### Exercises

**Solve each proportion.**

1.  $\frac{1}{2} = \frac{28}{x}$  **56**      2.  $\frac{3}{8} = \frac{y}{24}$  **9**      3.  $\frac{x+22}{x+2} = \frac{30}{10}$  **8**  
 4.  $\frac{3}{18.2} = \frac{9}{y}$  **54.6**      5.  $\frac{2x+3}{8} = \frac{5}{4}$  **3.5**      6.  $\frac{x+1}{x-1} = \frac{3}{4}$  **-7**

**Use a proportion to solve each problem.**

- If 3 cassettes cost \$44.85, find the cost of one cassette. **\$14.95**
- The ratio of the sides of a triangle are 8:15:17. If the perimeter of the triangle is 480 inches, find the length of each side of the triangle. **96 in., 180 in., 204 in.**
- The scale on a map indicates that one inch equals 4 miles. If two towns are 3.5 inches apart on the map, what is the actual distance between the towns? **14 mi**

## Lesson 6-1

## 6-1 Study Guide and Intervention

### Proportions

**Write Ratios** A ratio is a comparison of two quantities. The ratio  $a$  to  $b$ , where  $b$  is not zero, can be written as  $\frac{a}{b}$  or  $a:b$ . The ratio of two quantities is sometimes called a **scale factor**. For a scale factor, the units for each quantity are the same.

**Example 1** In 2002, the Chicago Cubs baseball team won 67 games out of 162. Write a ratio for the number of games won to the total number of games played. To find the ratio, divide the number of games won by the total number of games played. The result is  $\frac{67}{162}$ , which is about 0.41. The Chicago Cubs won about 41% of their games in 2002.

**Example 2** A doll house that is 15 inches tall is a scale model of a real house with a height of 20 feet. What is the ratio of the height of the doll house to the height of the real house?

To start, convert the height of the real house to inches.

$$20 \text{ feet} \times 12 \text{ inches per foot} = 240 \text{ inches}$$

To find the ratio or scale factor of the heights, divide the height of the doll house by the height of the real house. The ratio is 15 inches:240 inches or 1:16. The height of the doll house is  $\frac{1}{16}$  the height of the real house.

### Exercises

- In the 2002 Major League baseball season, Sammy Sosa hit 49 home runs and was at bat 556 times. Find the ratio of home runs to the number of times he was at bat.  
 $\frac{49}{556}$
- There are 182 girls in the sophomore class of 305 students. Find the ratio of girls to total students.  
 $\frac{182}{305}$
- The length of a rectangle is 8 inches and its width is 5 inches. Find the ratio of length to width.  
 $\frac{8}{5}$
- The sides of a triangle are 3 inches, 4 inches, and 5 inches. Find the scale factor between the longest and the shortest sides.  
 $\frac{5}{3}$
- The length of a model train is 18 inches. It is a scale model of a train that is 48 feet long. Find the scale factor.  
 $\frac{1}{32}$

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## 6-1 Skills Practice

### Proportions

1. **FOOTBALL** A tight end scored 6 touchdowns in 14 games. Find the ratio of touchdowns per game.  
**0.43:1**
2. **EDUCATION** In a schedule of 6 classes, Marta has 2 elective classes. What is the ratio of elective to non-elective classes in Marta's schedule?  
**1:2**

## Lesson 6-1

3. **BIOLOGY** Out of 274 listed species of birds in the United States, 78 species made the endangered list. Find the ratio of endangered species of birds to listed species in the United States.  
 **$\frac{39}{137}$**
4. **ART** An artist in Portland, Oregon, makes bronze sculptures of dogs. The ratio of the height of a sculpture to the actual height of the dog is 2:3. If the height of the sculpture is 14 inches, find the height of the dog.  
**21 in.**

5. **SCHOOL** The ratio of male students to female students in the drama club at Campbell High School is 3:4. If the number of male students in the club is 18, what is the number of female students?  
**24**

Solve each proportion.

6.  $\frac{2}{5} = \frac{x}{40}$     **16**                      7.  $\frac{7}{10} = \frac{21}{x}$     **30**                      8.  $\frac{20}{5} = \frac{4x}{6}$     **6**
9.  $\frac{5x}{4} = \frac{35}{8}$     **3.5**                      10.  $\frac{x+1}{3} = \frac{7}{2}$     **9.5**                      11.  $\frac{15}{3} = \frac{x-3}{5}$     **28**

Find the measures of the sides of each triangle.

12. The ratio of the measures of the sides of a triangle is 3:5:7, and its perimeter is 450 centimeters.  
**90 cm, 150 cm, 210 cm**
13. The ratio of the measures of the sides of a triangle is 5:6:9, and its perimeter is 220 meters.  
**55 m, 66 m, 99 m**
14. The ratio of the measures of the sides of a triangle is 4:6:8, and its perimeter is 126 feet.  
**28 ft, 42 ft, 56 ft**
15. The ratio of the measures of the sides of a triangle is 5:7:8, and its perimeter is 40 inches.  
**10 in., 14 in., 16 in.**

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## 6-1 Practice (Average)

### Proportions

1. **NUTRITION** One ounce of cheddar cheese contains 9 grams of fat. Six of the grams of fat are saturated fats. Find the ratio of saturated fats to total fat in an ounce of cheese.  
**2:3**
2. **FARMING** The ratio of goats to sheep at a university research farm is 4:7. The number of sheep at the farm is 28. What is the number of goats?  
**16**
3. **ART** Edward Hopper's oil on canvas painting *Nighthawks* has a length of 60 inches and a width of 30 inches. A print of the original has a length of 2.5 inches. What is the width of the print?  
**1.25 in.**

Solve each proportion.

4.  $\frac{5}{8} = \frac{x}{12}$     **7.5**                      5.  $\frac{x}{1.12} = \frac{1}{5}$     **0.224**                      6.  $\frac{6x}{27} = \frac{4}{3}$     **6**
7.  $\frac{x+2}{3} = \frac{8}{9}$      **$\frac{2}{3}$**                       8.  $\frac{3x-5}{4} = \frac{-5}{7}$      **$\frac{5}{7}$**                       9.  $\frac{x-2}{4} = \frac{x+4}{2}$     **-10**

Find the measures of the sides of each triangle.

10. The ratio of the measures of the sides of a triangle is 3:4:6, and its perimeter is 104 feet.  
**24 ft, 32 ft, 48 ft**
11. The ratio of the measures of the sides of a triangle is 7:9:12, and its perimeter is 84 inches.  
**21 in., 27 in., 36 in.**
12. The ratio of the measures of the sides of a triangle is 6:7:9, and its perimeter is 77 centimeters.  
**21 cm, 24.5 cm, 31.5 cm**

Find the measures of the angles in each triangle.

13. The ratio of the measures of the angles is 4:5:6.  
**48, 60, 72**
14. The ratio of the measures of the angles is 5:7:8.  
**45, 63, 72**

15. **BRIDGES** The span of the Benjamin Franklin suspension bridge in Philadelphia, Pennsylvania, is 1750 feet. A model of the bridge has a span of 42 inches. What is the ratio of the span of the model to the span of the actual Benjamin Franklin Bridge?  
 **$\frac{1}{500}$**

NAME \_\_\_\_\_

DATE \_\_\_\_\_

PERIOD \_\_\_\_\_

6-1

Reading to Learn Mathematics  
Proportions

Pre-Activity How do artists use ratios?

Read the introduction to Lesson 6-1 at the top of page 282 in your textbook. Estimate the ratio of length to width for the background rectangles in Tiffany's Clematiss Skylight. **Sample answer: about 5 to 2**

Reading the Lesson

- Match each description in the first column with a word or phrase from the second column.
  - The ratio of two corresponding quantities **iv**
  - $r$  and  $u$  in the equation  $\frac{r}{s} = \frac{t}{u}$  **v**
  - a comparison of two quantities **vi**
  - $ru$  and  $st$  in the equation  $\frac{r}{s} = \frac{t}{u}$  **ii**
  - an equation stating that two ratios are equal **i**
  - $s$  and  $t$  in the equation  $\frac{r}{s} = \frac{t}{u}$  **iii**
  - proportion
  - cross products
  - means
  - scale factor
  - extremes
  - ratio

2. If  $m$ ,  $n$ ,  $p$ , and  $q$  are nonzero numbers such that  $\frac{m}{n} = \frac{p}{q}$ , which of the following statements could be false? **B, C, E**

- A.  $np = mq$       B.  $\frac{p}{n} = \frac{q}{m}$   
 C.  $mp = nq$       D.  $qm = pn$   
 E.  $\frac{m}{n} = \frac{p}{q}$       F.  $\frac{q}{p} = \frac{n}{m}$   
 G.  $m:p = n:q$       H.  $m:n = p:q$

3. Write two proportions that match each description.

- Means are 5 and 8; extremes are 4 and 10.  
**any two of**  $\frac{4}{5} = \frac{8}{10}$ ,  $\frac{4}{8} = \frac{5}{10}$ ,  $\frac{10}{5} = \frac{8}{4}$ ,  $\frac{10}{8} = \frac{5}{4}$
- Means are 5 and 4; extremes are positive integers that are different from means.  
**any two of**  $\frac{2}{5} = \frac{4}{10}$ ,  $\frac{2}{4} = \frac{5}{10}$ ,  $\frac{1}{4} = \frac{5}{20}$ ,  $\frac{1}{5} = \frac{4}{20}$ ,  $\frac{10}{5} = \frac{4}{2}$ ,  $\frac{10}{4} = \frac{5}{2}$ ,  $\frac{5}{4} = \frac{2}{1}$ ,  
 or  $\frac{20}{5} = \frac{4}{1}$

Helping You Remember

4. Sometimes it is easier to remember a mathematical idea if you put it into words without using any mathematical symbols. How can you use this approach to remember the concept of equality of cross products? **Sample answer: The product of the means equals the product of the extremes.**

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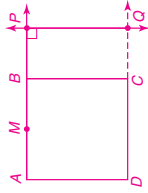
6-1

Enrichment

Golden Rectangles

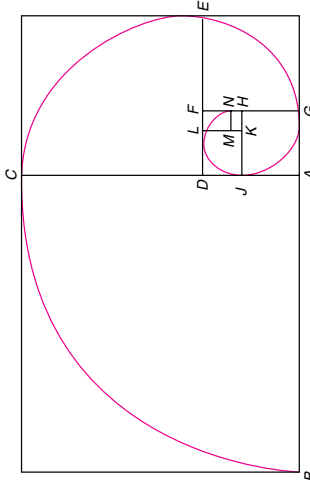
Use a straightedge, compass, and the instructions below to construct a golden rectangle.

