GLENCOE MATHEMATICS

Geometry

Chapter 12
Resource Masters

Glencoe

New York, New York
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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 12 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Geometry
Chapter 12 Resource Masters

1 2 3 4 5 6 7 8 9 10 009 11 10 09 08 07 06 05 04 03

Contents

Teacher's Guide to Using the Chapter 12 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 12 Resource Masters* includes the core materials needed for Chapter 12. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 12-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 12 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 684–685. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

NAME ____ DATE ____________ PERIOD _____

12 Reading to Learn Mathematics

Vocabulary Builder

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 12. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
altitude		
axis		
cone		
cylinder		
great circle		
hemisphere		
lateral area		
lateral edges		
lateral faces		
net		

(continued on the next page)

NAME ______________________________ DATE ____________ PERIOD _____

12 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

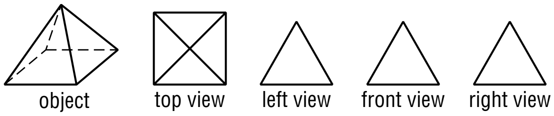
Vocabulary Term	Found on Page	Definition/Description/Example
oblique		
orthogonal drawing ohr·THAHG·uhn·uhl		
polyhedrons		
prism PRIZ·uhm		
pyramid		
regular polyhedron		
regular prism		
right		
sphere SFIR		
surface area		

NAME _______________________________________ DATE ____________ PERIOD _____

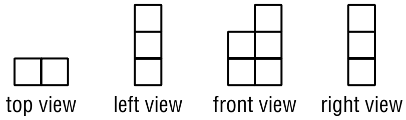
12-1 Study Guide and Intervention

Three-Dimensional Figures

Drawings of Three-Dimensional Figures To work with a three-dimensional object, a useful skill is the ability to make an **orthogonal drawing**, which is a set of two-dimensional drawings of the different sides of the object. For a square pyramid, you would show the top view, the left view, the front view, and the right view.

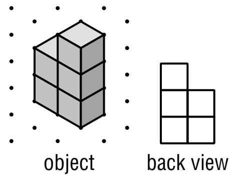


Example **Draw the back view of the figure given the orthogonal drawing.**



- The top view indicates two columns.
- The left view indicates that the height of figure is three blocks.
- The front view indicates that the columns have heights 2 and 3 blocks.
- The right view indicates that the height of the figure is three blocks.

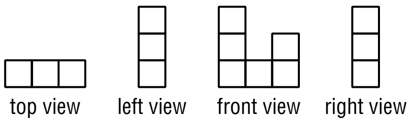
Use blocks to make a model of the object. Then use your model to draw the back view. The back view indicates that the columns have heights 3 and 2 blocks.



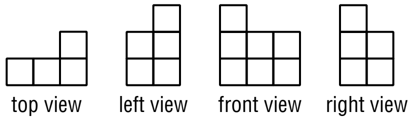
Exercises

Draw the back view and corner view of a figure given each orthogonal drawing.

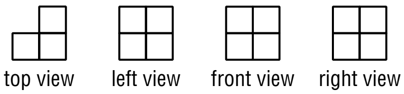
1.



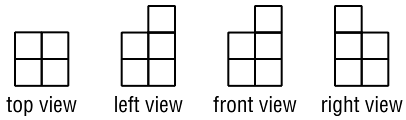
2.



3.



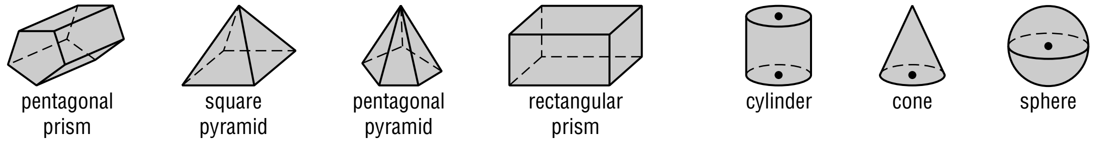
4.



12-1 Study Guide and Intervention *(continued)*

Three-Dimensional Figures

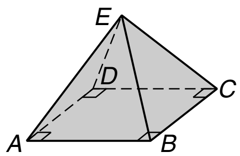
Identify Three-Dimensional Figures A **polyhedron** is a solid with all flat surfaces. Each surface of a polyhedron is called a **face**, and each line segment where faces intersect is called an **edge**. Two special kinds of polyhedra are **prisms**, for which two faces are congruent, parallel **bases**, and **pyramids**, for which one face is a base and all the other faces meet at a point called the **vertex**. Prisms and pyramids are named for the shape of their bases, and a regular polyhedron has a **regular** polygon as its base.



Other solids are a **cylinder**, which has congruent circular bases in parallel planes, a **cone**, which has one circular base and a vertex, and a **sphere**.

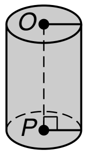
Example **Identify each solid. Name the bases, faces, edges, and vertices.**

a.



The figure is a rectangular pyramid. The base is rectangle *ABCD*, and the four faces △*ABE*, △*BCE*, △*CDE*, and △*ADE* meet at vertex *E*. The edges are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$. The vertices are *A*, *B*, *C*, *D*, and *E*.

b.

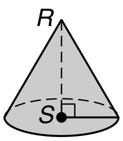


This solid is a cylinder. The two bases are ⊙*O* and ⊙*P*.

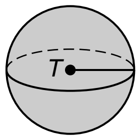
Exercises

Identify each solid. Name the bases, faces, edges, and vertices.

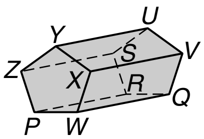
1.



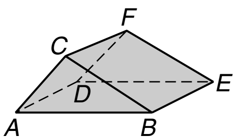
2.



3.



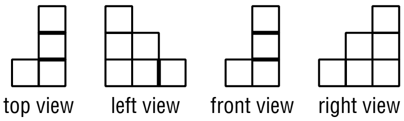
4.

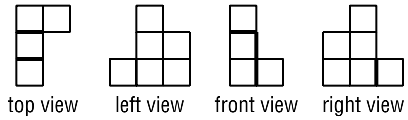


12-1 Skills Practice

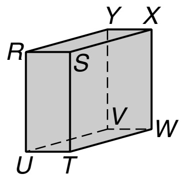
Three-Dimensional Figures

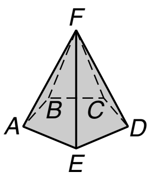
Draw the back view and corner view of a figure given each orthogonal drawing.

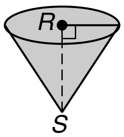
1. 

2. 

Identify each solid. Name the bases, faces, edges, and vertices.

3. 

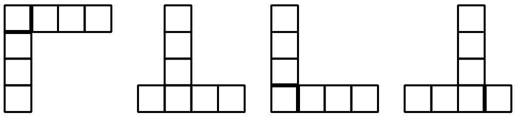
4. 

5. 

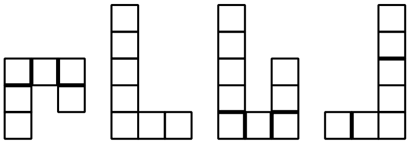
12-1 Practice

Three-Dimensional Figures

Draw the back view and corner view of a figure given each orthogonal drawing.

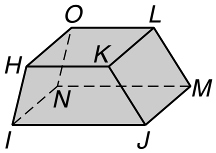
1. 

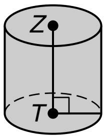
top view left view front view right view

2. 

top view left view front view right view

Identify each solid. Name the bases, faces, edges, and vertices.

3. 

4. 

5. MINERALS Pyrite, also known as fool's gold, can form crystals that are perfect cubes. Suppose a gemologist wants to cut a cube of pyrite to get a square and a rectangular face. What cuts should be made to get each of the shapes? Illustrate your answers.

NAME ____ DATE ____________ PERIOD _____

12-1 Reading to Learn Mathematics

Three-Dimensional Figures

Pre-Activity **Why are drawings of three-dimensional structures valuable to archeologists?**

Read the introduction to Lesson 12-1 at the top of page 636 in your textbook.

Why do you think archeologists would want to know the details of the sizes and shapes of ancient three-dimensional structures?

Lesson 12-1

Reading the Lesson

1. Match each description from the first column with one of the terms from the second column. (Some of the terms may be used more than once or not at all.)

a. a polyhedron with two parallel congruent bases
b. the set of points in space that are a given distance from a given point
c. a regular polyhedron with eight faces
d. a polyhedron that has all faces but one intersecting at one point
e. a line segment where two faces of a polyhedron intersect
f. a solid with congruent circular bases in a pair of parallel planes
g. a regular polyhedron whose faces are squares
h. a flat surface of a polyhedron

i. octahedron
ii. face
iii. icosahedron
iv. edge
v. prism
vi. dodecahedron
vii. cylinder
viii. sphere
ix. cone
x. hexahedron
xi. pyramid
xii. tetrahedron

2. Fill in the missing numbers, words, or phrases to complete each sentence.

a. A triangular prism has ____ vertices. It has ____ faces: ____ bases are congruent _________________, and ____ faces are parallelograms.

b. A regular octahedron has ____ vertices and ____ faces. Each face is a(n) __________________________.

c. A hexagonal prism has ____ vertices. It has ____ faces: ____ of them are the bases, which are congruent _________________, and the other ____ faces are parallelograms.

d. An octagonal pyramid has ____ vertices and ____ faces. The base is a(n) _________________, and the other ____ faces are _________________.

e. There are exactly ____ types of regular polyhedra. These are called the _________________ solids. A polyhedron whose faces are regular pentagons is a _________________, which has ____ faces.

Helping You Remember

3. A good way to remember the characteristics of geometric solids is to think about how different solids are alike. Name a way in which pyramids and cones are alike.

NAME ____ DATE ____________ PERIOD _____

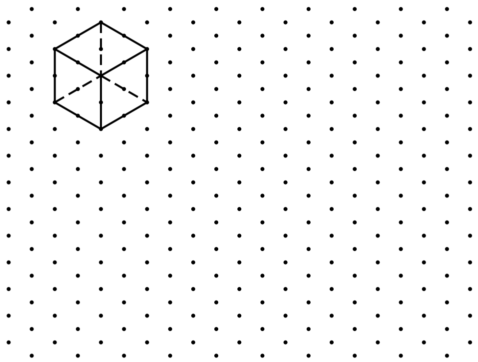
12-1 Enrichment

Drawing Solids on Isometric Dot Paper

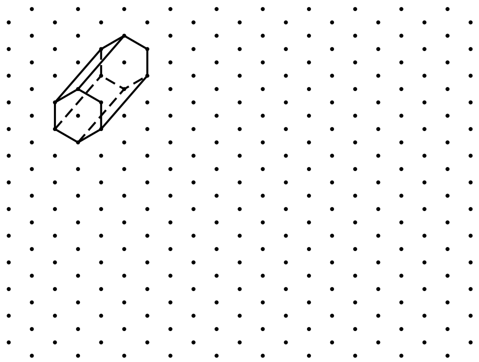
Isometric dot paper is helpful for drawing solids. Remember to use dashed lines for hidden edges.

For each solid shown, draw another solid whose dimensions are twice as large.

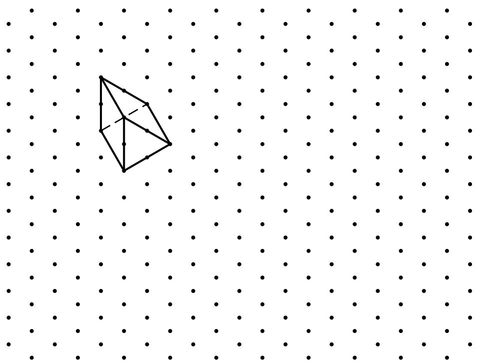
1.



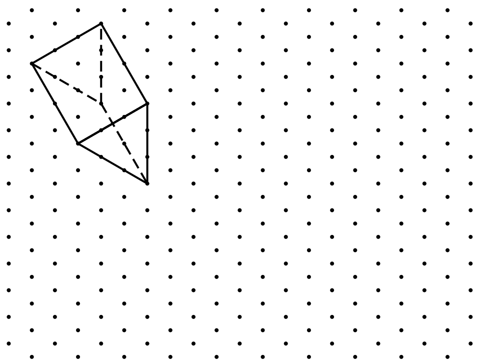
2.



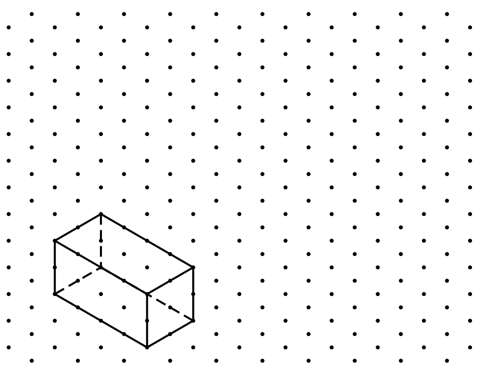
3.



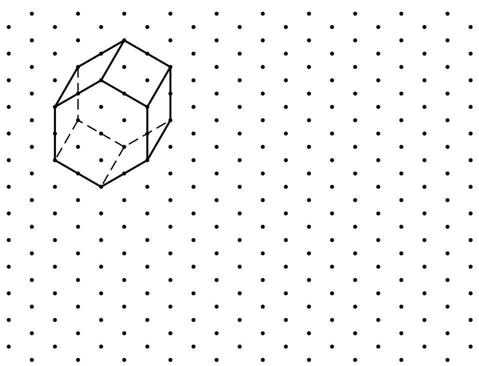
4.



5.



6.

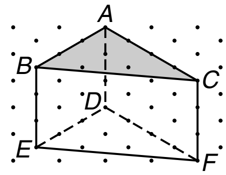


12-2 Study Guide and Intervention

Nets and Surface Area

Models for Three-Dimensional Figures One way to relate a three-dimensional figure and a two-dimensional drawing is to use isometric dot paper. Another way is to make a flat pattern, called a *net*, for the surfaces of a solid.

Example 1 **Use isometric dot paper to sketch a triangular prism with 3-4-5 right triangles as bases and with a height of 3 units.**

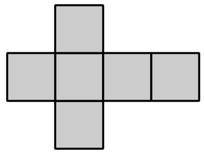


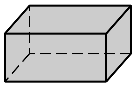
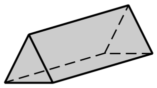
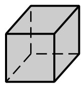
Step 1 Draw $\overline{AB}$ at 3 units and draw $\overline{AC}$ at 4 units.

Step 2 Draw $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, each at 3 units.

Step 3 Draw $\overline{BC}$ and $\triangle DEF$.

Example 2 **Match the net at the right with one of the solids below.**



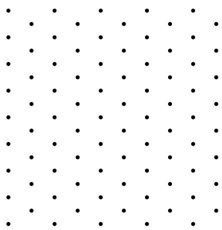
a. **b.** **c.**

The six squares of the net can be folded into a cube. The net represents solid c.

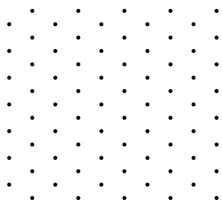
Exercises

Sketch each solid using isometric dot paper.

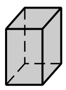
1. cube with edge 4

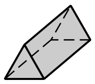


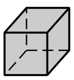
2. rectangular prism 1 unit high, 5 units long, and 4 units wide

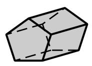


Draw a net for each solid.

3. 

4. 

5. 

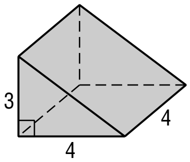
6. 

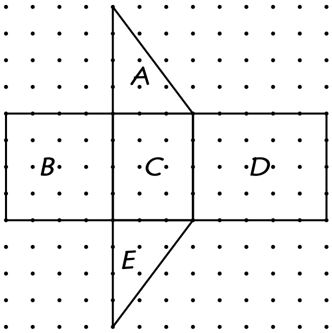
12-2 Study Guide and Intervention *(continued)*

Nets and Surface Area

Surface Area The **surface area** of a solid is the sum of the areas of the faces of the solid. Nets are useful in visualizing each face and calculating the area of the faces.

Example **Find the surface area of the triangular prism.**





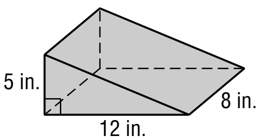
First draw a net using rectangular dot paper. Using the Pythagorean Theorem, the hypotenuse of the right triangle is $\sqrt{3^2 + 4^2}$ or 5.

$$\begin{aligned}\text{Surface area} &= A + B + C + D + E \\ &= \frac{1}{2}(4 \cdot 3) + 4 \cdot 4 + 4 \cdot 3 + 4 \cdot 5 + \frac{1}{2}(4 \cdot 3) \\ &= 60 \text{ square units}\end{aligned}$$

Exercises

Find the surface area of each solid. Round to the nearest tenth if necessary.

1.



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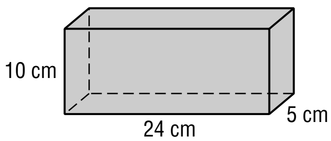

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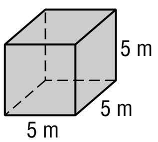

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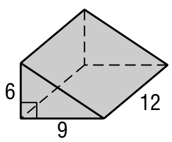

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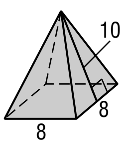

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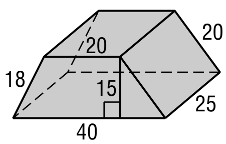


12-2 Skills Practice

Nets and Surface Area

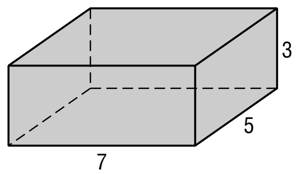
Sketch each solid using isometric dot paper.

1. cube 2 units on each edge

2. rectangular prism 2 units high, 5 units long, and 2 units wide

For each solid, draw a net and find the surface area.

3.



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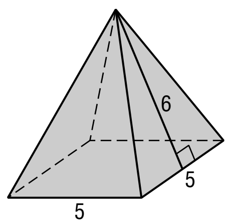


NAME ____ DATE ____________ PERIOD _____

12-2 Practice

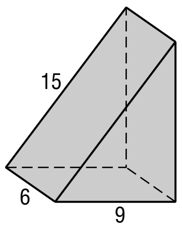
Nets and Surface Area

Sketch each solid using isometric dot paper.

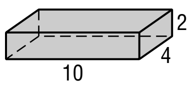
1. rectangular prism 3 units high, 3 units long, and 2 units wide

2. triangular prism 3 units high, whose bases are right triangles with legs 2 units and 4 units long

3. For the solid, draw a net and find the surface area.



4. SHIPPING Rawanda needs to wrap a package to ship to her aunt. The rectangular package measures 2 inches high, 10 inches long, and 4 inches wide. Draw a net of the package. How much wrapping paper does Rawanda need to cover the package?



12-2 Reading to Learn Mathematics

Nets and Surface Area

Pre-Activity **Why is surface area important to car manufacturers?**

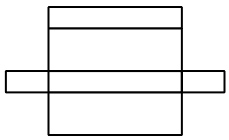
Read the introduction to Lesson 12-2 at the top of page 643 in your textbook.

Suppose you are a passenger in a car that is moving at 50 miles per hour on the highway. If you hold your right hand just outside the right window, in what position does your hand encounter the greatest air resistance?

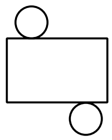
Reading the Lesson

1. Name the solid that can be formed by folding each net without any overlap.

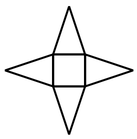
a.



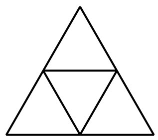
b.



c.



d.



2. Supply the missing number, word, or phrase to make a true statement. Be as specific as possible.

a. To find the surface area of a cube, add the areas of ____ congruent squares.

b. To find the surface area of a cylinder, add the areas of two congruent

_________________ and one _________________.

c. To find the surface area of a dodecahedron, add the areas of 12 congruent

_________________ _________________.

d. To find the surface area of an icosahedron, add the areas of ____ congruent

_________________ triangles.

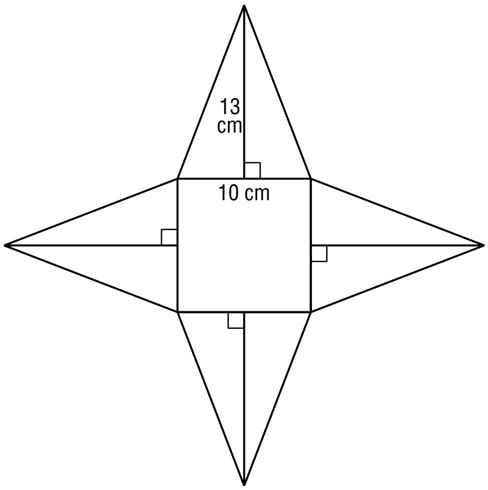
Helping You Remember

3. A good way to remember a mathematical concept is to think about how the concept relates to everyday life. How could you explain the concept of *surface area* using an everyday situation?

12-2 Enrichment

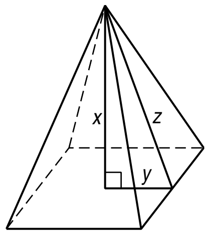
Pyramids

1. On a sheet of paper, draw the figure at the right so that the measurements are accurate. Cut out the shape and fold it to make a pyramid.



2. Measure the height from the highest point straight down (perpendicular) to the base. What is the height of the pyramid?

The drawing at the right shows the pyramid you made. Answer each of the following.



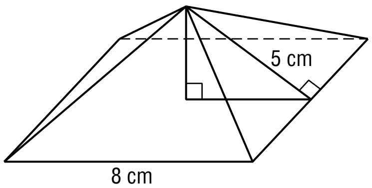
3. Measure the length of y in your pyramid.

4. Measure the length of z, the height of one of the triangular faces.

5. Use the Pythagorean Theorem to find x, the height of the pyramid. Is this equal to the height of your model?

Make a paper pattern for each pyramid below. Cut out your pattern and fold it to make a model. Measure the height of the pyramid and use the Pythagorean Theorem to verify this measure.

6.



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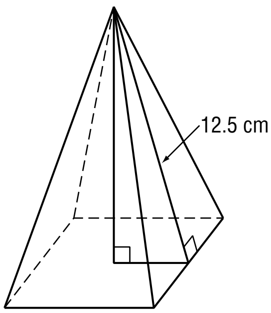
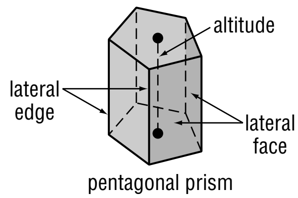


12-3 Study Guide and Intervention

Surface Areas of Prisms

Lateral Areas of Prisms Here are some characteristics of prisms.



- The bases are parallel and congruent.
- The **lateral faces** are the faces that are not bases.
- The lateral faces intersect at **lateral edges**, which are parallel.
- The **altitude** of a prism is a segment that is perpendicular to the bases with an endpoint in each base.
- For a **right prism**, the lateral edges are perpendicular to the bases. Otherwise, the prism is **oblique**.

Lateral Area of a Prism	If a prism has a lateral area of L square units, a height of h units, and each base has a perimeter of P units, then $L = Ph$.

Example **Find the lateral area of the regular pentagonal prism above if each base has a perimeter of 75 centimeters and the altitude is 10 centimeters.**

$L = Ph$ Lateral area of a prism

$= 75(10)$ $P = 75$, $h = 10$

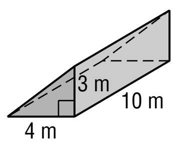
$= 750$ Multiply.

The lateral area is 750 square centimeters.

Exercises

Find the lateral area of each prism.

1.



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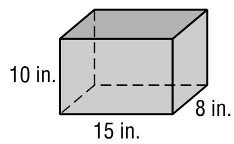

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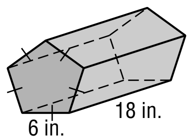

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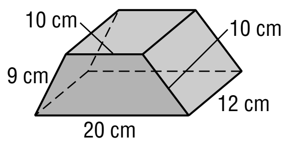

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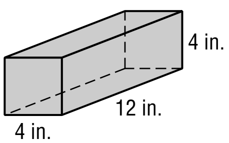

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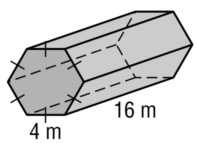
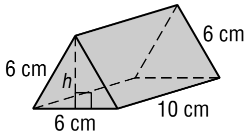


Lesson 12-3

12-3 Study Guide and Intervention *(continued)*

Surface Areas of Prisms

Surface Areas of Prisms The surface area of a prism is the lateral area of the prism plus the areas of the bases.



Surface Area of a Prism	If the total surface area of a prism is T square units, its height is h units, and each base has an area of B square units and a perimeter of P units, then $T = L + 2B$.

Example **Find the surface area of the triangular prism above.**

Find the lateral area of the prism.

$L = Ph$ Lateral area of a prism
$= (18)(10)$ $P = 18$, $h = 10$
$= 180 \text{ cm}^2$ Multiply.

Find the area of each base. Use the Pythagorean Theorem to find the height of the triangular base.

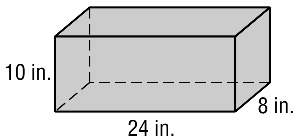
$h^2 + 3^2 = 6^2$ Pythagorean Theorem
$h^2 = 27$ Simplify.
$h = 3\sqrt{3}$ Take the square root of each side.
$B = \frac{1}{2} \times \text{base} \times \text{height}$ Area of a triangle
$= \frac{1}{2}(6)(3\sqrt{3})$ or 15.6 cm^2

The total area is the lateral area plus the area of the two bases.