- Construct square  $ABCD$  with sides of 2 cm.
- Construct the midpoint of  $\overline{AB}$ . Call the midpoint  $M$ .
- Draw  $\overline{MB}$ . Set your compass at an opening equal to  $MC$ . Use  $M$  as the center to draw an arc that intersects  $\overline{AB}$ . Call the point of intersection  $P$ .
- Construct a line through  $P$  that is perpendicular to  $\overline{AB}$ .
- Draw  $\overline{DC}$  so that it intersects the perpendicular line in step 4. Call the intersection point  $Q$ .  $APQD$  is a **golden rectangle** because the ratio of its length to its width is 1.618. Check this conclusion by finding the value of  $\frac{PQ}{AP}$ . Rectangles whose sides have this ratio are, it is said, the most pleasing to the human eye.



A figure consisting of similar golden rectangles is shown below. Use a compass and the instructions below to draw quarter-circle arcs that form a spiral like that found in the shell of a chambered nautilus.

- Using  $A$  as a center, draw an arc that passes through  $B$  and  $C$ .
- Using  $D$  as a center, draw an arc that passes through  $C$  and  $E$ .
- Using  $F$  as a center, draw an arc that passes through  $E$  and  $G$ .
- Using  $H$  as a center, draw an arc that passes through  $G$  and  $J$ .
- Using  $K$  as a center, draw an arc that passes through  $J$  and  $L$ .
- Using  $M$  as a center, draw an arc that passes through  $L$  and  $N$ .



NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## 6-2 Study Guide and Intervention

### Similar Polygons

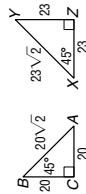
#### Identify Similar Figures

**Example 1** Determine whether the triangles are similar.

Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.

$\angle C \cong \angle Z$  because they are right angles, and  $\angle B \cong \angle X$ . By the Third Angle Theorem,  $\angle A \cong \angle Y$ .

For the sides,  $\frac{BC}{XZ} = \frac{20}{23} = \frac{20\sqrt{2}}{23\sqrt{2}} = \frac{20}{23}$ , and  $\frac{AC}{YZ} = \frac{20}{23}$ . The side lengths are proportional. So  $\triangle BCA \sim \triangle XZY$ .



**Example 2** Is polygon WXYZ ~ polygon PQRS?

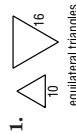
For the sides,  $\frac{WX}{PQ} = \frac{12}{8} = \frac{3}{2}$ ,  $\frac{XY}{QR} = \frac{18}{12} = \frac{3}{2}$ ,  $\frac{YZ}{RS} = \frac{15}{10} = \frac{3}{2}$ , and  $\frac{ZW}{SP} = \frac{9}{6} = \frac{3}{2}$ . So corresponding sides are proportional.

Also,  $\angle W \cong \angle P$ ,  $\angle X \cong \angle Q$ ,  $\angle Y \cong \angle R$ , and  $\angle Z \cong \angle S$ , so corresponding angles are congruent. We can conclude that polygon WXYZ ~ polygon PQRS.

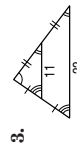


#### Exercises

Determine whether each pair of figures is similar. If they are similar, give the ratio of corresponding sides.



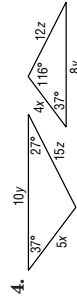
yes;  $\frac{5}{8}$



yes;  $\frac{1}{2}$



no



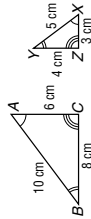
yes;  $\frac{5}{4}$

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

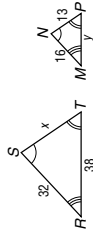
## 6-2 Study Guide and Intervention

### Similar Polygons

**Scale Factors** When two polygons are similar, the ratio of the lengths of corresponding sides is called the **scale factor**. At the right,  $\triangle ABC \sim \triangle XYZ$ . The scale factor of  $\triangle ABC$  to  $\triangle XYZ$  is 2 and the scale factor of  $\triangle XYZ$  to  $\triangle ABC$  is  $\frac{1}{2}$ .



**Example 1** The two polygons are similar. Find  $x$  and  $y$ .



Use the congruent angles to write the corresponding vertices in order.

$\triangle RST \sim \triangle MNP$

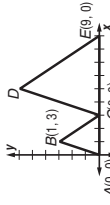
Write proportions to find  $x$  and  $y$ .

$$\frac{32}{16} = \frac{x}{8} \quad \frac{38}{13} = \frac{y}{16}$$

$$16x = 32(8) \quad 32y = 38(16)$$

$$x = 26 \quad y = 19$$

**Example 2**  $\triangle ABC \sim \triangle CDE$ . Find the scale factor and find the lengths of  $\overline{CD}$  and  $\overline{DE}$ .



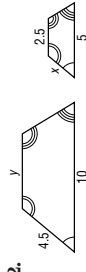
$AC = 3 - 0 = 3$  and  $CE = 9 - 3 = 6$ . The scale factor of  $\triangle CDE$  to  $\triangle ABC$  is  $6:3$  or  $2:1$ . Using the Distance Formula,  $AB = \sqrt{1^2 + 9} = \sqrt{10}$  and  $BC = \sqrt{4^2 + 9} = \sqrt{13}$ . The lengths of the sides of  $\triangle CDE$  are twice those of  $\triangle ABC$ , so  $DC = 2(BC)$  or  $2\sqrt{10}$  and  $DE = 2(AC)$  or  $2\sqrt{13}$ .

#### Exercises

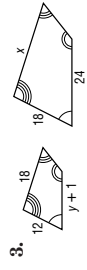
Each pair of polygons is similar. Find  $x$  and  $y$ .



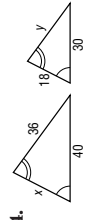
$x = 6$ ;  $y = 15$



$x = 2.25$ ;  $y = 5$



$x = 27$ ;  $y = 15$



$x = 24$ ;  $y = 27$

5. In Example 2 above, point  $D$  has coordinates  $(5, 6)$ . Use the Distance Formula to verify the lengths of  $\overline{CD}$  and  $\overline{DE}$ .

$CD = 2\sqrt{10}$ ,  $DE = 2\sqrt{13}$

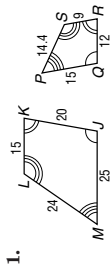
NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**6-2 Practice (Average)**  
**Similar Polygons**

Determine whether each pair of figures is similar. Justify your answer.



$\triangle ABC \sim \triangle TUV$ ;  $\angle A \cong \angle U$ ,  
 $\angle B \cong \angle V$ , and  $\angle C \cong \angle T$  by the  
Third Angle Theorem and  
 $\frac{AB}{UV} = \frac{BC}{TU} = \frac{CA}{TV} = \frac{12}{18} = \frac{2}{3}$ .

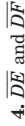


$\triangle JKL \sim \triangle QPS$ ;  
 $\angle J \cong \angle Q$ ,  $\angle K \cong \angle P$ , and  
 $\angle L \cong \angle S$ ,  $\angle M \cong \angle R$ , and  
 $\frac{JK}{PQ} = \frac{KL}{RS} = \frac{LM}{SP} = \frac{MJ}{RQ} = \frac{5}{3}$  or 1.67

Each pair of polygons is similar. Write a similarity statement, and find  $x$ , the measure(s) of indicated side(s), and the scale factor.



$ABCD \sim LMNP$ ;  
 $\frac{3}{3} = \frac{10}{10} = \frac{14}{x+6} = \frac{4}{x+9}$



$\triangle ABC \sim \triangle DEF$ ;  
 $7; 4; 8; \frac{3}{2}$

5. **COORDINATE GEOMETRY** Triangle  $ABC$  has vertices  $A(0, 0)$ ,  $B(-4, 0)$ , and  $C(-2, 4)$ . The coordinates of each vertex are multiplied by 3 to create  $\triangle AEF$ . Show that  $\triangle AEF$  is similar to  $\triangle ABC$ .

$AB = 4$ ,  $BC = CA = 2\sqrt{5}$  and  $AE = 12$ ,  $EF = FA = 6\sqrt{5}$ .  
 $\frac{AB}{AE} = \frac{BC}{EF} = \frac{CA}{FA} = \frac{1}{3}$ ;  $\angle A \cong \angle A$  and since  $\overline{BC} \parallel \overline{EF}$ ,  $\angle B \cong \angle E$  and  
 $\angle C \cong \angle F$ . Since the corresponding angles are congruent and the  
corresponding sides are proportional,  $\triangle ABC \sim \triangle AEF$ .

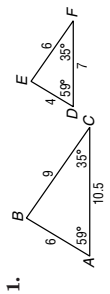
6. **INTERIOR DESIGN** Graham used the scale drawing of his living room to decide where to place furniture. Find the dimensions of the living room if the scale in the drawing is 1 inch = 4.5 feet.  
**18 ft by 11 ft 3 in.**

Lesson 6-2

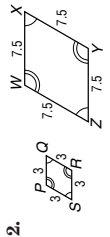
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**6-2 Skills Practice**  
**Similar Polygons**

Determine whether each pair of figures is similar. Justify your answer.

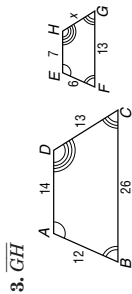


$\triangle ABC \sim \triangle DEF$ ;  $\angle A \cong \angle D$ ,  
 $\angle C \cong \angle F$ , and  $\angle B \cong \angle E$   
because if two angles of one  
triangle are congruent to two  
angles of a second triangle, then  
the third angles are congruent;  
 $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{2}$ .

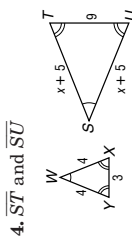


rhombus  $PQRS \sim$  rhombus  $WXYZ$ ;  
 $\angle P \cong \angle W$ ,  $\angle Q \cong \angle X$ ,  $\angle R \cong \angle Y$ ,  
 $\angle S \cong \angle Z$ ;  
 $\frac{PQ}{WX} = \frac{QR}{XY} = \frac{RS}{YZ} = \frac{SP}{ZW} = \frac{3}{7.5} = 0.4$

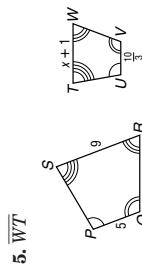
Each pair of polygons is similar. Write a similarity statement, and find  $x$ , the measure(s) of indicated side(s), and the scale factor.



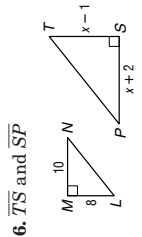
$ABCD \sim EFGH$ ;  
 $6\frac{1}{2}; 6\frac{1}{2}; 2$



$\triangle WXY \sim \triangle STU$  (or  $\triangle SUT$ )  
 $7; 12; 12; \frac{1}{3}$



$PQRS \sim UVWT$ ;  
 $5; 6; \frac{3}{2}$



$\triangle LMN \sim \triangle TSP$ ;  
 $13; 12; 15; \frac{2}{3}$



## 6-2 Reading to Learn Mathematics

### Similar Polygons

#### Pre-Activity How do artists use geometric patterns?

Read the introduction to Lesson 6-2 at the top of page 289 in your textbook.