$T = 180 + 2(15.6)$ Substitution
$= 211.2 \text{ cm}^2$ Simplify.

Exercises

Find the surface area of each prism. Round to the nearest tenth if necessary.

1.


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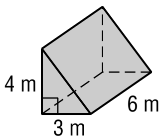

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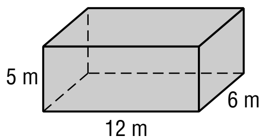

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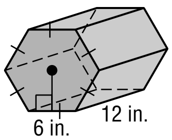

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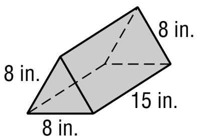

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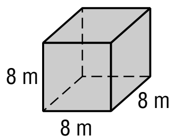
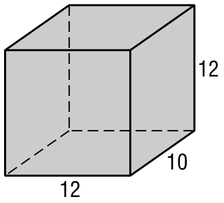


12-3 Skills Practice

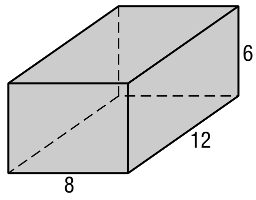
Surface Areas of Prisms

Find the lateral area of each prism.

1.



2.



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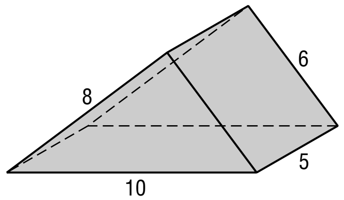

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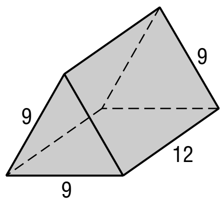
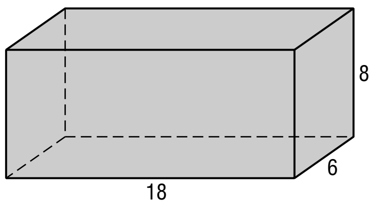


Find the surface area of each prism. Round to the nearest tenth if necessary.

5.



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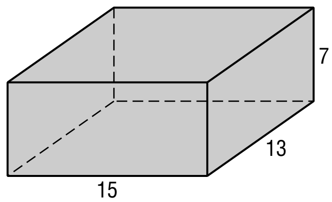

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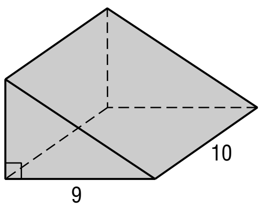

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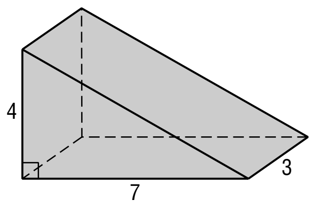


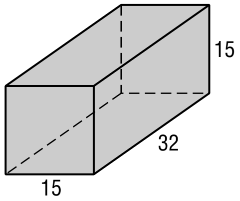
NAME ____ DATE ____________ PERIOD _____

12-3 Practice

Surface Areas of Prisms

Find the lateral area of each prism. Round to the nearest tenth if necessary.

1.



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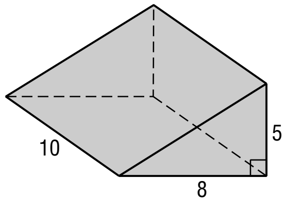

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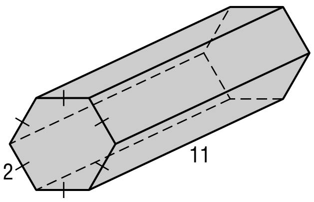

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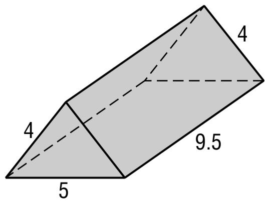
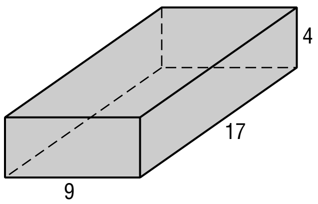


Find the surface area of each prism. Round to the nearest tenth if necessary.

5.



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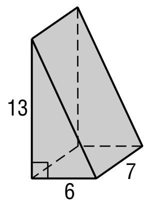

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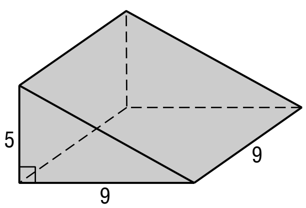

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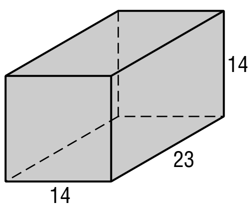


9. CRAFTS Becca made a rectangular jewelry box in her art class and plans to cover it in red silk. If the jewelry box is $6\frac{1}{2}$ inches long, $4\ \frac{1}{2}$ inches wide, and 3 inches high, find the surface area that will be covered.

NAME ____ DATE ____________ PERIOD _____

12-3 Reading to Learn Mathematics

Surface Areas of Prisms

Pre-Activity **How do brick masons know how many bricks to order for a project?**

Read the introduction to Lesson 12-3 at the top of page 649 in your textbook.

How could the brick mason figure out approximately how many bricks are needed for the garage?

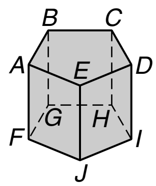
Reading the Lesson

1. Determine whether each sentence is *always*, *sometimes*, or *never* true.

a. A base of a prism is a face of the prism.
b. A face of a prism is a base of the prism.
c. The lateral faces of a prism are rectangles.
d. If a base of a prism has n vertices, then the prism has n faces.
e. If a base of a prism has n vertices, then the prism has n lateral edges.
f. In a right prism, the lateral edges are also altitudes.
g. The bases of a prism are congruent regular polygons.
h. Any two lateral edges of a prism are perpendicular to each other.
i. In a rectangular prism, any pair of opposite faces can be called the bases.
j. All of the lateral faces of a prism are congruent to each other.

2. Explain the difference between the *lateral area* of a prism and the *surface area* of a prism. Your explanation should apply to both right and oblique prisms. Do not use any formulas in your explanation.

3. Refer to the figure.



a. Name this solid with as specific a name as possible.

b. Name the bases of the solid.
c. Name the lateral faces.

d. Name the edges.
e. Name an altitude of the solid.
f. If a represents the area, P represents the perimeter of one of the bases, and $x = AF$, write an expression for the surface area of the solid that involves a, P, and x.

Helping You Remember

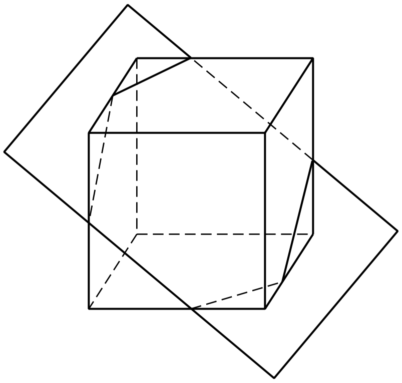
4. A good way to remember a new mathematical term is to relate it to an everyday use of the same word. How can the way the word *lateral* is used in sports help you remember the meaning of the *lateral area* of a solid?

Lesson 12-3

12-3 Enrichment

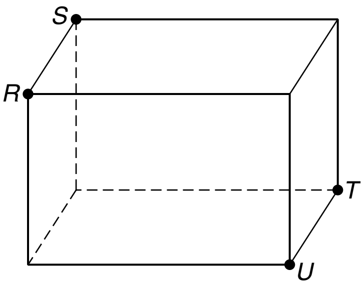
Cross Sections of Prisms

When a plane intersects a solid figure to form a two-dimensional figure, the results is called a **cross section**. The figure at the right shows a plane intersecting a cube. The cross section is a hexagon.



For each right prism, connect the labeled points in alphabetical order to show a cross section. Then identify the polygon.

1.



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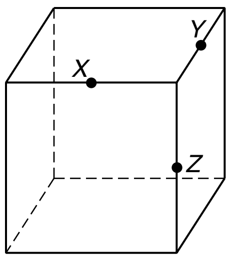

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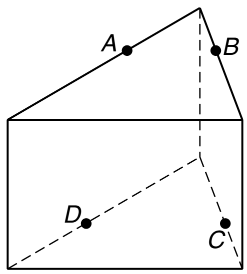
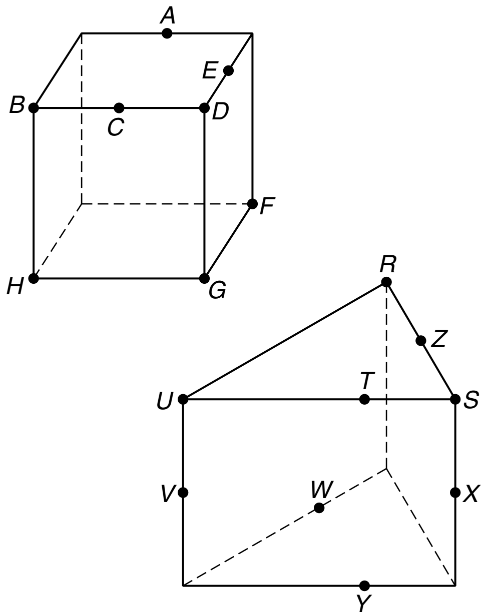


Refer to the right prisms shown at the right. In the rectangular prism, *A* and *C* are midpoints. Identify the cross-section polygon formed by a plane containing the given points.



4. A, C, H

5. C, E, G

6. H, C, E, F

7. H, A, E

8. B, D, F

9. V, X, R

10. R, T, Y

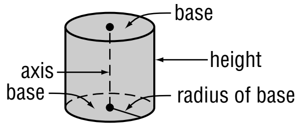
11. R, S, W

NAME ____ DATE ____________ PERIOD _____

12-4 Study Guide and Intervention

Surface Areas of Cylinders

Lateral Areas of Cylinders A **cylinder** is a solid whose bases are congruent circles that lie in parallel planes. The **axis** of a cylinder is the segment whose endpoints are the centers of these circles. For a **right cylinder**, the axis and the altitude of the cylinder are equal. The lateral area of a right cylinder is the circumference of the cylinder multiplied by the height.



Lateral Area of a Cylinder	If a cylinder has a lateral area of L square units, a height of h units, and the bases have radii of r units, then $L = 2\pi rh$.

Example **Find the lateral area of the cylinder above if the radius of the base is 6 centimeters and the height is 14 centimeters.**

$L = 2\pi rh$ Lateral area of a cylinder

$= 2\pi(6)(14)$ Substitution

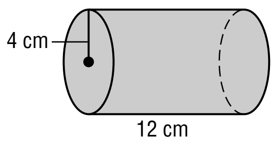
≈ 527.8 Simplify.

The lateral area is about 527.8 square centimeters.

Exercises

Find the lateral area of each cylinder. Round to the nearest tenth.

1.



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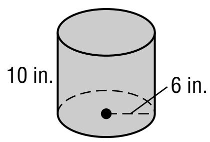

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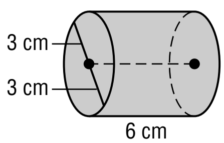

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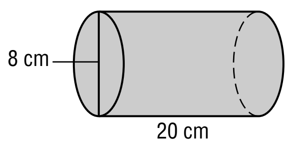

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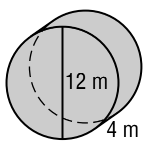

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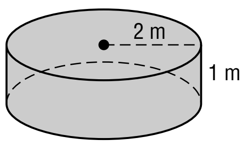
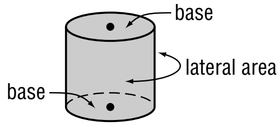


12-4 Study Guide and Intervention *(continued)*

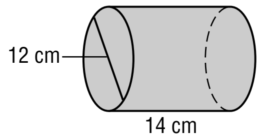
Surface Areas of Cylinders

Surface Areas of Cylinders The surface area of a cylinder is the lateral area of the cylinder plus the areas of the bases.



Surface Area of a Cylinder	If a cylinder has a surface area of T square units, a height of h units, and the bases have radii of r units, then $T = 2\pi rh + 2\pi r^2$.

Example **Find the surface area of the cylinder.**



Find the lateral area of the cylinder. If the diameter is 12 centimeters, then the radius is 6 centimeters.

$L = Ph$ Lateral area of a cylinder
$= (2\pi r)h$ $P = 2\pi r$
$= 2\pi(6)(14)$ $r = 6$, $h = 14$
≈ 527.8 Simplify.

Find the area of each base.