- Describe the figures that have similar shapes. **Sample answer:** Light and dark figures are arranged in alternating patterns. Figures that have their feet pointing toward the center of the circle have the same shape but different sizes.
- What happens to the figures as your eyes move from the center to the outer edge? **Sample answer:** The figures become smaller.

#### Reading the Lesson

- Complete each sentence.
  - Two polygons that have exactly the same shape, but not necessarily the same size, are **similar**.
  - Two polygons are congruent if they have exactly the same shape and the same **size**.
  - Two polygons are similar if their corresponding angles are **congruent** and their corresponding sides are **proportional**.
  - Two polygons are congruent if their corresponding angles are **congruent** and their corresponding sides are **congruent**.
  - The ratio of the lengths of corresponding sides of two similar figures is called the **scale factor**.
  - Multiplying the coordinates of all points of a figure in the coordinate plane by a scale factor to get a similar figure is called a **dilation**.
  - If two polygons are similar with a scale factor of 1, then the polygons are **congruent**.
- Determine whether each statement is *always*, *sometimes*, or *never* true.
  - Two similar triangles are congruent. **sometimes**
  - Two equilateral triangles are congruent. **sometimes**
  - An equilateral triangle is similar to a scalene triangle. **never**
  - Two rectangles are similar. **sometimes**
  - Two isosceles right triangles are congruent. **sometimes**
  - Two isosceles right triangles are similar. **always**
  - A square is similar to an equilateral triangle. **never**
  - Two acute triangles are similar. **sometimes**
  - Two rectangles in which the length is twice the width are similar. **always**
  - Two congruent polygons are similar. **always**

#### Helping You Remember

- A good way to remember a new mathematical vocabulary term is to relate it to words used in everyday life. The word *scale* has many meanings in English. Give three phrases that include the word *scale* in a way that is related to proportions. **Sample answer:** scale drawing, scale model, scale of miles (on a map)

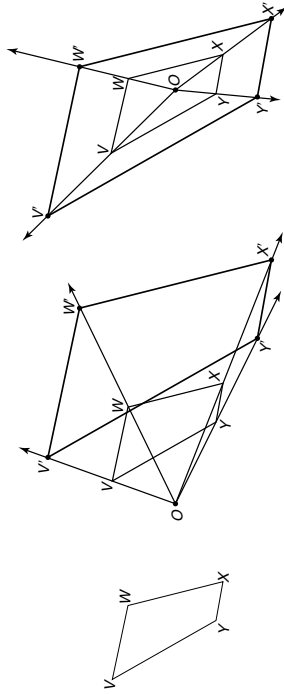
## 6-2 Enrichment

### Constructing Similar Polygons

Here are four steps for constructing a polygon that is similar to and with sides twice as long as those of an existing polygon.

- Choose any point either inside or outside the polygon and label it  $O$ .
- Draw rays from  $O$  through each vertex of the polygon.
- For vertex  $V$ , set the compass to length  $OV$ . Then locate a new point  $V'$  on ray  $OV$  such that  $VV' = OV$ . Thus,  $OV' = 2(OV)$ .
- Repeat Step 3 for each vertex. Connect points  $V', W', X', Y'$  and  $Z'$  to form the new polygon.

Two constructions of polygons similar to and with sides twice those of  $VWXYZ$  are shown below. Notice that the placement of point  $O$  does not affect the size or shape of  $V'W'X'Y'Z'$ , only its location.



Trace each polygon. Then construct a similar polygon with sides twice as long as those of the given polygon. See students' constructions.



- Explain how to construct a similar polygon with sides three times the length of those of polygon  $HJKL$ . Then do the construction. See students' work.
- Explain how to construct a similar polygon with sides  $1\frac{1}{2}$  times the length of those of polygon  $MNPQRS$ . Then do the construction. See students' work.



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## 6-3 Study Guide and Intervention

### Similar Triangles

**Identify Similar Triangles** Here are three ways to show that two triangles are similar.

<b>AA Similarity</b>	Two angles of one triangle are congruent to two angles of another triangle.
<b>SSS Similarity</b>	The measures of the corresponding sides of two triangles are proportional.
<b>SAS Similarity</b>	The measures of two sides of one triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent.

**Example 1** Determine whether the triangles are similar.



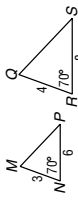
$$\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$$

$\triangle ABC \sim \triangle DEF$  by SSS Similarity.

**Example 2** Determine whether the triangles are similar.



$$\frac{6}{4} = \frac{8}{8} = \frac{7}{7}$$

$$m\angle N = m\angle R, \text{ so } \angle N \cong \angle R.$$

$\triangle MNP \sim \triangle QRS$  by SAS Similarity.

**Example 1** Determine whether the triangles are similar.



$$\frac{AC}{DF} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{AB}{DE} = \frac{10}{15} = \frac{2}{3}$$

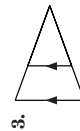
$\triangle ABC \sim \triangle DEF$  by SSS Similarity.

### Exercises

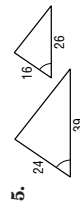
Determine whether each pair of triangles is similar. Justify your answer.



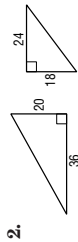
yes; AA Similarity



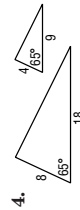
yes; AA Similarity



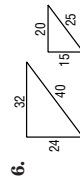
yes; SAS Similarity



no;  $36 \neq 20$   
 $24 \neq 18$



yes; SAS Similarity



yes; SSS Similarity

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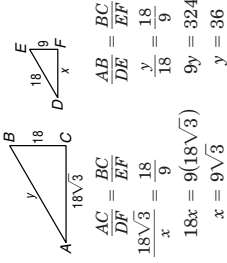
PERIOD \_\_\_\_\_

## 6-3 Study Guide and Intervention

### Similar Triangles

**Use Similar Triangles** Similar triangles can be used to find measurements.

**Example 1**  $\triangle ABC \sim \triangle DEF$ . Find  $x$  and  $y$ .



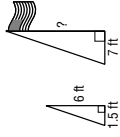
$$\frac{AC}{DF} = \frac{BC}{EF}$$

$$\frac{y}{18} = \frac{10}{9}$$

$$9y = 180$$

$$y = 20$$

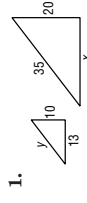
**Example 2** A person 6 feet tall casts a 1.5-foot-long shadow at the same time that a flagpole casts a 7-foot-long shadow. How tall is the flagpole?



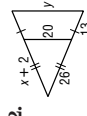
The sun's rays form similar triangles. Using  $x$  for the height of the pole,  $\frac{6}{1.5} = \frac{x}{7}$ , so  $1.5x = 42$  and  $x = 28$ . The flagpole is 28 feet tall.

### Exercises

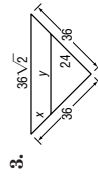
Each pair of triangles is similar. Find  $x$  and  $y$ .



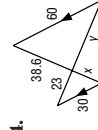
$x = 26$ ;  $y = 17.5$



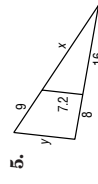
$x = 24$ ;  $y = 30$



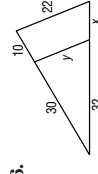
$x = 12$ ;  $y = 24\sqrt{2}$



$x = 19.3$ ;  $y = 46$



$x = 18$ ;  $y = 10.8$



$x = 10\frac{2}{3}$ ;  $y = 16\frac{1}{2}$

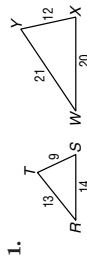
7. The heights of two vertical posts are 2 meters and 0.45 meter. When the shorter post casts a shadow that is 0.85 meter long, what is the length of the longer post's shadow to the nearest hundredth? **3.78 m**

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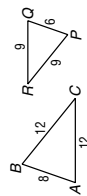
### 6-3 Skills Practice

#### Similar Triangles

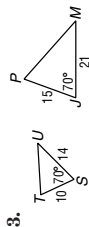
Determine whether each pair of triangles is similar. Justify your answer.



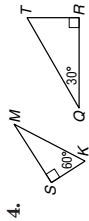
**No; the sides are not proportional.**



**yes;  $\triangle ABC \sim \triangle PQR$  (or  $\triangle QPR$ ); SSS Similarity**



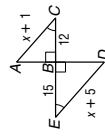
**yes;  $\triangle STU \sim \triangle JPM$ ; SAS Similarity**



**yes;  $\triangle SKM \sim \triangle RTQ$ ; AA Similarity**

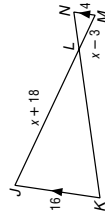
ALGEBRA Identify the similar triangles, and find  $x$  and the measures of the indicated sides.

5.  $\overline{AC}$  and  $\overline{ED}$



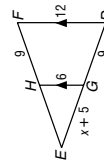
**$\triangle ABC \sim \triangle DBE$ ; 15; 16; 20**

6.  $\overline{JL}$  and  $\overline{LM}$



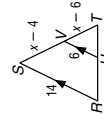
**$\triangle JKL \sim \triangle MNL$ ; 10; 28; 7**

7.  $\overline{EH}$  and  $\overline{EF}$



**$\triangle DEF \sim \triangle GEH$ ; 4; 9; 18**

8.  $\overline{UT}$  and  $\overline{RT}$



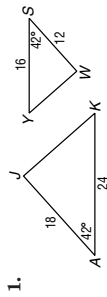
**$\triangle RST \sim \triangle UVT$ ; 12; 6; 14**

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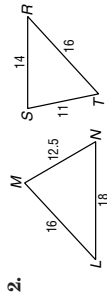
### 6-3 Practice (Average)

#### Similar Triangles

Determine whether each pair of triangles is similar. Justify your answer.



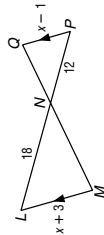
**yes;  $\triangle JAK \sim \triangle WSY$ ; SAS Similarity**



**No; the sides are not proportional.**

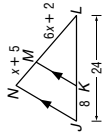
ALGEBRA Identify the similar triangles, and find  $x$  and the measures of the indicated sides.

3.  $\overline{LM}$  and  $\overline{QP}$



**$\triangle LMN \sim \triangle PQN$ ; 9; 12; 8**

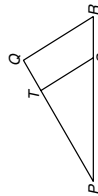
4.  $\overline{NL}$  and  $\overline{ML}$



**$\triangle JLN \sim \triangle KLM$ ; 2; 21; 14**

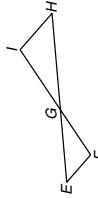
Use the given information to find each measure.