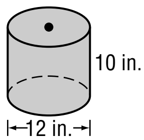
$B = \pi r^2$ Area of a circle
$= \pi(6)^2$ $r = 6$
≈ 113.1 Simplify.

The total area is the lateral area plus the area of the two bases.
$T = 527.8 + 113.1 + 113.1$ or 754 square centimeters.

Exercises

Find the surface area of each cylinder. Round to the nearest tenth.

1.



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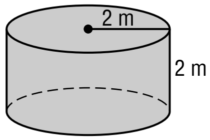

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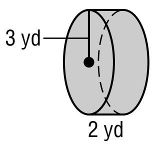

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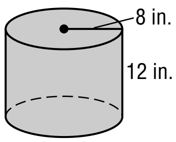

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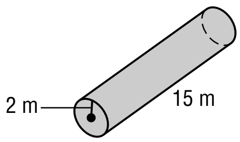

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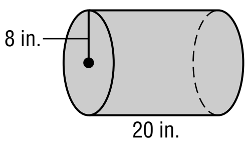


12-4 Skills Practice

Surface Areas of Cylinders

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. $r = 10$ in., $h = 12$ in.

2. $r = 8$ cm, $h = 15$ cm

3. $r = 5$ ft, $h = 20$ ft

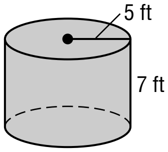
4. $d = 20$ yd, $h = 5$ yd

5. $d = 8$ m, $h = 7$ m

6. $d = 24$ mm, $h = 20$ mm

Find the surface area of each cylinder. Round to the nearest tenth.

7.



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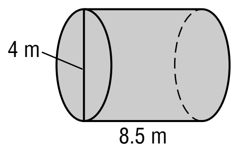


Find the radius of the base of each cylinder.

9. The surface area is 603.2 square meters, and the height is 10 meters.

10. The surface area is 100.5 square inches, and the height is 6 inches.

11. The surface area is 226.2 square centimeters, and the height is 5 centimeters.

12. The surface area is 1520.5 square yards, and the height is 14.2 yards.

Lesson 12-4

NAME ______________________________ DATE __________ PERIOD _____

12-4 Practice

Surface Areas of Cylinders

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. $r = 8$ cm, $h = 9$ cm

2. $r = 12$ in., $h = 14$ in.

3. $d = 14$ mm, $h = 32$ mm

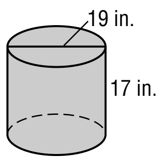
4. $d = 6$ yd, $h = 12$ yd

5. $r = 2.5$ ft, $h = 7$ ft

6. $d = 13$ m, $h = 20$ m

Find the surface area of each cylinder. Round to the nearest tenth.

7.



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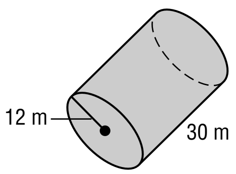


Find the radius of the base of each right cylinder.

9. The surface area is 628.3 square millimeters, and the height is 15 millimeters.

10. The surface area is 892.2 square feet, and the height is 4.2 feet.

11. The surface area is 158.3 square inches, and the height is 5.4 inches.

12. KALEIDOSCOPES Nathan built a kaleidoscope with a 20-centimeter barrel and a 5-centimeter diameter. He plans to cover the barrel with embossed paper of his own design. How many square centimeters of paper will it take to cover the barrel of the kaleidoscope?

NAME ____ DATE ____________ PERIOD _____

12-4 Reading to Learn Mathematics

Surface Areas of Cylinders

Pre-Activity **How are cylinders used in extreme sports?**

Read the introduction to Lesson 12-4 at the top of page 655 in your textbook.

Why is the surface area of a half-pipe more than half the surface area of a complete pipe?

Reading the Lesson

1. Underline the correct word or phrase to form a true statement.

a. The bases of a cylinder are (rectangles/regular polygons/circles).

b. The (axis/radius/diameter) of a cylinder is the segment whose endpoints are the centers of the bases.

c. The net of a cylinder is composed of two congruent (rectangles/circles) and one (rectangle/semicircle).

d. In a right cylinder, the axis of the cylinder is also a(n) (base/lateral edge/altitude).

e. A cylinder that is not a right cylinder is called an (acute/obtuse/oblique) cylinder.

2. Match each description from the first column with an expression from the second column that represents its value.

a. the lateral area of a right cylinder in which the radius of each base is x cm and the length of the axis is y cm

b. the surface area of a right prism with square bases in which the length of a side of a base is x cm and the length of a lateral edge is y cm

c. the surface area of a right cylinder in which the radius of a base is x cm and the height is y cm

d. the surface area of regular hexahedron (cube) in which the length of each edge is x cm

e. the lateral area of a triangular prism in which the bases are equilateral triangles with side length x cm and the height is y cm

f. the surface area of a right cylinder in which the diameter of the base is x cm and the length of the axis is y cm

i. $(2x^2 + 4xy)$ cm^2

ii. $(2\pi xy + 2\pi x^2)$ cm^2

iii. $3xy$ cm^2

iv. $6x^2$ cm^2

v. $2\pi xy$ cm^2

vi. $\left(\frac{\pi x^2}{2} + \pi xy\right)$ cm^2

Helping You Remember

3. Often the best way to remember a mathematical formula is to think about where the different parts of the formula come from. How can you use this approach to remember the formula for the surface area of a cylinder?

Lesson 12-4

NAME ____ DATE ____________ PERIOD _____

12-4 Enrichment

Can-didly Speaking

Beverage companies have mathematically determined the ideal shape for a 12-ounce soft-drink can. Why won't any shape do? Some shapes are more economical than others. In order to hold 12 ounces of soft drink, an extremely skinny can would have to be very tall. The total amount of aluminum used for such a shape would be greater than the amount used for the conventional shape, thus costing the company more to make the skinny can. The radius r chosen determines the height h needed to hold 12 ounces of liquid. This also determines the amount of aluminum needed to make the can. Companies also have to keep in mind that the top of the can is three times thicker than the bottom and sides. Why? So you won’t tear off the entire top when you open the can! The following formulas can be used to find the height and amount of aluminum a needed for a 12-ounce soft-drink can.

$$h = \frac{17.89}{\pi r^2}$$

$$a = 0.02\pi\left[\frac{r^2 + 35.78}{r}\right]$$

The values of r and h are measured in inches.

Find the height needed for a 12-ounce can for each radius. Round to the nearest tenth.

1. $2\frac{2}{3}$ in. **2.** 1 in. **3.** 3 in. **4.** $1\frac{3}{4}$ in.

Sketch the shape of each can in Exercises 1–4.

5. Exercise 1 **6.** Exercise 2 **7.** Exercise 3 **8.** Exercise 4

9. a. Measure the radius of a soft-drink can.

b. Use the formula to find the height of the can.

c. Measure the height of a soft-drink can.

d. How does this measure compare to your findings in part a?

10. Find the amount of aluminum used in making a soft-drink can.

12-5 Study Guide and Intervention

Surface Areas of Pyramids

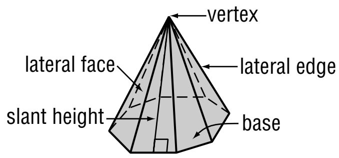
Lateral Areas of Regular Pyramids Here are some properties of pyramids.

- The base is a polygon.
- All of the faces, except the base, intersect in a common point known as the **vertex**.
- The faces that intersect at the vertex, which are called **lateral faces**, are triangles.

For a **regular pyramid**, the base is a regular polygon and the **slant height** is the height of each lateral face.

Lateral Area of a Regular Pyramid	If a regular pyramid has a lateral area of L square units, a slant height of ℓ units, and its base has a perimeter of P units, then $L = \frac{1}{2}P\ell$.

Example **The roof of a barn is a regular octagonal pyramid. The base of the pyramid has sides of 12 feet, and the slant height of the roof is 15 feet. Find the lateral area of the roof.**



The perimeter of the base is 8(12) or 96 feet.

$L = \frac{1}{2}P\ell$ Lateral area of a pyramid

$= \frac{1}{2}(96)(15)$ $P = 96$, $\ell = 15$

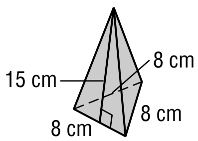
$= 720$ Multiply.

The lateral area is 720 square feet.

Exercises

Find the lateral area of each regular pyramid. Round to the nearest tenth if necessary.

1.



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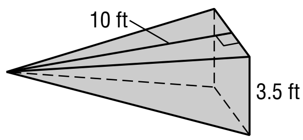

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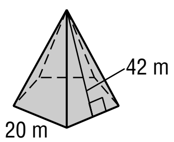

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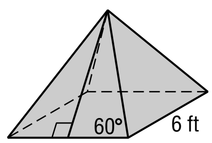

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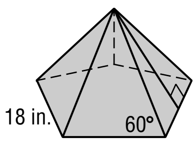

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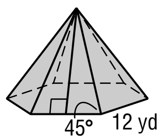
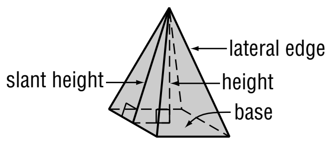


Lesson 12-5

12-5 Study Guide and Intervention *(continued)*

Surface Areas of Pyramids

Surface Areas of Regular Pyramids The surface area of a regular pyramid is the lateral area plus the area of the base.



Surface Area of a Regular Pyramid	If a regular pyramid has a surface area of T square units, a slant height of ℓ units, and its base has a perimeter of P units and an area of B square units, then $T = \frac{1}{2}P\ell + B$.

Example **For the regular square pyramid above, find the surface area to the nearest tenth if each side of the base is 12 centimeters and the height of the pyramid is 8 centimeters.**

Look at the pyramid above. The slant height is the hypotenuse of a right triangle. One leg of that triangle is the height of the pyramid, and the other leg is half the length of a side of the base. Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = 6^2 + 8^2$ Pythagorean Theorem

$= 100$ Simplify.

$\ell = 10$ Take the square root of each side.

$T = \frac{1}{2}P\ell + B$ Surface area of a pyramid

$= \frac{1}{2}(4)(12)(10) + 12^2$ $P = (4)(12)$, $\ell = 10$, $B = 12^2$

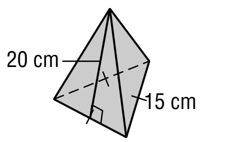
$= 384$ Simplify.

The surface area is 384 square centimeters.

Exercises

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.



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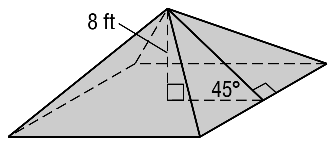

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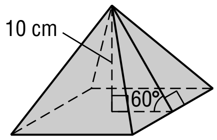

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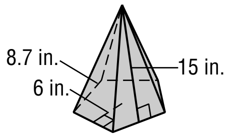

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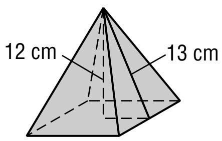

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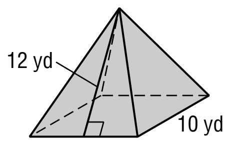


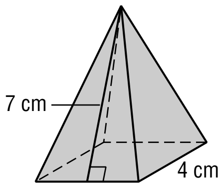
NAME ______________________________ DATE ____________ PERIOD _____

12-5 Skills Practice

Surface Area of Pyramids

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.



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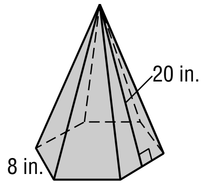

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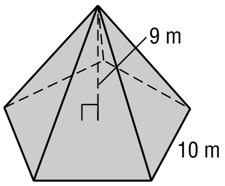

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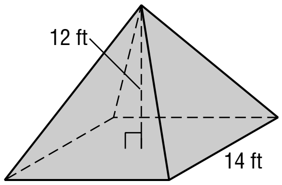

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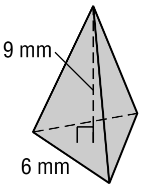

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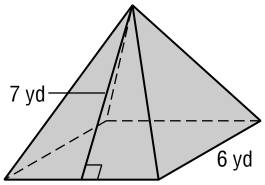

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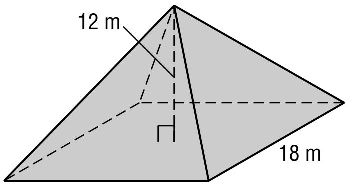

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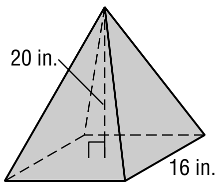


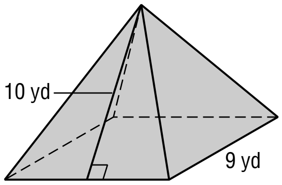
NAME ____ DATE ____________ PERIOD _____

12-5 Practice

Surface Area of Pyramids

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.