5. If  $\overline{TS} \parallel \overline{QR}$ ,  $TS = 6$ ,  $PS = x + 7$ ,  $QR = 8$ , and  $SR = x - 1$ , find  $PS$  and  $PR$ .



**$PS = 12$ ;  $PR = 16$**

6. If  $\overline{EF} \parallel \overline{HI}$ ,  $EF = 3$ ,  $EG = x + 1$ ,  $HI = 4$ , and  $HG = x + 3$ , find  $EG$  and  $HG$ .



**$EG = 6$ ;  $HG = 8$**

INDIRECT MEASUREMENT For Exercises 7 and 8, use the following information.

A lighthouse casts a 128-foot shadow. A nearby lamppost that measures 5 feet 3 inches casts an 8-foot shadow.

7. Write a proportion that can be used to determine the height of the lighthouse.

**Sample answer: If  $x = \text{height of lighthouse}$ ,  $\frac{x}{128} = \frac{5.25}{8}$ .**

8. What is the height of the lighthouse? **84 ft**

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## 6-3 Enrichment

### Ratio Puzzles with Triangles

If you know the perimeter of a triangle and the ratios of the sides, you can find the lengths of the sides.

#### Example

The perimeter of a triangle is 84 units. The sides have lengths  $r$ ,  $s$ , and  $t$ . The ratio of  $s$  to  $r$  is 5:3, and the ratio of  $t$  to  $r$  is 2:1. Find the length of each side.

Since both ratios contain  $r$ , rewrite one or both ratios to make  $r$  the same. You can write the ratio of  $t$  to  $r$  as 6:3. Now you can write a three-part ratio.

$$r : s : t = 3 : 5 : 6$$

There is a number  $x$  such that  $r = 3x$ ,  $s = 5x$ , and  $t = 6x$ . Since you know the perimeter, 84, you can use algebra to find the lengths of the sides.

$$r + s + t = 84$$

$$3x + 5x + 6x = 84$$

$$14x = 84$$

$$x = 6$$

$$3x = 18, 5x = 30, 6x = 36$$

$$\text{So } r = 18, s = 30, \text{ and } t = 36.$$

#### Find the lengths of the sides of each triangle.

1. The perimeter of a triangle is 75 units. The sides have lengths  $a$ ,  $b$ , and  $c$ . The ratio of  $b$  to  $a$  is 3:5, and the ratio of  $c$  to  $a$  is 7:5. Find the length of each side.

$$a = 25, b = 15, c = 35$$

2. The perimeter of a triangle is 88 units. The sides have lengths  $d$ ,  $e$ , and  $f$ . The ratio of  $e$  to  $d$  is 3:1, and the ratio of  $f$  to  $e$  is 10:9. Find the length of each side.

$$d = 12, e = 36, f = 40$$

3. The perimeter of a triangle is 91 units. The sides have lengths  $p$ ,  $q$ , and  $r$ . The ratio of  $p$  to  $r$  is 3:1, and the ratio of  $q$  to  $r$  is 5:2. Find the length of each side.

$$p = 42, q = 35, r = 14$$

4. The perimeter of a triangle is 68 units. The sides have lengths  $g$ ,  $h$ , and  $j$ . The ratio of  $j$  to  $g$  is 2:1, and the ratio of  $h$  to  $g$  is 5:4. Find the length of each side.

$$g = 16, h = 20, j = 32$$

5. Write a problem similar to those above involving ratios in triangles.

See students' work.

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## 6-3 Reading to Learn Mathematics

### Similar Triangles

#### Pre-Activity How do engineers use geometry?

Read the introduction to Lesson 6-3 at the top of page 298 in your textbook.

- What does it mean to say that triangular shapes result in rigid construction? **Sample answer: A triangular structure will not change its shape or size if you push on its sides without breaking or bending them.**
- What would happen if the shapes used in the construction were quadrilaterals? **Sample answer: The structure might collapse, because quadrilaterals are not rigid.**

#### Reading the Lesson

1. State whether each condition guarantees that two triangles are *congruent* or *similar*.

If the condition guarantees that the triangles are both similar and congruent, write *congruent*. If there is not enough information to guarantee that the triangles will be congruent or similar, write *neither*.

- Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle. **congruent**
  - The measures of all three pairs of corresponding sides are proportional. **similar**
  - Two angles of one triangle are congruent to two angles of the other triangle. **similar**
  - Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle. **congruent**
  - The measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle, and the included angles are congruent. **similar**
  - The three sides of one triangle are congruent to the three sides of the other triangle. **congruent**
  - The three angles of one triangle are congruent to the three angles of the other triangle. **similar**
  - One acute angle of a right triangle is congruent to one acute angle of another right triangle. **similar**
  - The measures of two sides of a triangle are proportional to the measures of two sides of another triangle. **neither**
2. Identify each of the following as an example of a *reflexive*, *symmetric*, or *transitive* property.
- If  $\triangle RST \sim \triangle UVW$ , then  $\triangle UVW \sim \triangle RST$ . **symmetric**
  - If  $\triangle RST \sim \triangle UVW$  and  $\triangle UVW \sim \triangle OPQ$ , then  $\triangle RST \sim \triangle OPQ$ . **transitive**
  - $\triangle RST \sim \triangle RST$ . **reflexive**

#### Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose one of your classmates is having trouble understanding the difference between SAS for congruent triangles and SAS for similar triangles. How can you explain the difference to him?

**Sample answer: In SAS for congruence, corresponding sides are congruent, while in SAS for similarity, corresponding sides are proportional. (In both, the included angles are congruent.)**

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Lesson 6-3

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## 6-4 Study Guide and Intervention (continued)

### Parallel Lines and Proportional Parts

**Proportional Parts of Triangles** In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. The converse is also true.

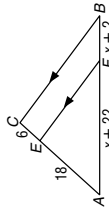
If  $X$  and  $Y$  are the midpoints of  $\overline{RT}$  and  $\overline{ST}$ , then  $\overline{XY}$  is a midsegment of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

$$\text{If } \overline{XY} \parallel \overline{RS}, \text{ then } \frac{RX}{XT} = \frac{SY}{YT}.$$

$$\text{If } \frac{RX}{XT} = \frac{SY}{YT}, \text{ then } \overline{XY} \parallel \overline{RS}.$$

If  $\overline{XY}$  is a midsegment, then  $\overline{XY} \parallel \overline{RS}$  and  $XY = \frac{1}{2}RS$ .

**Example 1** In  $\triangle ABC$ ,  $\overline{EF} \parallel \overline{CB}$ . Find  $x$ .



$$\text{Since } \overline{EF} \parallel \overline{CB}, \frac{AF}{FB} = \frac{AE}{EC}.$$

$$\frac{x+22}{x+2} = \frac{18}{6}$$

$$6x + 132 = 18x + 36$$

$$96 = 12x$$

$$8 = x$$

**Example 2**

A triangle has vertices  $D(3, 6)$ ,  $E(-3, -2)$ , and  $F(7, -2)$ .

Midsegment  $\overline{GH}$  is parallel to  $\overline{EF}$ . Find the length of  $\overline{GH}$ .

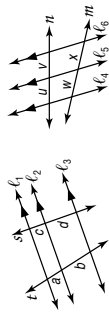
$\overline{GH}$  is a midsegment, so its length is one-half that of  $\overline{EF}$ . Points  $E$  and  $F$  have the same  $y$ -coordinate, so  $EF = 7 - (-3) = 10$ . The length of midsegment  $\overline{GH}$  is 5.

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## 6-4 Study Guide and Intervention (continued)

### Parallel Lines and Proportional Parts

**Divide Segments Proportionally** When three or more parallel lines cut two transversals, they separate the transversals into proportional parts. If the ratio of the parts is 1, then the parallel lines separate the transversals into congruent parts.



$$\text{If } l_1 \parallel l_2 \parallel l_3, \text{ then } \frac{a}{b} = \frac{c}{d}.$$

$$\text{If } l_4 \parallel l_5 \parallel l_6 \text{ and } \frac{u}{v} = 1, \text{ then } \frac{w}{x} = 1.$$

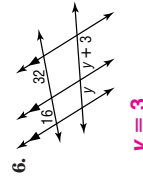
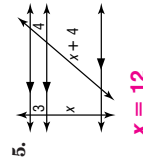
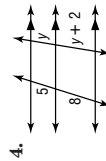
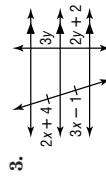
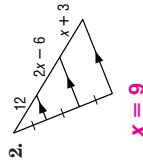
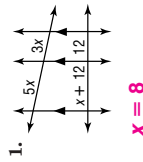
**Example**

Refer to lines  $l_1, l_2$ , and  $l_3$  above. If  $a = 3$ ,  $b = 8$ , and  $c = 5$ , find  $d$ .

$$l_1 \parallel l_2 \parallel l_3 \text{ so } \frac{3}{8} = \frac{5}{d}. \text{ Then } 3d = 40 \text{ and } d = 13\frac{1}{3}.$$

**Exercises**

Find  $x$  and  $y$ .



NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## 6-4 Study Guide and Intervention

### Parallel Lines and Proportional Parts

**Proportional Parts of Triangles** In any triangle, a line parallel to one side of a triangle separates the other two sides proportionally. The converse is also true.

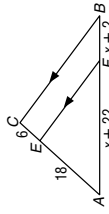
If  $X$  and  $Y$  are the midpoints of  $\overline{RT}$  and  $\overline{ST}$ , then  $\overline{XY}$  is a midsegment of the triangle. The Triangle Midsegment Theorem states that a midsegment is parallel to the third side and is half its length.

$$\text{If } \overline{XY} \parallel \overline{RS}, \text{ then } \frac{RX}{XT} = \frac{SY}{YT}.$$

$$\text{If } \frac{RX}{XT} = \frac{SY}{YT}, \text{ then } \overline{XY} \parallel \overline{RS}.$$

If  $\overline{XY}$  is a midsegment, then  $\overline{XY} \parallel \overline{RS}$  and  $XY = \frac{1}{2}RS$ .

**Example 1** In  $\triangle ABC$ ,  $\overline{EF} \parallel \overline{CB}$ . Find  $x$ .



$$\text{Since } \overline{EF} \parallel \overline{CB}, \frac{AF}{FB} = \frac{AE}{EC}.$$

$$\frac{x+22}{x+2} = \frac{18}{6}$$

$$6x + 132 = 18x + 36$$

$$96 = 12x$$

$$8 = x$$

**Example 2**

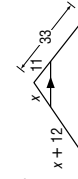
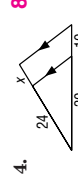
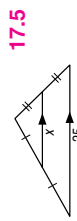
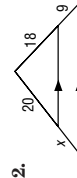
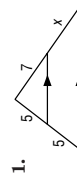
A triangle has vertices  $D(3, 6)$ ,  $E(-3, -2)$ , and  $F(7, -2)$ .

Midsegment  $\overline{GH}$  is parallel to  $\overline{EF}$ . Find the length of  $\overline{GH}$ .

$\overline{GH}$  is a midsegment, so its length is one-half that of  $\overline{EF}$ . Points  $E$  and  $F$  have the same  $y$ -coordinate, so  $EF = 7 - (-3) = 10$ . The length of midsegment  $\overline{GH}$  is 5.

**Exercises**

Find  $x$ .



7. In Example 2, find the slope of  $\overline{EF}$  and show that  $\overline{EF} \parallel \overline{GH}$ .

The endpoints of  $\overline{GH}$  are at  $(0, 2)$  and  $(5, 2)$ . The slope of  $\overline{GH}$  is 0 and the slope of  $\overline{EF}$  is 0.  $\overline{GH}$  and  $\overline{EF}$  are parallel.

NAME \_\_\_\_\_

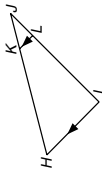
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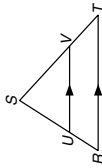
**6-4 Skills Practice**

**Parallel Lines and Proportional Parts**

- If  $JK = 7$ ,  $KH = 21$ , and  $JL = 6$ , find  $LI$ .
- Find  $x$  and  $TV$  if  $RU = 8$ ,  $US = 14$ ,  $TV = x - 1$  and  $VS = 17.5$ .



**18**



**11; 10**

Determine whether  $\overline{BC} \parallel \overline{DE}$ .

- $AD = 15$ ,  $DB = 12$ ,  $AE = 10$ , and  $EC = 8$

**yes**

- $BD = 9$ ,  $BA = 27$ , and  $CE$  is one third of  $EA$

**no**

- $AE = 30$ ,  $AC = 45$ , and  $AD$  is twice  $DB$

**yes**

**COORDINATE GEOMETRY** For Exercises 6-8, use the following information.

Triangle  $ABC$  has vertices  $A(-5, 2)$ ,  $B(1, 8)$ , and  $C(4, 2)$ . Point  $D$  is the midpoint of  $AB$  and  $E$  is the midpoint of  $AC$ .

- Identify the coordinates of  $D$  and  $E$ .

**$D(-2, 5)$ ;  $E(-0.5, 2)$**

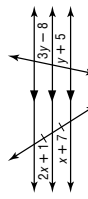
- Show that  $\overline{BC}$  is parallel to  $\overline{DE}$ .

**Since the slope of  $\overline{BC}$  is  $-2$  and the slope of  $\overline{DE}$  is  $-2$ ,  $\overline{BC}$  is parallel to  $\overline{DE}$ .**

- Show that  $DE = \frac{1}{2}BC$ .

**The length of  $\overline{DE}$  is  $\sqrt{11.25}$  or  $\frac{\sqrt{45}}{2}$  and the length of  $\overline{BC}$  is  $\sqrt{45}$ .**

- Find  $x$  and  $y$ .



**$x = 6$ ,  $y = 6.5$**

- Find  $x$  and  $y$ .



**$x = 2$ ,  $y = 4$**

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Glencoe Geometry

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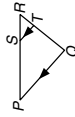
**6-4 Practice (Average)**

**Parallel Lines and Proportional Parts**

- If  $AD = 24$ ,  $DB = 27$ , and  $EB = 18$ , find  $CE$ .
- Find  $x$ ,  $QT$ , and  $TR$  if  $QT = x + 6$ ,  $SR = 12$ ,  $PS = 27$ , and  $TR = x - 4$ .



**16**



**12; 18; 8**

Determine whether  $\overline{JK} \parallel \overline{NM}$ .

- $JN = 18$ ,  $JL = 30$ ,  $KM = 21$ , and  $ML = 35$

**no**

- $KM = 24$ ,  $KL = 44$ , and  $NL = \frac{5}{6}JN$

**yes**



**COORDINATE GEOMETRY** For Exercises 5 and 6, use the following information.

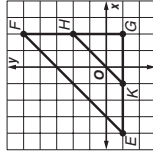
Triangle  $EFG$  has vertices  $E(-4, -1)$ ,  $F(2, 5)$ , and  $G(2, -1)$ . Point  $K$  is the midpoint of  $EG$  and  $H$  is the midpoint of  $FG$ .

- Show that  $\overline{EF}$  is parallel to  $\overline{KH}$ .

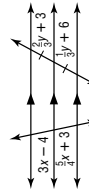
**The slope of  $\overline{EF}$  is 1 and the slope of  $\overline{KH}$  is 1, so  $\overline{EF}$  is parallel to  $\overline{KH}$ .**

- Show that  $KH = \frac{1}{2}EF$ .

**The length of  $\overline{KH}$  is  $3\sqrt{2}$ , and the length of  $\overline{EF}$  is  $6\sqrt{2}$ .**

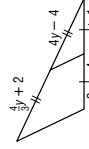


- Find  $x$  and  $y$ .



**$x = 4$ ,  $y = 9$**

- Find  $x$  and  $y$ .



**$x = \frac{5}{2}$ ,  $y = \frac{9}{4}$**

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**9. MAPS** The distance from Wilmington to Ash Grove along Kendall is 820 feet and along Magnolia, 660 feet. If the distance between Beech and Ash Grove along Magnolia is 280 feet, what is the distance between the two streets along Kendall?

**about 348 ft**

## 6-4 Reading to Learn Mathematics

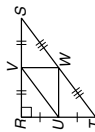
### Parallel Lines and Proportional Parts

#### Pre-Activity

How do city planners use geometry? Read the introduction to Lesson 6-4 at the top of page 307 in your textbook. Use a geometric idea to explain why the distance between Chicago Avenue and Ontario Street is shorter along Michigan Avenue than along Lake Shore Drive. **Sample answer: The shortest distance between two parallel lines is the perpendicular distance.**

#### Reading the Lesson

- Provide the missing words to complete the statement of each theorem. Then state the name of the theorem.
  - If a line intersects two sides of a triangle and separates the sides into corresponding segments of **proportional** lengths, then the line is **parallel** to the third side. **Converse of the Triangle Proportionality Theorem**
  - A midsegment of a triangle is **parallel** to one side of the triangle and its length is **one-half** the length of that side. **Triangle Midsegment Theorem**
  - If a line is **parallel** to one side of a triangle and intersects the other two sides in **two** distinct points, then it separates these sides into **segments** of proportional length. **Triangle Proportionality Theorem**
- Refer to the figure at the right.
  - Name the three midsegments of  $\triangle RST$ .  **$\overline{UV}$ ,  $\overline{VW}$ ,  $\overline{UW}$**
  - If  $RS = 8$ ,  $RU = 3$ , and  $TW = 5$ , find the length of each of the midsegments.  **$UV = 5$ ,  $VW = 3$ ,  $UW = 4$**
  - What is the perimeter of  $\triangle RST$ ? **24**
  - What is the perimeter of  $\triangle UVW$ ? **12**
  - What are the perimeters of  $\triangle RUV$ ,  $\triangle SVW$ , and  $\triangle TUW$ ? **12; 12; 12**
  - How are the perimeters of each of the four small triangles related to the perimeter of the large triangle? **The perimeter of each small triangle is one-half the perimeter of the large triangle.**
  - Would the relationship that you found in part f apply to any triangle in which the midpoints of the three sides are connected? **yes**



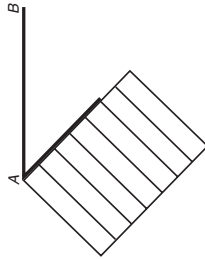
## 6-4 Enrichment

### Parallel Lines and Congruent Parts

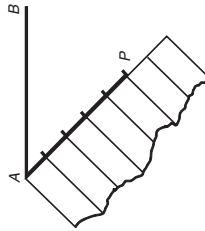
There is a theorem stating that if three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal. This can be shown for any number of parallel lines. The following drafting technique uses this fact to divide a segment into congruent parts.

$\overline{AB}$  is to be separated into five congruent parts. This can be done very accurately without using a ruler. All that is needed is a compass and a piece of notebook paper.

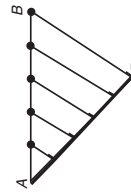
**Step 1** Hold the corner of a piece of notebook paper at point A.



**Step 2** From point A, draw a segment along the paper that is five spaces long. Mark where the lines of the notebook paper meet the segment. Label the fifth point, P.



**Step 3** Draw  $\overline{PB}$ . Through each of the other marks on  $\overline{AP}$ , construct a line parallel to  $\overline{BP}$ . The points where these lines intersect  $\overline{AB}$  will divide  $\overline{AB}$  into five congruent segments.



Use a compass and a piece of notebook paper to divide each segment into the given number of congruent parts. **See student's work.**

- six congruent parts
- seven congruent parts



## 6-5 Study Guide and Intervention

### Parts of Similar Triangles

**Perimeters** If two triangles are similar, their perimeters have the same proportion as the corresponding sides.

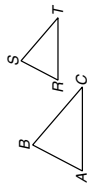
If  $\triangle ABC \sim \triangle RST$ , then

$$\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$$

**Example** Use the diagram above with  $\triangle ABC \sim \triangle RST$ . If  $AB = 24$  and  $RS = 15$ , find the ratio of their perimeters.

Since  $\triangle ABC \sim \triangle RST$ , the ratio of the perimeters of  $\triangle ABC$  and  $\triangle RST$  is the same as the ratio of corresponding sides.

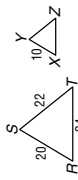
$$\begin{aligned} \text{Therefore } \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle RST} &= \frac{24}{15} \\ &= \frac{8}{5} \end{aligned}$$



### Exercises

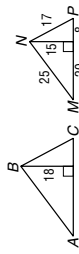
Each pair of triangles is similar. Find the perimeter of the indicated triangle.

1.  $\triangle XYZ$



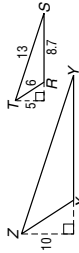
33

3.  $\triangle ABC$



84

4.  $\triangle XYZ$



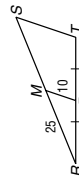
55.4

5.  $\triangle ABC$



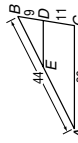
54

6.  $\triangle RST$



110

2.  $\triangle BDE$



45

## 6-5 Study Guide and Intervention

### Parts of Similar Triangles

**Special Segments of Similar Triangles** When two triangles are similar, corresponding altitudes, angle bisectors, and medians are proportional to the corresponding sides. Also, in any triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides of the triangle.