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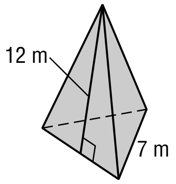

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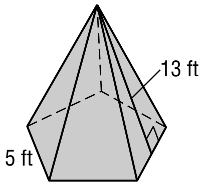

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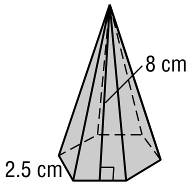

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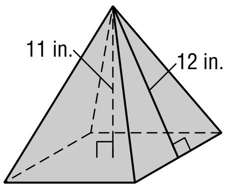

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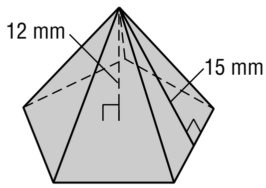

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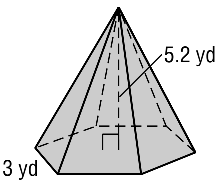

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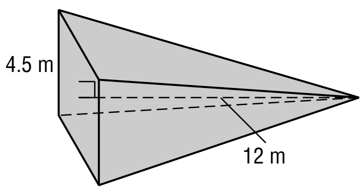

9. GAZEBOS The roof of a gazebo is a regular octagonal pyramid. If the base of the pyramid has sides of 0.5 meters and the slant height of the roof is 1.9 meters, find the area of the roof.

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12-5 Reading to Learn Mathematics

Surface Areas of Pyramids

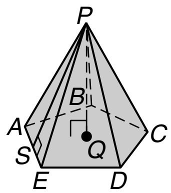
Pre-Activity **How are pyramids used in architecture?**

Read the introduction to Lesson 12-5 at the top of page 660 in your textbook.

Why do you think that the architect for the new entrance to the Louvre decided to use a pyramid rather than a rectangular prism?

Reading the Lesson

1. In the figure, *ABCDE* has congruent sides and congruent angles.



a. Describe this pyramid with as specific a name as possible.

b. Use the figure to name the base of this pyramid.

c. Describe the base of the pyramid.

d. Name the vertex of the pyramid.

e. Name the lateral faces of the pyramid.

f. Describe the lateral faces.

g. Name the lateral edges of the pyramid.

h. Name the altitude of the pyramid.

i. Write an expression for the height of the pyramid.

j. Write an expression for the slant height of the pyramid.

2. In a regular square pyramid, let s represent the side length of the base, h represent the height, a represent the apothem, and ℓ represent the slant height. Also, let L represent the lateral area and let T represent the surface area. Which of the following relationships are correct?

A. $s = 2a$ **B.** $a^2 + \ell^2 = h^2$ **C.** $L = 4\ell s$

D. $h = \sqrt{\ell^2 - a^2}$ **E.** $\left(\frac{s}{2}\right)^2 + h^2 = \ell^2$ **F.** $T = s^2 + 2\ell s$

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that one of your classmates is having trouble remembering the difference between the *height* and the *slant height* of a regular pyramid. How can you explain this concept?

Lesson 12-5

12-5 Enrichment

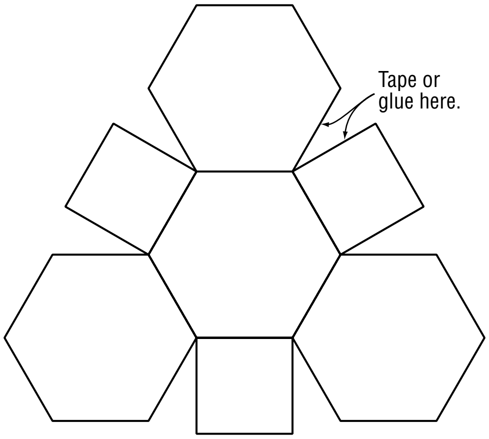
Two Truncated Solids

To create a truncated solid, you could start with an ordinary solid and then cut off the corners. Another way to make such a shape is to use the patterns on this page.

The Truncated Octahedron

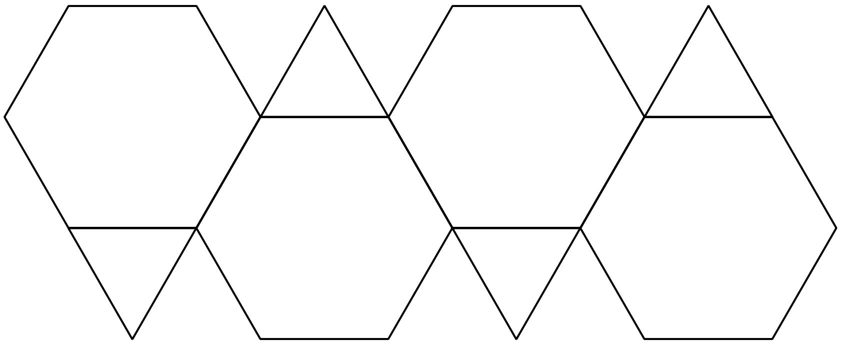
1. Two copies of the pattern at the right can be used to make a *truncated octahedron*, a solid with 6 square faces and 8 regular hexagonal faces.

Each pattern makes half of the truncated octahedron. Attach adjacent faces using glue or tape to make a cup-shaped figure.



The Truncated Tetrahedron

2. The pattern below will make a *truncated tetrahedron*, a solid with 8 polygonal faces: 4 hexagons and 4 equilateral triangles.



Solve.

3. Find the surface area of the truncated octahedron if each polygon in the pattern has sides of 3 inches.

4. Find the surface area of the truncated tetrahedron if each polygon in the pattern has sides of 3 inches.

Area Formulas for Regular Polygons (s is the length of one side)	
triangle	$A = \frac{s^2}{4}\sqrt{3}$
hexagon	$A = \frac{3s^2}{2}\sqrt{3}$
octagon	$A = 2s^2(\sqrt{2} + 1)$

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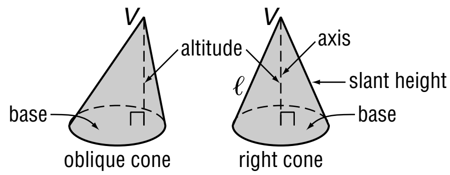
12-6 Study Guide and Intervention

Surface Areas of Cones

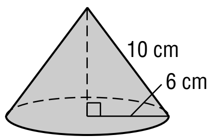
Lateral Areas of Cones Cones have the following properties.



- A cone has one circular base and one vertex.
- The segment whose endpoints are the vertex and the center of the base is the **axis** of the cone.
- The segment that has one endpoint at the vertex, is perpendicular to the base, and has its other endpoint on the base is the **altitude** of the cone.
- For a **right cone** the axis is also the altitude, and any segment from the circumference of the base to the vertex is the **slant height** ℓ. If a cone is not a right cone, it is oblique.

Lateral Area of a Cone	If a cone has a lateral area of L square units, a slant height of ℓ units, and the radius of the base is r units, then $L = \pi r\ell$.

Example **Find the lateral area of a cone with slant height of 10 centimeters and a base with a radius of 6 centimeters.**



$L = \pi r\ell$ Lateral area of a cone

$= \pi(6)(10)$ $r = 6$, $\ell = 10$

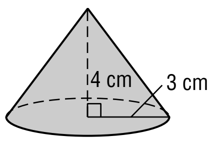
≈ 188.5 Simplify.

The lateral area is about 188.5 square centimeters.

Exercises

Find lateral area of each circular cone. Round to the nearest tenth.

1.



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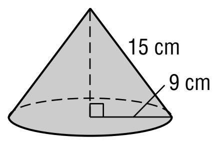

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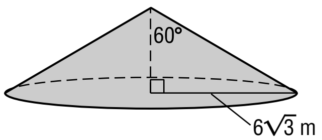

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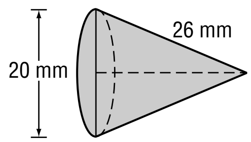

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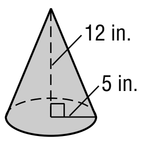

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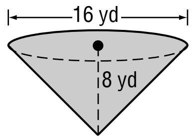

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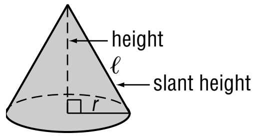
12-6 Study Guide and Intervention *(continued)*

Surface Areas of Cones

Surface Areas of Cones The surface area of a cone is the lateral area of the cone plus the area of the base.



Surface Area of a Right Cone	If a cone has a surface area of T square units, a slant height of ℓ units, and the radius of the base is r units, then $T = \pi r\ell + \pi r^2$.

Example **For the cone above, find the surface area to the nearest tenth if the radius is 6 centimeters and the height is 8 centimeters.**

The slant height is the hypotenuse of a right triangle with legs of length 6 and 8. Use the Pythagorean Theorem.

$\ell^2 = 6^2 + 8^2$ Pythagorean Theorem
$\ell^2 = 100$ Simplify.
$\ell = 10$ Take the square root of each side.

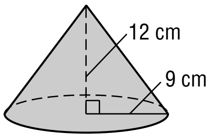
$T = \pi r\ell + \pi r^2$ Surface area of a cone
$= \pi(6)(10) + \pi \cdot 6^2$ $r = 6$, $\ell = 10$
≈ 301.6 Simplify.

The surface area is about 301.6 square centimeters.

Exercises

Find the surface area of each cone. Round to the nearest tenth.

1.



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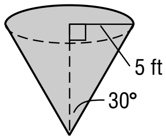

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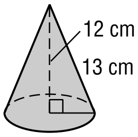

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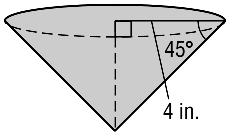

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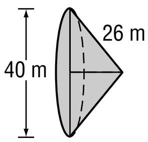

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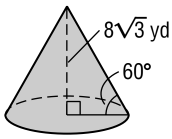

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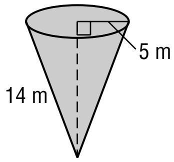
12-6 Skills Practice

Surface Areas of Cones

Lesson 12-6

Find the surface area of each cone. Round to the nearest tenth if necessary.

1.



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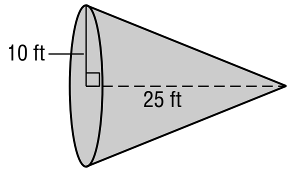

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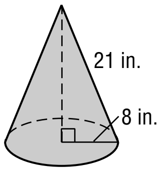

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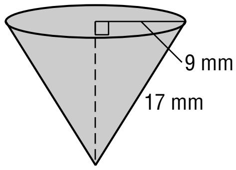

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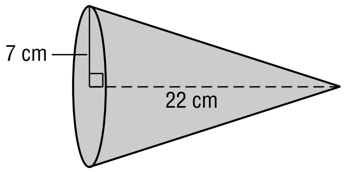

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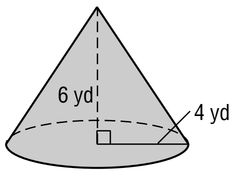


7. Find the surface area of a cone if the height is 12 inches and the slant height is 15 inches.

8. Find the surface area of a cone if the height is 9 centimeters and the slant height is 12 centimeters.

9. Find the surface area of a cone if the height is 10 meters and the slant height is 14 meters.

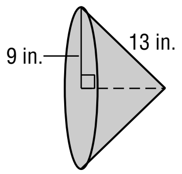
10. Find the surface area of a cone if the height is 5 feet and the slant height is 7 feet.

12-6 Practice

Surface Areas of Cones

Find the surface area of each cone. Round to the nearest tenth if necessary.

1.



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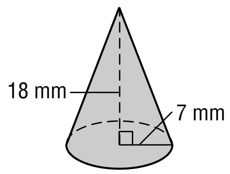

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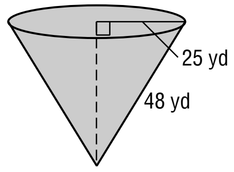

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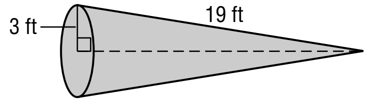

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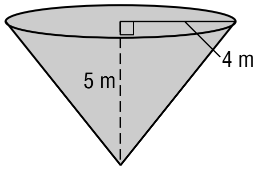

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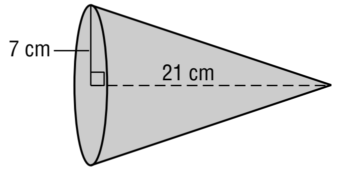

7. Find the surface area of a cone if the height is 8 feet and the slant height is 10 feet.

8. Find the surface area of a cone if the height is 14 centimeters and the slant height is 16.4 centimeters.

9. Find the surface area of a cone if the height is 12 inches and the diameter is 27 inches.

10. HATS Cuong bought a conical hat on a recent trip to central Vietnam. The basic frame of the hat is 16 hoops of bamboo that gradually diminish in size. The hat is covered in palm leaves. If the hat has a diameter of 50 centimeters and a slant height of 32 centimeters, what is the lateral area of the conical hat?