#### Example 1

In the figure,  $\triangle ABC \sim \triangle XYZ$ , with angle bisectors as shown. Find  $x$ .

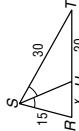


Since  $\triangle ABC \sim \triangle XYZ$ , the measures of the angle bisectors are proportional to the measures of a pair of corresponding sides.

$$\begin{aligned} \frac{AB}{XY} &= \frac{BD}{YW} \\ \frac{24}{x} &= \frac{10}{8} \\ x &= 8 \\ 10x &= 24(8) \\ 10x &= 192 \\ x &= 19.2 \end{aligned}$$

#### Example 2

$\overline{SU}$  bisects  $\angle RST$ . Find  $x$ .



Since  $\overline{SU}$  is an angle bisector,  $\frac{RU}{TU} = \frac{RS}{TS}$ .

$$\begin{aligned} \frac{x}{20-x} &= \frac{15}{30} \\ 30x &= 20(15) \\ 30x &= 300 \\ x &= 10 \end{aligned}$$

### Exercises

Find  $x$  for each pair of similar triangles.

1.



10

2.



8

3.



2.25

4.



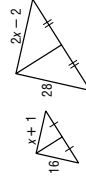
8.75

5.



$12\frac{5}{6}$

6.



15



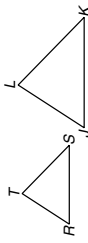
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**6-5 Skills Practice**

**Parts of Similar Triangles**

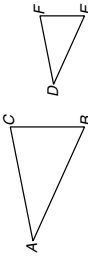
Find the perimeter of the given triangle.

1.  $\triangle JKL$ , if  $\triangle JKL \sim \triangle RST$ ,  $RS = 14$ ,  $ST = 12$ ,  $TR = 10$ , and  $LJ = 14$



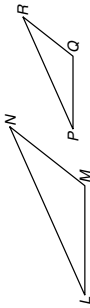
50.4

2.  $\triangle DEF$ , if  $\triangle DEF \sim \triangle ABC$ ,  $AB = 27$ ,  $BC = 16$ ,  $CA = 25$ , and  $FD = 15$



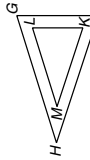
40.8

3.  $\triangle PQR$ , if  $\triangle PQR \sim \triangle LMN$ ,  $LM = 16$ ,  $MN = 14$ ,  $NL = 27$ , and  $RP = 18$



38

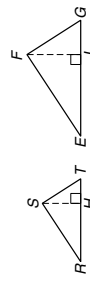
4.  $\triangle KLM$ , if  $\triangle KLM \sim \triangle FGH$ ,  $FG = 30$ ,  $GH = 38$ ,  $HF = 38$ , and  $KL = 24$



84.8

Use the given information to find each measure.

5. Find  $FG$  if  $\triangle RST \sim \triangle EFG$ ,  $SH$  is an altitude of  $\triangle RST$ ,  $FJ$  is an altitude of  $\triangle EFG$ ,  $ST = 6$ ,  $SH = 5$ , and  $FJ = 7$ .



8.4

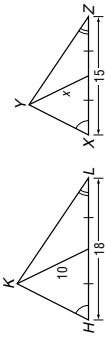
6. Find  $MN$  if  $\triangle ABC \sim \triangle MNP$ ,  $AD$  is an altitude of  $\triangle ABC$ ,  $MQ$  is an altitude of  $\triangle MNP$ ,  $AB = 24$ ,  $AD = 14$ , and  $MQ = 10.5$ .



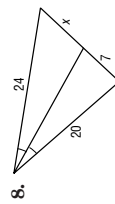
18

Find  $x$ .

7.  $\triangle HKL \sim \triangle XYZ$



8.1



8.4

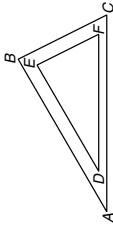
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**6-5 Practice (Average)**

**Parts of Similar Triangles**

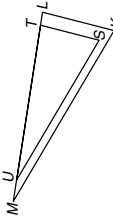
Find the perimeter of the given triangle.

1.  $\triangle DEF$ , if  $\triangle ABC \sim \triangle DEF$ ,  $AB = 36$ ,  $BC = 20$ ,  $CA = 40$ , and  $DE = 35$



93.3

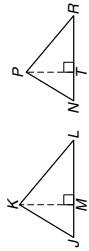
2.  $\triangle STU$ , if  $\triangle STU \sim \triangle KLM$ ,  $KL = 12$ ,  $LM = 31$ ,  $MK = 32$ , and  $US = 28$



65.625

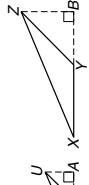
Use the given information to find each measure.

3. Find  $PR$  if  $\triangle JKL \sim \triangle NPR$ ,  $KM$  is an altitude of  $\triangle JKL$ ,  $PT$  is an altitude of  $\triangle NPR$ ,  $KL = 28$ ,  $KM = 18$ , and  $PT = 15.75$ .



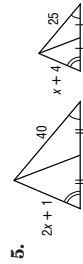
24.5

4. Find  $ZY$  if  $\triangle STU \sim \triangle XYZ$ ,  $UA$  is an altitude of  $\triangle STU$ ,  $ZB$  is an altitude of  $\triangle XYZ$ ,  $UT = 8.5$ ,  $UA = 6$ , and  $ZB = 11.4$ .

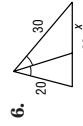


16.15

Find  $x$ .



13.5



16.8

**PHOTOGRAPHY For Exercises 7 and 8, use the following information.**  
Francine has a camera in which the distance from the lens to the film is 24 millimeters.

7. If Francine takes a full-length photograph of her friend from a distance of 3 meters and the height of her friend is 140 centimeters, what will be the height of the image on the film? (*Hint:* Convert to the same unit of measure.) **11.2 mm**

8. Suppose the height of the image on the film of her friend is 15 millimeters. If Francine took a full-length shot, what was the distance between the camera and her friend? **2.24 m**

## 6-5

## Reading to Learn Mathematics

### Parts of Similar Triangles

#### Pre-Activity

How is geometry related to photography?

- Read the introduction to Lesson 6-5 at the top of page 316 in your textbook.
- How is similarity involved in the process of making a photographic print from a negative?

**Sample answer:** The print is an enlarged version of the negative. The image on the print is similar to the image on the negative, so when you look at the print, you will see exactly the same things in the same proportions as on the negative, just larger.

- Why do photographers place their cameras on tripods?

**Sample answer:** Three points determine a plane, so the three feet of the tripod are coplanar and the tripod will not wobble.

#### Reading the Lesson

1. In the figure,  $\triangle RST \sim \triangle UVW$ . Complete each proportion involving the lengths of segments in this figure by replacing the question mark. Then identify the definition or theorem from the list below that the completed proportion illustrates.



- Definition of congruent polygons
- Definition of similar polygons
- Proportional Perimeters Theorem
- Angle Bisectors Theorem
- Similar triangles have corresponding altitudes proportional to corresponding sides.
- Similar triangles have corresponding medians proportional to corresponding sides.
- Similar triangles have corresponding angle bisectors proportional to corresponding sides.

- a.  $\frac{RS + ST + TR}{?} = \frac{RS}{UV} = \frac{ST}{VW} = \frac{RT}{UW} = \frac{SX}{?} = \frac{VY}{?} = \frac{VZ}{?}$   
 b.  $\frac{RM}{UN} = \frac{?}{VW}$     **ST; vi**  
 c.  $\frac{RS}{UV} = \frac{?}{VW}$     **ST; vi**  
 d.  $\frac{RS}{UV} = \frac{ST}{?}$     **VW; ii**  
 e.  $\frac{RP}{PS} = \frac{?}{ST}$     **RT; iv**  
 f.  $\frac{UN}{?} = \frac{UW}{RT}$     **RM; vi**  
 g.  $\frac{TP}{WQ} = \frac{RT}{?}$     **UW; vii**  
 h.  $\frac{UW}{VW} = \frac{?}{QV}$     **UQ; iv**

#### Helping You Remember

2. A good way to remember a large amount of information is to remember key words. What key words will help you remember the features of similar triangles that are proportional to the lengths of the corresponding sides?  
**perimeters, altitudes, angle bisectors, medians**

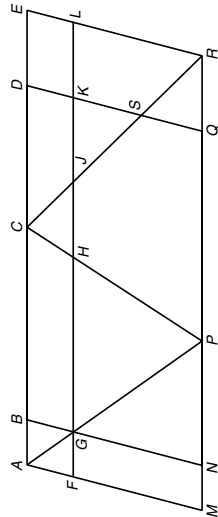
## 6-5

## Enrichment

### Proportions for Similar Triangles

Recall that if a line crosses two sides of a triangle and is parallel to the third side, then the line separates the two sides that it crosses into segments of proportional lengths.

You can write many proportions by identifying similar triangles in the following diagram. In the diagram,  $\overline{AM} \parallel \overline{BN}$ ,  $\overline{AE} \parallel \overline{FL}$ ,  $\overline{MR} \parallel \overline{DQ}$ , and  $\overline{DQ} \parallel \overline{ER}$ .



Answer each question. Use the diagram above.

- Name a triangle similar to  $\triangle GNP$ .     **$\triangle AMP$  or  $\triangle GBA$  or  $\triangle AFG$**
- Name a triangle similar to  $\triangle CJH$ .     **$\triangle CRP$**
- Name two triangles similar to  $\triangle JKS$ .     **$\triangle GHP$**
- Name a triangle similar to  $\triangle ACP$ .     **$\triangle GHP$**

Complete each proportion.

5.  $\frac{AG}{AP} = \frac{AF}{?}$     **AM**    6.  $\frac{CP}{CH} = \frac{CR}{?}$     **CJ**    7.  $\frac{JS}{JR} = \frac{?}{JL}$     **JK**  
 8.  $\frac{PH}{PC} = \frac{PG}{?}$     **PA**    9.  $\frac{ER}{LR} = \frac{?}{JR}$     **CR**    10.  $\frac{MN}{MP} = \frac{?}{AP}$     **GA**

Solve.

- If  $CJ = 16$ ,  $JR = 48$ , and  $LR = 30$ , find  $EL$ .    **10**
  - If  $DK = 5$ ,  $KS = 7$ , and  $CJ = 8$ , find  $JS$ .    **11.2**
  - If  $MN = 12$ ,  $NP = 32$ , and  $AP = 48$ , find  $AG$ . Round to the nearest tenth.    **13.1**
  - If  $CH = 18$ ,  $HP = 82$ , and  $CR = 130$ , find  $CJ$ .    **23.4**
15. Write three more problems that can be solved using the diagram above.  
**See students' work.**

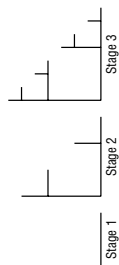
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## 6-6 Study Guide and Intervention *(continued)*

### Fractals and Self-Similarity

**Characteristics of Fractals** The act of repeating a process over and over, such as finding a third of a segment, then a third of the new segment, and so on, is called **iteration**. When the process of iteration is applied to some geometric figures, the results are called **fractals**. For objects such as fractals, when a portion of the object has the same shape or characteristics as the entire object, the object can be called **self-similar**.