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12-6 Reading to Learn Mathematics

Surface Areas of Cones

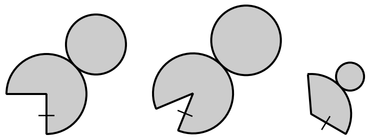
Pre-Activity **How is the lateral area of a cone used to cover tepees?**

Read the introduction to Lesson 12-6 at the top of page 666 in your textbook.

If you wanted to build a tepee of a certain size, how would it help you to know the formula for the lateral area of a cone?

Reading the Lesson

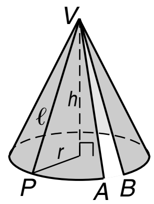
1.



a. Which net will give the cone with the greatest lateral area?

b. Which net will give the tallest cone?

2. Refer to the figure at the right. Suppose you have removed the circular base of the cone and cut from V to A so that you can unroll the lateral surface onto a flat table.



a. How can you be sure that the flattened-out piece is a sector of a circle?

b. How do you know that the flattened-out piece is not a full circle?

3. Suppose you have a right cone with radius r, diameter d, height h, and slant height ℓ. Which of the following relationships involving these lengths are correct?

A. $r = 2d$ **B.** $r + h = \ell$ **C.** $r^2 + h^2 = \ell^2$

D. $r^2 + \ell^2 = h^2$ **E.** $r = \sqrt{\ell^2 - h^2}$ **F.** $h = \pm\sqrt{\ell^2 - r^2}$

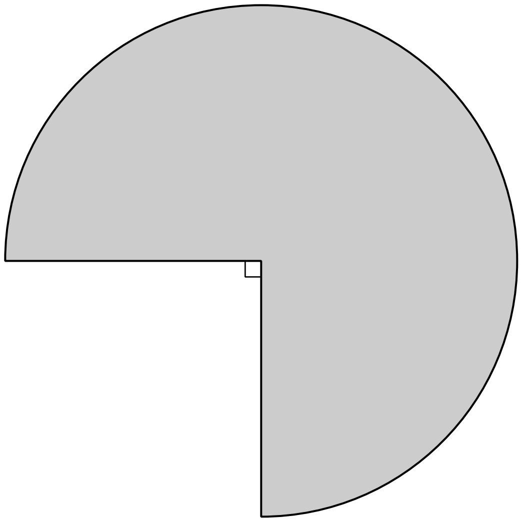
Helping You Remember

4. One way to remember a new formula is to relate it to a formula you already know. Explain how the formulas for the lateral areas of a pyramid and a cone are similar.

12-6 Enrichment

Cone Patterns

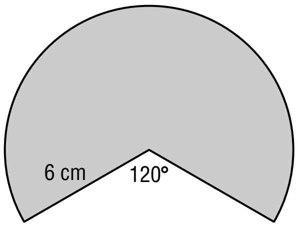
The pattern at the right is made from a circle. It can be folded to make a cone.



1. Measure the radius of the circle to the nearest centimeter.
2. The pattern is what fraction of the complete circle?
3. What is the circumference of the complete circle?
4. How long is the circular arc that is the outside of the pattern?
5. Cut out the pattern and tape it together to form a cone.
6. Measure the diameter of the circular base of the cone.
7. What is the circumference of the base of the cone?
8. What is the slant height of the cone?
9. Use the Pythagorean Theorem to calculate the height of the cone. Use a decimal approximation. Check your calculation by measuring the height with a metric ruler.
10. Find the lateral area.
11. Find the total surface area.

Make a paper pattern for each cone with the given measurements. Then cut the pattern out and make the cone. Find the measurements.

12.

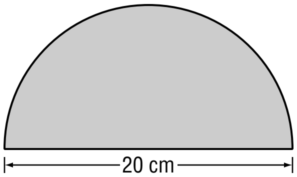


diameter of base =

lateral area =

height of cone =
(to nearest tenth of a centimeter)

13.



diameter of base =

lateral area =

height of cone =
(to nearest tenth of a centimeter)

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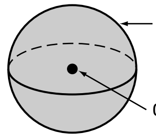
12-7 Study Guide and Intervention

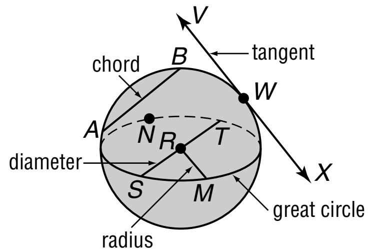
Surface Areas of Spheres

Properties of Spheres A **sphere** is the locus of all points that are a given distance from a given point called its **center**.



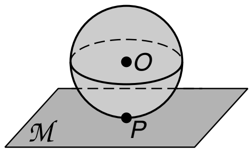
Here are some terms associated with a sphere.

- A **radius** is a segment whose endpoints are the center of the sphere and a point on the sphere.
- A **chord** is a segment whose endpoints are points on the sphere.
- A **diameter** is a chord that contains the sphere's center.
- A **tangent** is a line that intersects the sphere in exactly one point.
- A **great circle** is the intersection of a sphere and a plane that contains the center of the sphere.
- A **hemisphere** is one-half of a sphere. Each great circle of a sphere determines two hemispheres.

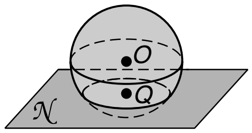


$\overline{RS}$ is a radius. $\overline{AB}$ is a chord.
$\overline{ST}$ is a diameter. $\overleftrightarrow{VX}$ is a tangent.
The circle that contains points *S*, *M*, *T*, and *N* is a great circle; it determines two hemispheres.

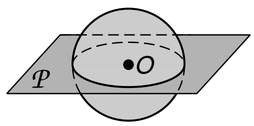
Example **Determine the shapes you get when you intersect a plane with a sphere.**



The intersection of plane ℳ and sphere *O* is point *P*.



The intersection of plane 𝒩 and sphere *O* is circle *Q*.



The intersection of plane 𝒫 and sphere *O* is circle *O*.

A plane can intersect a sphere in a point, in a circle, or in a great circle.

Exercises

Describe each object as a model of a *circle*, a *sphere*, a *hemisphere*, or *none of these*.

1. a baseball

2. a pancake

3. the Earth

4. a kettle grill cover

5. a basketball rim

6. cola can

Determine whether each statement is *true* or *false*.

7. All lines intersecting a sphere are tangent to the sphere.

8. Every plane that intersects a sphere makes a great circle.

9. The eastern hemisphere of Earth is congruent to the western hemisphere.

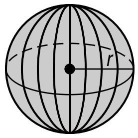
10. The diameter of a sphere is congruent to the diameter of a great circle.

Lesson 12-7

12-7 Study Guide and Intervention *(continued)*

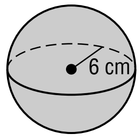
Surface Areas of Spheres

Surface Areas of Spheres You can think of the surface area of a sphere as the total area of all of the nonoverlapping strips it would take to cover the sphere. If r is the radius of the sphere, then the area of a great circle of the sphere is πr^2. The total surface area of the sphere is four times the area of a great circle.



Surface Area of a Sphere	If a sphere has a surface area of T square units and a radius of r units, then $T = 4\pi r^2$.

Example **Find the surface area of a sphere to the nearest tenth if the radius of the sphere is 6 centimeters.**



$T = 4\pi r^2$ Surface area of a sphere

$= 4\pi \cdot 6^2$ $r = 6$

≈ 452.4 Simplify.

The surface area is 452.4 square centimeters.

Exercises

Find the surface area of each sphere with the given radius or diameter to the nearest tenth.

1. $r = 8$ cm

2. $r = 2\sqrt{2}$ ft

3. $r = \pi$ cm

4. $d = 10$ in.

5. $d = 6\pi$ m

6. $d = 16$ yd

7. Find the surface area of a hemisphere with radius 12 centimeters.

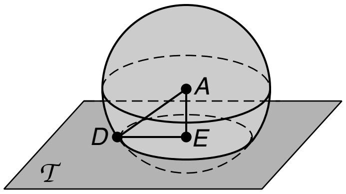
8. Find the surface area of a hemisphere with diameter π centimeters.

9. Find the radius of a sphere if the surface area of a hemisphere is 192π square centimeters.

12-7 Skills Practice

Surface Areas of Spheres

In the figure, *A* is the center of the sphere, and plane $\mathcal{T}$ intersects the sphere in circle *E*. Round to the nearest tenth if necessary.



1. If $AE = 5$ and $DE = 12$, find AD.

2. If $AE = 7$ and $DE = 15$, find AD.

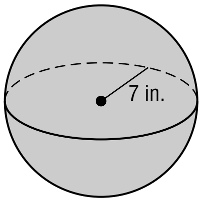
3. If the radius of the sphere is 18 units and the radius of $\odot E$ is 17 units, find AE.

4. If the radius of the sphere is 10 units and the radius of $\odot E$ is 9 units, find AE.

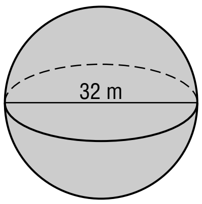
5. If M is a point on $\odot E$ and $AD = 23$, find AM.

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

6.



7.



8. a hemisphere with a radius of the great circle 8 yards

9. a hemisphere with a radius of the great circle 2.5 millimeters

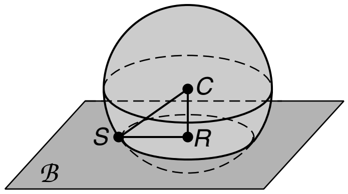
10. a sphere with the area of a great circle 28.6 inches

NAME ____ DATE ____________ PERIOD _____

12-7 Practice

Surface Areas of Spheres

In the figure, C is the center of the sphere, and plane $\mathcal{B}$ intersects the sphere in circle R. Round to the nearest tenth if necessary.



1. If $CR = 4$ and $SR = 14$, find CS.

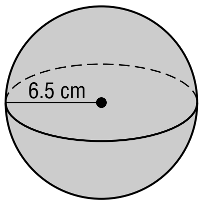
2. If $CR = 7$ and $SR = 24$, find CS.

3. If the radius of the sphere is 28 units and the radius of $\odot R$ is 26 units, find CR.

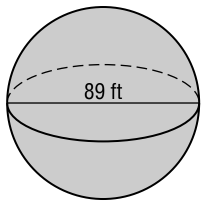
4. If J is a point on $\odot R$ and $CS = 7.3$, find CJ.

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

5.



6.



7. a sphere with the area of a great circle 29.8 meters

8. a hemisphere with a radius of the great circle 8.4 inches

9. a hemisphere with the circumference of a great circle 18 millimeters

10. SPORTS A standard size 5 soccer ball for ages 13 and older has a circumference of 27–28 inches. Suppose Breck is on a team that plays with a 28-inch soccer ball. Find the surface area of the ball.

12-7 Reading to Learn Mathematics

Surface Areas of Spheres

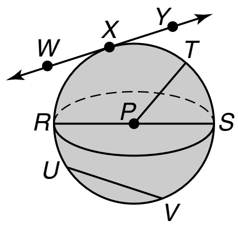
Pre-Activity **How do manufacturers of sports equipment use the surface areas of spheres?**

Read the introduction to Lesson 12-7 at the top of page 671 in your textbook.

How would knowing the formula for the surface area of a sphere help manufacturers of beach balls?

Reading the Lesson

1. In the figure, P is the center of the sphere. Name each of the following in the figure.



a. three radii of the sphere

b. a diameter of the sphere

c. two chords of the sphere

d. a great circle of the sphere

e. a tangent to the sphere

f. the point of tangency

2. Determine whether each sentence is *sometimes*, *always*, or *never* true.

a. If a sphere and a plane intersect in more than one point, their intersection will be a great circle.

b. A great circle has the same center as the sphere.

c. The endpoints of a radius of a sphere are two points on the sphere.

d. A chord of a sphere is a diameter of the sphere.

e. A radius of a great circle is also a radius of the sphere.

3. Match each surface area formula with the name of the appropriate solid.

a. $T = \pi r\ell + \pi r^2$	**i.** regular pyramid
b. $T = Ph + 2B$	**ii.** hemisphere
c. $T = 4\pi r^2$	**iii.** cylinder
d. $T = \frac{1}{2}P\ell + B$	**iv.** prism
e. $T = 2\pi rh + 2\pi r^2$	**v.** sphere
f. $T = 3\pi r^2$	**vi.** cone

Helping You Remember

4. Many students have trouble remembering all of the formulas they have learned in this chapter. What is an easy way to remember the formula for the surface area of a sphere?

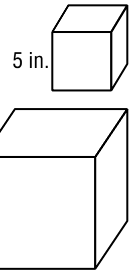
NAME ____ DATE ____________ PERIOD _____

12-7 Enrichment

Doubling Sizes

Consider what happens to surface area when the sides of a figure are doubled.

The sides of the large cube are twice the size of the sides of the small cube.