**Example** In the diagram at the right, notice that the details at each stage are similar to the details at Stage 1.

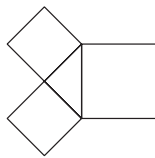


#### Exercises

1. Follow the iteration process below to produce a fractal.

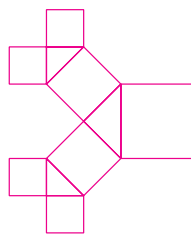
##### Stage 1

- Draw a square.
- Draw an isosceles right triangle on the top side of the square. Use the side of the square as the hypotenuse of the triangle.
- Draw a square on each leg of the right triangle.



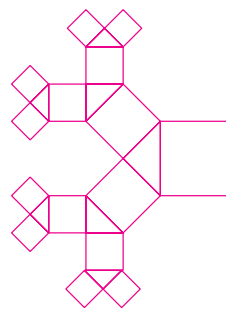
##### Stage 2

Repeat the steps in Stage 1, drawing an isosceles triangle and two small squares for each of the small squares from Stage 1.



##### Stage 3

Repeat the steps in Stage 1 for each of the smallest squares in Stage 2.



2. Is the figure produced in Stage 3 self-similar? **yes**

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## 6-6 Study Guide and Intervention *(continued)*

### Fractals and Self-Similarity

**Nongometric Iteration** An iterative process can be applied to an algebraic expression or equation. The result is called a **recursive formula**.

**Example** Find the value of  $x^n$ , where the initial value of  $x$  is 2. Repeat the process three times and describe the pattern.

Initial value: 2

First time:  $2^3 = 8$

Second time:  $8^3 = 512$

Third time:  $512^3 = 134,217,728$

The result of each step of the iteration is used for the next step. For this example, the  $x$  values are greater with each iteration. There is no maximum value, so the values are described as *approaching infinity*.

#### Exercises

For Exercises 1–5, find the value of each expression. Then use that value as the next  $x$  in the expression. Repeat the process three times, and describe your observations.

1.  $\sqrt{x}$ , where  $x$  initially equals 5

**$\sqrt{5} = 2.24$ ; 1.50, 1.22, 1.11; the  $x$  values get smaller with each iteration.**

2.  $\frac{1}{x}$ , where  $x$  initially equals 2

**$\frac{1}{2}$ ; 2,  $\frac{1}{2}$ ; 2; the  $x$  values alternate between  $\frac{1}{2}$  and 2.**

3.  $3^x$ , where  $x$  initially equals 1

**$3^1 = 3$ ;  $3^3 = 27$ ;  $3^{27} = 7.6 \times 10^{12}$ ; the  $x$  values get greater with each iteration, approaching infinity.**

4.  $x - 5$  where  $x$  initially equals 10

**$10 - 5 = 5$ ; 0, -5, -10; the  $x$  values decrease by 5 with each iteration.**

5.  $x^2 - 4$ , where  $x$  initially equals 16

**$16^2 - 4 = 252$ ;  $252^2 - 4 = 63,500$ ;  $63,500^2 - 4 = 4.0 \times 10^9$ ;  $(4.0 \times 10^9)^2 - 4 = 1.6 \times 10^{19}$ ; the  $x$  values increase with each iteration, approaching infinity.**

6. Harpesh paid \$1000 for a savings certificate. It earns interest at an annual rate of 2.8%, and interest is added to the certificate each year. What will the certificate be worth after four years? **\$1116.79**

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**6-6 Skills Practice**  
*Fractals and Self-Similarity*

Stages 1 and 2 of a fractal known as the Cantor set are shown. To get Stage 2, the segment in Stage 1 is trisected, and the interior of the middle segment is removed. (The interior of a segment is the segment with its endpoints removed.) The process is repeated for subsequent stages.

Stage 1 \_\_\_\_\_

Stage 2 \_\_\_\_\_

1. Draw stages 3 and 4 of the Cantor set.

Stage 3 

Stage 4 

2. How many segments are there in Stage 3? Stage 4?  
**4 segments; 8 segments**

3. What happens to the length of the line segments in each stage?  
**The line segments decrease in length by two-thirds at each stage.**

4. The Cantor set is the set of points after infinitely many iterations. Is the Cantor set self-similar?  
**yes**

Find the value of each expression. Then, use that value as the next  $x$  in the expression. Repeat the process three times, and describe your observations.

5.  $2x - 3$ , where  $x$  initially equals 5

**$2(5) - 3 = 7$ ;  $2(7) - 3 = 11$ ;  $2(11) - 3 = 19$ ;  $2(19) - 3 = 35$**   
**The iterates increase by consecutive powers of 2.**

6.  $\frac{x}{2}$ , where  $x$  initially equals 1

**$\frac{1}{2} = 0.5$ ;  $\frac{0.5}{2} = 0.25$ ;  $\frac{0.25}{2} = 0.125$ ;  $\frac{0.125}{2} = 0.0625$**   
**The iterates are halved with each iteration, approaching zero.**

Find the first three iterates of each expression.

7.  $x^2 - 1$ , where  $x$  initially equals 2

**3, 8, 63**

8.  $\frac{1}{x}$ , where  $x$  initially equals 8

**0.125, 8, 0.125**

**Lesson 6-6**

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

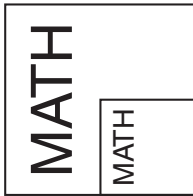
**6-6 Practice (Average)**  
*Fractals and Self-Similarity*

An artist is designing a book cover and wants to show a copy of the book cover in the lower left corner of the cover. After Stages 1 and 2 of the design, he realizes that the design is developing into a fractal!

Stage 1

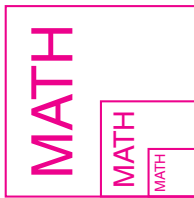


Stage 2

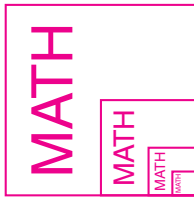


1. Draw Stages 3 and 4 of the book cover fractal.

Stage 3



Stage 4



2. After infinitely many iterations, will the result be a self-similar fractal? **yes**

3. On what part of the book cover should you focus your attention to be sure you can find a copy of the entire figure? **the lower left corner**

Find the value of each expression. Then, use that value as the next  $x$  in the expression. Repeat the process three times and describe your observations.

4.  $2(x + 3)$ , where  $x$  initially equals 0

**6, 18, 42, 90**

**The difference between values doubles with each term.**

5.  $\frac{x}{2} - 3$ , where  $x$  initially equals 10

**2, -2, -4, -5**

**The difference between values is multiplied by  $\frac{1}{2}$  with each term.**

Find the first three iterates of each expression.

6.  $\frac{x}{2} + 1$ , where  $x$  initially equals 1

**1.5; 1.75; 1.875**

7.  $2x^2$ , where  $x$  initially equals 2

**8; 128; 32,768**

8. **HOUSING** The Andrews purchased a house for \$96,000. The real estate agent who sold the house said that comparable houses in the area appreciate at a rate of 4.5% per year. If this pattern continues, what will be the value of the house in three years? Round to the nearest whole number. **\$109,552**

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## 6-6 Reading to Learn Mathematics

### Fractals and Self-Similarity

#### Pre-Activity How is mathematics found in nature?

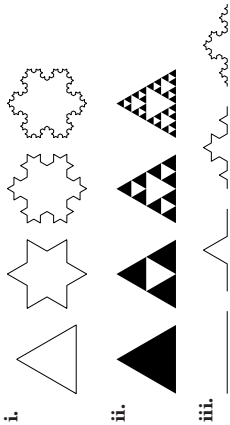
Read the introduction to Lesson 6-6 at the top of page 325 in your textbook. Name two objects from nature other than broccoli in which a small piece resembles the whole. **Sample answer: rings of a tree trunk; spiral shell**

#### Reading the Lesson

1. Match each definition from the first column with a term from the second column. (Some words of phrases in the second column may be used more than once or not at all.)

Phrase	Term
a. a geometric figure that is created using iteration	<b>iii</b> similar
b. a pattern in which smaller and smaller details of a shape have the same geometric characteristics as the original shape	<b>iv</b> fractal
c. the result of translating an iterative process into a formula or algebraic equation	<b>v</b> self-similar recursion formula
d. the process of repeating the same procedure over and over again	<b>ii</b> congruent

2. Refer to the three patterns below.



3. Give the special name for each of the fractals you obtain by continuing the pattern without end. **i. Koch snowflake; ii. Sierpinski triangle; iii. Koch curve**
4. Which of these fractals are self-similar? **Sierpinski triangle, Koch curve**
5. Which of these fractals are strictly self-similar? **Sierpinski triangle**

#### Helping You Remember

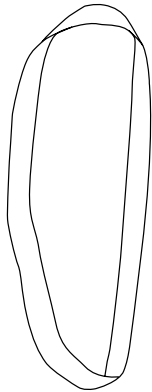
6. A good way to remember a new mathematical term is to relate it to everyday English words. The word *fractal* is related to *fraction* and *fragment*. Use your dictionary to find at least one definition for each of these words that you think is related to the meaning of the word *fractal* and explain the connection. **Sample answer: Fraction: a small part; a disconnected piece; fragment: a piece detached or broken off; something incomplete; in a fractal, the smaller pieces of the shape resemble the whole.**

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

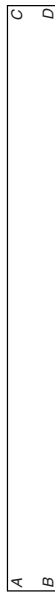
## 6-6 Enrichment

### The Möbius Strip

A Möbius strip is a special surface with only one side. It was discovered by August Ferdinand Möbius, a German astronomer and mathematician.



1. To make a Möbius strip, cut a strip of paper about 16 inches long and 1 inch wide. Mark the ends with the letters *A*, *B*, *C*, and *D* as shown below.



Twist the paper once, connecting *A* to *D* and *B* to *C*. Tape the ends together on both sides.

**See students' work.**

2. Use a crayon or pencil to shade one side of the paper. Shade around the strip until you get back to where you started. What happens?

**The entire strip is shaded on both sides.**

3. What do you think will happen if you cut the Möbius strip down the middle? Try it.

**Instead of two loops, you get one loop that is twice as long as the original one.**

4. Make another Möbius strip. Starting a third of the way in from one edge, cut around the strip, staying always the same distance in from the edge. What happens?