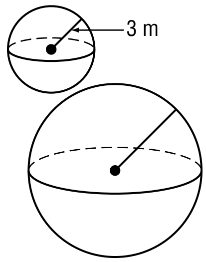
1. How long are the edges of the large cube?

2. What is the surface area of the small cube?

3. What is the surface area of the large cube?

4. The surface area of the large cube is how many times greater than that of the small cube?

The radius of the large sphere at the right is twice the radius of the small sphere.



5. What is the surface area of the small sphere?

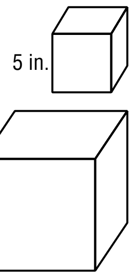
6. What is the surface area of the large sphere?

7. The surface area of the large sphere is how many times greater than the surface area of the small sphere?

8. It appears that if the dimensions of a solid are doubled, the surface area is multiplied by ______.

Now consider how doubling the dimensions affects the volume of a cube.

The sides of the large cube are twice the size of the small cube.



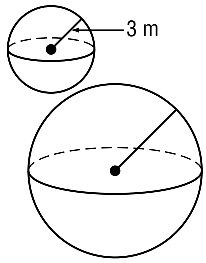
9. How long are the edges of the large cube?

10. What is the volume of the small cube?

11. What is the volume of the large cube?

12. The volume of the large cube is how many times greater than that of the small cube?

The large sphere at the right has twice the radius of the small sphere.



13. What is the volume of the small sphere?

14. What is the volume of the large sphere?

15. The volume of the large sphere is how many times greater than the volume of the small sphere?

16. It appears that if the dimensions of a solid are doubled, the volume is multiplied by ______.

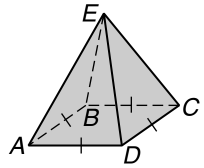
12 Chapter 12 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Which of these is part of an orthogonal drawing? **1.** ______
A. a perspective view **B.** a corner view
C. a two-dimensional top view **D.** a three-dimensional view

For Questions 2–4, use the figure.

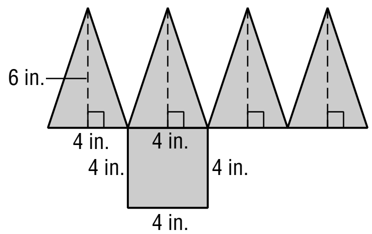


2. Identify this solid figure. **2.** ______
A. square pyramid **B.** square prism
C. triangular pyramid **D.** triangular prism

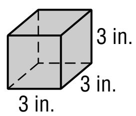
3. Name the base. **3.** ______
A. $\triangle ABE$ **B.** $\square ABCD$ **C.** $\triangle CDE$ **D.** E

4. How many edges does this figure have? **4.** ______
A. 3 **B.** 4 **C.** 6 **D.** 8

5. This net could be folded into a __?__. **5.** ______
A. tetrahedron
B. square pyramid
C. square prism
D. triangular prism



6. Find the surface area of this solid. **6.** ______
A. 9 in^2 **B.** 27 in^2
C. 36 in^2 **D.** 54 in^2



7. The areas of how many faces of a rectangular prism would be included in the lateral area? **7.** ______
A. 2 **B.** 4 **C.** 6 **D.** 8

8. Find the surface area of a rectangular prism with a length of 8 inches, a width of 5 inches, and a height of 2 inches. **8.** ______
A. 15 in^2 **B.** 66 in^2 **C.** 80 in^2 **D.** 132 in^2

9. The area of each face of a cube is 60 square centimeters. Find the surface area of the cube. **9.** ______
A. 120 cm^2 **B.** 240 cm^2 **C.** 360 cm^2 **D.** 3600 cm^2

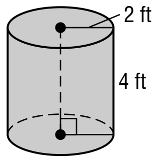
10. The lateral area of a right cylinder with a radius of 10 feet is 320π square feet. Find the surface area of the cylinder. **10.** ______
A. 220π ft^2 **B.** 360π ft^2 **C.** 420π ft^2 **D.** 520π ft^2

Assessments

NAME ______________________ DATE __________ PERIOD ______

12 Chapter 12 Test, Form 1 *(continued)*

For Questions 11 and 12, use the figure.



11. Find the lateral area to the nearest tenth. **11.** ______

A. 75.4 ft^2 **B.** 62.8 ft^2
C. 50.3 ft^2 **D.** 25.1 ft^2

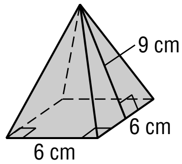
12. Find the surface area to the nearest tenth. **12.** ______

A. 75.4 ft^2 **B.** 62.8 ft^2 **C.** 50.3 ft^2 **D.** 25.1 ft^2

13. The lateral area of a regular pyramid is 300 square units. The perimeter of its base is 100 units. Find the slant height of the pyramid. **13.** ______

A. 3 units **B.** 6 units **C.** 12 units **D.** 30 units

For Questions 14 and 15, use the figure.



14. Find the lateral area. **14.** ______

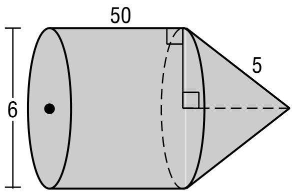
A. 108 cm^2 **B.** 144 cm^2
C. 162 cm^2 **D.** 324 cm^2

15. Find the surface area. **15.** ______

A. 108 cm^2 **B.** 144 cm^2 **C.** 162 cm^2 **D.** 324 cm^2

16. Find the surface area to the nearest tenth. **16.** ______

A. 546.6 $units^2$ **B.** 989.6 $units^2$
C. 1017.9 $units^2$ **D.** 1046.2 $units^2$



17. The radius of a right circular cone is 6 inches and the height is 8 inches. Find the slant height. **17.** ______

A. 2 in. **B.** 4 in. **C.** 10 in. **D.** 14 in.

18. A cone has a radius 17 inches long and slant height 20 inches long. Find the surface area to the nearest tenth. **18.** ______

A. 18,158.4 in^2 **B.** 1976.1 in^2 **C.** 1068.1 in^2 **D.** 340 in^2

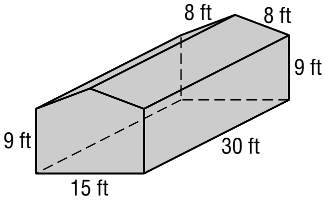
19. Which could be the intersection of a sphere and a plane? **19.** ______

A. line **B.** square **C.** oval **D.** point

20. A sphere has a diameter 42 centimeters long. Find the surface area to the nearest tenth. **20.** ______

A. 5541.8 cm^2 **B.** 2770.9 cm^2 **C.** 2167.1 cm^2 **D.** 527.8 cm^2

Bonus Find the amount of glass needed to cover the sides of the greenhouse shown. The bottom, front, and back are not glass. **B:** ______________



NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Test, Form 2A

SCORE ______

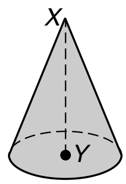
Write the letter for the correct answer in the blank at the right of each question.

1. What do the dark segments represent in an orthogonal drawing? **1.** ______

A. a change in color **B.** where paper should be folded

C. a design on the surface **D.** a break in the surface

For Questions 2 and 3, use the figure.



2. Identify the figure. **2.** ______

A. pyramid **B.** prism

C. cone **D.** cylinder

3. Name the base. **3.** ______

A. X **B.** Y **C.** $\overline{XY}$ **D.** $\odot Y$

4. What name is given to a prism having five faces? **4.** ______

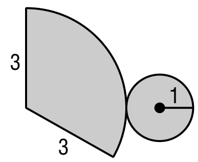
A. pentagonal prism **B.** square prism

C. triangular prism **D.** none of these

5. This net could be folded into a ___?___. **5.** ______

A. cone **B.** cylinder

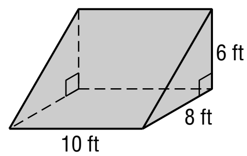
C. sphere **D.** triangular prism



6. Find the surface area of the solid. **6.** ______

A. 188 ft^2 **B.** 240 ft^2

C. 288 ft^2 **D.** 480 ft^2



7. The lateral area of a cube is 36 square inches. How long is each edge? **7.** ______

A. $\sqrt{6}$ in. **B.** 3 in. **C.** 6 in. **D.** 9 in.

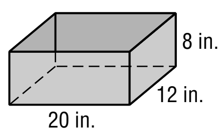
8. The lateral area of a prism is 56 square inches and the area of each base is 17 square inches. Find the surface area of the prism. **8.** ______

A. 952 in^2 **B.** 90 in^2 **C.** 73 in^2 **D.** 22 in^2

9. Find the surface area of the outside of the open box. **9.** ______

A. 1920 in^2 **B.** 998 in^2

C. 752 in^2 **D.** 400 in^2



10. The surface area of a right cylinder is 200π square centimeters and the radius is 4 centimeters. Find the height. **10.** ______

A. 42 cm **B.** 25 cm **C.** 23 cm **D.** 21 cm

12 Chapter 12 Test, Form 2A *(continued)*

For Questions 11 and 12, use a right cylinder with a radius of 3 inches and a height of 17 inches. Round to the nearest tenth.

11. Find the lateral area. **11.** ______

A. 320.4 in^2 **B.** 348.7 in^2 **C.** 377.0 in^2 **D.** 537.2 in^2

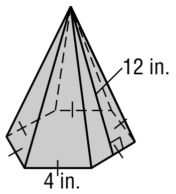
12. Find the surface area. **12.** ______

A. 320.4 in^2 **B.** 348.7 in^2 **C.** 377.0 in^2 **D.** 537.2 in^2

13. The lateral area of a regular pentagonal pyramid is 75 square inches and the slant height is 10 inches. Find the length of each side of the base. **13.** ______

A. 15 in. **B.** 14 in. **C.** 7.5 in. **D.** 3 in.

For Questions 14 and 15, use the figure.



14. Find the lateral area. **14.** ______

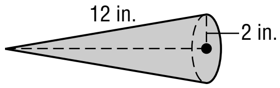
A. 144 cm^2 **B.** $144 + 24\sqrt{3}$ cm^2

C. 196 cm^2 **D.** 288 cm^2

15. Find the surface area. **15.** ______

A. 144 cm^2 **B.** $144 + 24\sqrt{3}$ cm^2 **C.** 196 cm^2 **D.** 288 cm^2

For Questions 16 and 17, use the figure. Round to the nearest tenth.



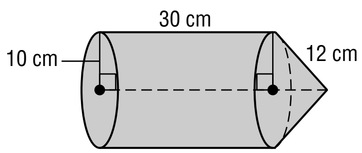
16. Find the lateral area. **16.** ______

A. 44.0 in^2 **B.** 75.4 in^2 **C.** 88.0 in^2 **D.** 100.5 in^2

17. Find the surface area. **17.** ______

A. 44.0 in^2 **B.** 75.4 in^2 **C.** 88.0 in^2 **D.** 100.5 in^2

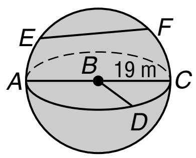
18. Find the surface area of this model rocket to the nearest tenth. **18.** ______



A. 2890.3 cm^2 **B.** 2576.1 cm^2

C. 2513.3 cm^2 **D.** 2261.9 cm^2

For Questions 19 and 20, use the figure.



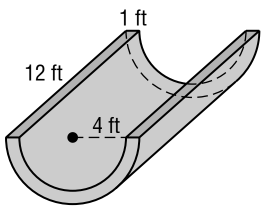
19. Identify a chord. **19.** ______

A. $\overline{EF}$ **B.** $\odot B$ **C.** $\overline{BD}$ **D.** $\widehat{AD}$

20. Find the surface area to the nearest tenth. **20.** ______

A. 4536.5 m^2 **B.** 2268.2 m^2 **C.** 477.5 m^2 **D.** 238.8 m^2

Bonus Find the surface area of the figure to the nearest tenth. **B:** ________________



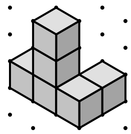
NAME ______________________ DATE ____________ PERIOD ______

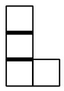
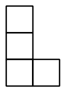
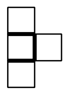
12 Chapter 12 Test, Form 2B

SCORE ______

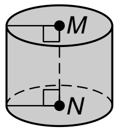
Write the letter for the correct answer in the blank at the right of each question.

1. Given the corner view of a figure, which is the top view? **1.** ______



A.  **B.**  **C.**  **D.** 

For Questions 2 and 3, use the figure.