**Two interlocking loops are formed.**

5. Start with another long strip of paper. Twist the paper twice and connect the ends. What happens when you cut down the center of this strip?

**Two interlocking loops are formed.**

6. Start with another long strip of paper. Twist the paper three times and connect the ends. What happens when you cut down the center of this strip?

**One two-sided loop with a knot is formed.**

# Chapter 6 Assessment Answer Key

Form 1  
Page 331

1.   A
2.   D
3.   B
4.   B
5.   B
6.   D
7.   B
8.   C
9.   A
10.  B

Page 332

11.   D
12.   D
13.   A
14.   B
15.   A
16.   B
17.   D
18.   C
19.   A
- B:       9.6

Form 2A  
Page 333

1.   A
2.   A
3.   D
4.   C
5.   B
6.   C
7.   C
8.   A
9.   C
10.  B

*(continued on the next page)*

# Chapter 6 Assessment Answer Key

Form 2A (continued)

Page 334

11. D

12. D

13. B

14. B

15. C

16. A

17. A

18. A

19. C

B: 32

Form 2B

Page 335

1. A

2. C

3. B

4. D

5. B

6. B

7. D

8. B

9. B

10. C

Page 336

11. D

12. A

13. C

14. D

15. B

16. A

17. D

18. C

19. D

B: 26

# Chapter 6 Assessment Answer Key

Form 2C

Page 337

1. 3:7

2. Yes; corres.  $\angle$ s  
are  $\cong$ .

3. 30 ft

4. 18

5. 42.5 ft

6. 6.3

7. 18.2

8. No; the sides are  
not proportional.

9. 72

10.  $\triangle PQR \sim \triangle STR$

11.  $2\frac{1}{2}$

Page 338

12. 9.5

13. 42

14. 47

15. 2

16. 3

17. 14.4

18. 4

19. 123

B: 7



# Chapter 6 Assessment Answer Key

Form 2D

Page 339

1. 5:11

2. No; the corresponding sides are not proportional.

3. 48 ft

4.  $\frac{4}{3}$

5.  $183\frac{1}{3}$  ft

6. 5.5

7. 15

8. yes; SSS Similarity

9. 75

10.  $\triangle XYZ \sim \triangle XNM$

11. 7.5

Page 340

12. 2.2

13. 96

14. 85

15. 9.5

16. 2

17.  $5\frac{5}{8}$

18. 28

19. 472

B: 4.8

# Chapter 6 Assessment Answer Key

Form 3

Page 341

1. 4:3

2. No; corr. sides are not proportional.

3. yes; SAS

4. 30 cm by 7.5 cm

5. 5.85

6. 5

7. 104 in.

8. 82

9. 48

10.  $\frac{8}{3}$

11. 5

Page 342

12.  $11\frac{3}{7}$

13. 7.2

14.  $\frac{8}{5}$

15. 60

16. 24.8

17. 24 ft

18. 9

19. 10 in.

B: 25.6

# Chapter 6 Assessment Answer Key

## Page 343, Open-Ended Assessment Scoring Rubric

Score	General Description	Specific Criteria
4	<b>Superior</b> A correct solution that is supported by well-developed, accurate explanations	<ul style="list-style-type: none"> <li>Shows thorough understanding of the concepts of <i>ratios, properties of proportions, similar figures, similar triangles, dividing segments into parts, proportional parts of triangles, corresponding perimeters and altitudes, fractals, and iteration.</i></li> <li>Uses appropriate strategies to solve problems.</li> <li>Computations are correct.</li> <li>Written explanations are exemplary.</li> <li>Figures are accurate and appropriate.</li> <li>Goes beyond requirements of some or all problems.</li> </ul>
3	<b>Satisfactory</b> A generally correct solution, but may contain minor flaws in reasoning or computation	<ul style="list-style-type: none"> <li>Shows an understanding of the concepts of <i>ratios, properties of proportions, similar figures, similar triangles, dividing segments into parts, proportional parts of triangles, corresponding perimeters and altitudes, fractals, and iteration.</i></li> <li>Uses appropriate strategies to solve problems.</li> <li>Computations are mostly correct.</li> <li>Written explanations are effective.</li> <li>Figures are mostly accurate and appropriate.</li> <li>Satisfies all requirements of problems.</li> </ul>
2	<b>Nearly Satisfactory</b> A partially correct interpretation and/or solution to the problem	<ul style="list-style-type: none"> <li>Shows an understanding of most of the concepts of <i>ratios, properties of proportions, similar figures, similar triangles, dividing segments into parts, proportional parts of triangles, corresponding perimeters and altitudes, fractals, and iteration.</i></li> <li>May not use appropriate strategies to solve problems.</li> <li>Computations are mostly correct.</li> <li>Written explanations are satisfactory.</li> <li>Figures are mostly accurate.</li> <li>Satisfies the requirements of most of the problems.</li> </ul>
1	<b>Nearly Unsatisfactory</b> A correct solution with no supporting evidence or explanation	<ul style="list-style-type: none"> <li>Final computation is correct.</li> <li>No written explanations or work is shown to substantiate the final computation.</li> <li>Figures may be accurate but lack detail or explanation.</li> <li>Satisfies minimal requirements of some of the problems.</li> </ul>
0	<b>Unsatisfactory</b> An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	<ul style="list-style-type: none"> <li>Shows little or no understanding of most of the concepts of <i>ratios, properties of proportions, similar figures, similar triangles, dividing segments into parts, proportional parts of triangles, corresponding perimeters and altitudes, fractals, and iteration.</i></li> <li>Does not use appropriate strategies to solve problems.</li> <li>Computations are incorrect.</li> <li>Written explanations are unsatisfactory.</li> <li>Figures are inaccurate or inappropriate.</li> <li>Does not satisfy requirements of problems.</li> <li>No answer may be given.</li> </ul>

# Chapter 6 Assessment Answer Key

## Page 343, Open-Ended Assessment Sample Answers

*In addition to the scoring rubric found on page A25, the following sample answers may be used as guidance in evaluating open-ended assessment items.*

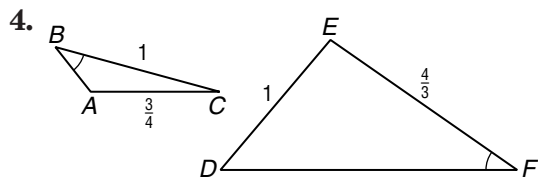
1.  $\frac{5}{3} = \frac{10}{6}$  or  $\frac{15}{3} = \frac{10}{2}$

2. a. 3:2:2 (Note: Check to be sure that the sum of any two sides of the triangle is greater than the third side. For example, a ratio of 1:1:2 would not be acceptable.)

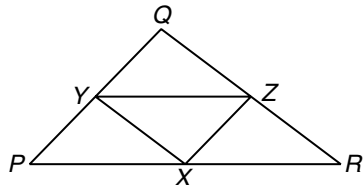
b. 18, 12, 12

c. 12, 8, 8

3.  $\triangle ABH \sim \triangle CDI \sim \triangle GFI \sim \triangle ADG \sim \triangle GDE \sim \triangle CFE \sim \triangle AGE$  by the AA Postulate.

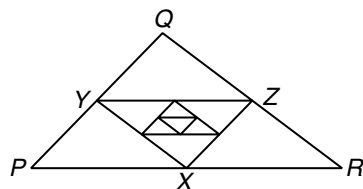


5. a.



Mark the midpoint of each side of  $\triangle PQR$ . Connect these points to form  $\triangle XYZ$ . Since each segment of  $\triangle XYZ$  is a midsegment of  $\triangle PQR$ , its length will be half the corresponding side, and therefore the perimeter will be half the perimeter of  $\triangle PQR$ .

b.



c. Each of the triangles are similar by SSS (since their sides are half the length of the preceding sides), so the figure is strictly self-similar and thus a fractal.

d. 6

# Chapter 6 Assessment Answer Key

## Vocabulary Test/Review Page 344

1. ratio
2. scale factor
3. midsegment
4. extremes
5. means
6. fractal
7. similar polygons
8. iteration
9. proportion
10. self-similar
11. Sample answer: Given a proportion  $\frac{a}{b} = \frac{c}{d}$ ,  
 $ad = bc$ .
12. Sample answer: Each part of the figure, no matter where it is located or what size is selected, contains the same figure as the whole.

## Quiz 1 Page 345

1. 1:30
2. 33 in.
3.  $\triangle BAC \sim \triangle QPR$  or  $\triangle ABC \sim \triangle QPR$  by SAS.
4.  $x = 4$ ;  
scale factor 5:4
5. B

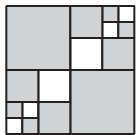
## Quiz 2 Page 345

1. yes; AA
2. No; the sides are not proportional.
3.  $\triangle ABD \sim \triangle CDE$  or  $\triangle ADB \sim \triangle CDF$ ;  
8
4.  $22\frac{1}{2}$  ft
5. 6

## Quiz 3 Page 346

1.  $22\frac{1}{2}$
2.  $13\frac{1}{3}$
3. 20
4. 9
5. 14

## Quiz 4 Page 346

1. 81
2.   
Stage 3
3. 7, 27, 107
4. 6, 30, 870

# Chapter 6 Assessment Answer Key

## Mid-Chapter Test Page 347

### Part I

1. B

2. D

3. C

4. D

5. B

### Part II

6. No; SAS does not apply.

7. yes; SAS

8.  $28; \frac{8}{3}$

9. 120

10. 3.6 in.

## Cumulative Review Page 348

1.  $(-5.5, -8)$

2. 11

3. true

4. 6

5. perpendicular

6.  $y = 4x - 47$

7. yes;  $TU = WX = \sqrt{40}$ ,  
 $UV = XY = \sqrt{53}$ , and  
 $VT = YW = \sqrt{41}$

8.  $\angle NMQ \cong \angle NPR$ ;  
 $\overline{LM} \cong \overline{LP}$

9.  $x = 0.5, y = 2, z = 1$

10.  $m\angle BEG < m\angle CEG$

$\angle H \cong \angle H, \overline{JL} \parallel \overline{KM}$ ,  
and  $\angle HJL \cong \angle HKM$ ,  
11. thus  $\triangle HJL \sim \triangle HKM$   
by AA Similarity.

12. 9

13.  $a = 1; b = 2$

# Chapter 6 Assessment Answer Key

## Standardized Test Practice

Page 349

Page 350

1.  A  B  C  D

2.  E  F  G  H

3.  A  B  C  D

4.  E  F  G  H

5.  A  B  C  D

6.  E  F  G  H

7.  A  B  C  D

8.  E  F  G  H

9.

1	1	9	
.	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

2	5		
.	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

1	2	8	
.	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12.

1	6		
.	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13.           -3          

14.           85.5          

15.            $11 < x < 53$           

16.           64.5 cm          

Answers