2. Identify the figure. **2.** ______

A. pyramid **B.** prism
C. cone **D.** cylinder

3. Name a base. **3.** ______

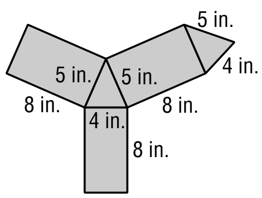
A. $\odot M$ **B.** N **C.** $\overline{MN}$ **D.** M

4. What name is given to a pyramid having seven faces? **4.** ______

A. heptagonal pyramid **B.** hexagonal pyramid
C. pentagonal pyramid **D.** octagonal pyramid

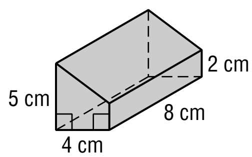
5. This net could be folded into a __?__. **5.** ______

A. rectangular pyramid
B. rectangular prism
C. triangular pyramid
D. triangular prism



6. Find the surface area of the solid. **6.** ______

A. 88 cm^2 **B.** 102 cm^2
C. 156 cm^2 **D.** 160 cm^2



7. Find the lateral area of an equilateral triangular prism if the area of each lateral face is 10 square centimeters. **7.** ______

A. $10\sqrt{3}$ cm^2 **B.** 30 cm^2 **C.** 50 cm^2 **D.** 100 cm^2

8. The surface area of a cube is 96 square inches. Find the length of an edge. **8.** ______

A. $\sqrt{24}$ in. **B.** 4 in. **C.** 8 in. **D.** 16 in.

9. The surface area of a rectangular prism is 190 square inches, the length is 10 inches, and the width 3 inches. Find the height. **9.** ______

A. 30 in. **B.** 20 in. **C.** 10 in. **D.** 5 in.

10. A right cylinder has a radius of 2 feet and a height of 5 feet. Find its surface area. **10.** ______

A. 20π ft^2 **B.** 28π ft^2 **C.** 36π ft^2 **D.** 40π ft^2

NAME ______________________________ DATE ____________ PERIOD ______

12 Chapter 12 Test, Form 2B *(continued)*

For Questions 11 and 12, use a right cylinder with a radius of 5 centimeters and a height of 22 centimeters. Round to the nearest tenth.

11. Find the lateral area. **11.** ______

A. 848.2 cm^2 **B.** 769.7 cm^2 **C.** 691.2 cm^2 **D.** 345.6 cm^2

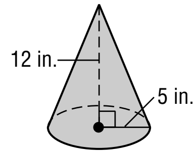
12. Find the surface area. **12.** ______

A. 848.2 cm^2 **B.** 769.7 cm^2 **C.** 691.2 cm^2 **D.** 345.6 cm^2

13. Find the lateral area of the conical hat to the nearest tenth. **13.** ______

A. 942.5 in^2 **B.** 408.4 in^2

C. 204.2 in^2 **D.** 188.5 in^2



For Questions 14 and 15, use a tetrahedron that has edges of length 12 centimeters.

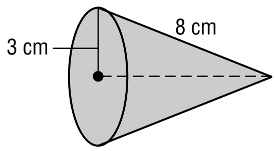
14. Find the lateral area. **14.** ______

A. $48\sqrt{3} \text{ cm}^2$ **B.** $96\sqrt{3} \text{ cm}^2$ **C.** $108\sqrt{3} \text{ cm}^2$ **D.** $144\sqrt{3} \text{ cm}^2$

15. Find the surface area. **15.** ______

A. $48\sqrt{3} \text{ cm}^2$ **B.** $96\sqrt{3} \text{ cm}^2$ **C.** $108\sqrt{3} \text{ cm}^2$ **D.** $144\sqrt{3} \text{ cm}^2$

For Questions 16 and 17, use the figure. Round to the nearest tenth.



16. Find the lateral area. **16.** ______

A. 75.4 cm^2 **B.** 103.7 cm^2

C. 131.9 cm^2 **D.** 150.8 cm^2

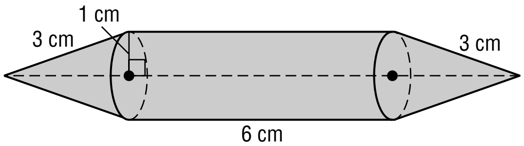
17. Find the surface area. **17.** ______

A. 75.4 cm^2 **B.** 103.7 cm^2 **C.** 131.9 cm^2 **D.** 150.8 cm^2

18. Find the surface area of the solid to the nearest tenth. **18.** ______

A. 62.8 cm^2 **B.** 56.5 cm^2

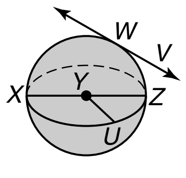
C. 47.1 cm^2 **D.** 37.7 cm^2



19. Name a tangent to the sphere. **19.** ______

A. $\overline{YU}$ **B.** $\overline{XZ}$

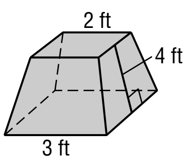
C. $\widehat{UZ}$ **D.** $\overleftrightarrow{WV}$



20. The surface area of a sphere is 64π square centimeters. Find the radius. **20.** ______

A. 16 cm **B.** 8 cm **C.** 4 cm **D.** 2 cm

Bonus Find the surface area of the frustum of a square pyramid. **B:** ________________

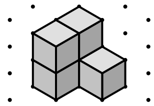


NAME ______________________ DATE __________ PERIOD ______

12 Chapter 12 Test, Form 2C

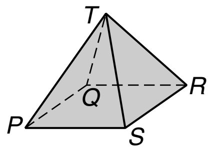
SCORE ______

1. Given the corner view of a figure, sketch the front view.



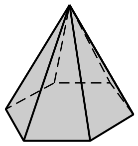
1. ______________

2. Name the faces of the solid.



2. ______________

3. Identify the solid.

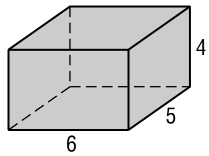


3. ______________

4. How many faces does a dodecahedron have?

4. ______________

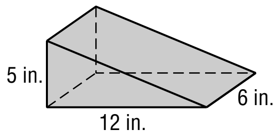
5. Draw a net for the solid.



5. ______________

Assessments

6. Find the surface area of the solid.

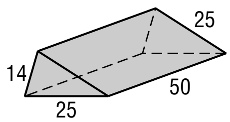


6. ______________

7. Find the lateral area of a triangular prism with a height of 8 centimeters, and with bases having sides that measure 4 centimeters, 5 centimeters, and 6 centimeters.

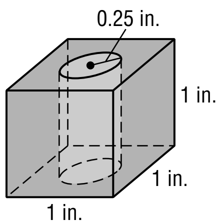
7. ______________

8. Find the surface area of the prism.



8. ______________

9. Find the surface area of the solid to the nearest tenth.



9. ______________

10. A gallon of paint costs $12.99 and covers 400 square feet. How many gallons are needed to paint two coats on the walls and ceiling (not the floor) of a rectangular room that is 30 feet long, 18 feet wide, and 8 feet high? Round to the next whole gallon.

10. ______________

NAME ________________________ DATE ____________ PERIOD ______

12 Chapter 12 Test, Form 2C *(continued)*

11. Find the lateral area of a right cylinder with a diameter of 8.6 yards and a height of 19.4 yards. Round to the nearest tenth. **11.** __________

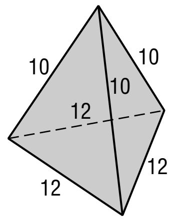
12. The surface area of a cylinder is 180π square inches and the height is 9 inches. Find the radius. **12.** __________

For Questions 13 and 14, use a regular hexagonal pyramid with base edges of 10 inches and a slant height of 9 inches.

13. Find the lateral area. **13.** __________

14. Find the surface area. **14.** __________

15. Find the lateral area of the triangular pyramid. **15.** __________



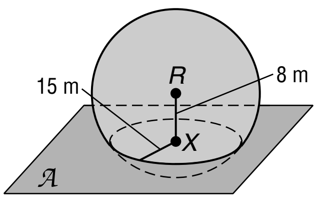
16. The surface area of a regular pyramid is 276 square centimeters, the slant height measures 8 centimeters, and the area of the base is 50 square centimeters. Find the perimeter of the base. **16.** __________

For Questions 17 and 18, use a right circular cone with a radius of 4 feet and a height of 3 feet. Round to the nearest tenth.

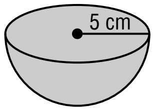
17. Find the lateral area. **17.** __________

18. Find the surface area. **18.** __________

19. Plane $\mathcal{A}$ intersects sphere R in $\odot X$. Find the radius of the sphere. **19.** __________



20. Find the surface area of this hemisphere to the nearest tenth. **20.** __________



Bonus The surface area of the exterior of a hollow rubber ball is 16π square inches. The rubber is $\frac{1}{4}$ inch thick. Find the surface area of the interior of the ball. **B:** __________

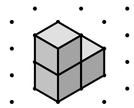
NAME ______________________ DATE __________ PERIOD ______

Chapter 12 Test, Form 2D

SCORE ______

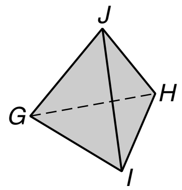
12

1. Given the corner view of a figure, sketch the back view.



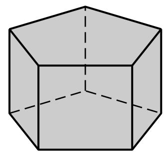
1. ____________

2. Name the edges of the solid.



2. ____________

3. Identify the solid.

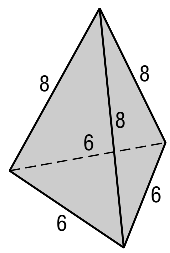


3. ____________

4. How many edges does a cube have?

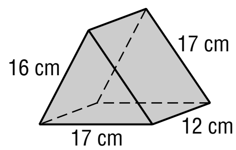
4. ____________

5. Draw a net for the solid.



5. ____________

6. Find the surface area of the solid.

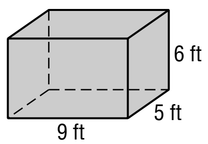


6. ____________

7. Find the lateral area of a regular pentagonal prism if the perimeter of the base is 50 inches and the height is 15 inches.

7. ____________

8. Find the surface area of the prism.

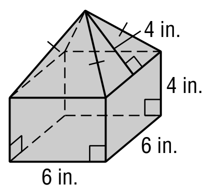


8. ____________

9. A gallon of paint costs $18 and covers 300 square feet. How many gallons are needed to paint the walls and bottom of a rectangular swimming pool whose length is 25 feet, width is 16 feet, and depth is 4 feet? Round to the next whole gallon.

9. ____________

10. Find the surface area of the solid.



10. ____________

11. A right cylinder has a diameter of 23.6 meters and a height of 11.4 meters. Find the lateral area to the nearest tenth. **11.** ______

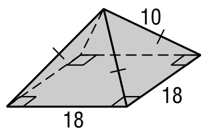
12. The surface area of a right cylinder is 252π square feet and the height is 11 feet. Find the radius. **12.** ______

For Questions 13 and 14, use a regular octagonal pyramid with base edges 9 feet long, slant height 15 feet, and a base with an apothem of 10.86 feet.

13. Find the lateral area. **13.** ______

14. Find the surface area to the nearest tenth. **14.** ______

15. Find the lateral area of the solid to the nearest tenth. **15.** ______



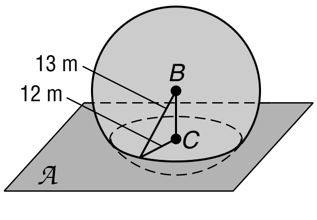
16. The surface area of a regular decagonal pyramid is 1800 square feet, the area of the base is 120 square feet, and the slant height is 15 feet. Find the length of each side of the base. **16.** ______

For Questions 17 and 18, use a cone with a radius of 5 centimeters and a height of 12 centimeters. Round to the nearest tenth.

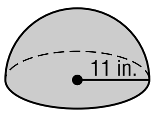
17. Find the lateral area. **17.** ______

18. Find the surface area. **18.** ______

19. Plane $\mathcal{A}$ intersects sphere B in $\odot C$. Find BC. **19.** ______



20. Find the surface area of this hemisphere to the nearest tenth. **20.** ______



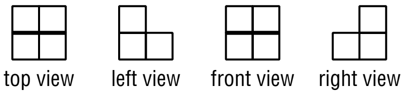
Bonus The length of each side of a cube is 6 inches long. Find the surface area of a sphere inscribed in the cube. **B:** ______

NAME ______________________ DATE __________ PERIOD ______

12 Chapter 12 Test, Form 3

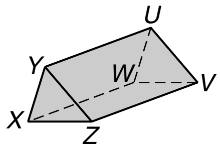
SCORE ______

1. Draw the back view of a figure given its orthogonal drawing.



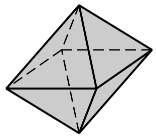
1. ______________

2. Name the bases of the solid.



2. ______________

3. Identify the solid.

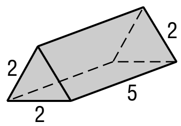


3. ______________

4. How many faces does an icosahedron have?

4. ______________

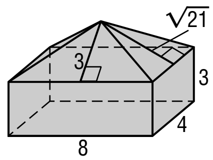
5. Draw a net for the solid.



5. ______________

Assessments

6. Find the surface area of the solid.

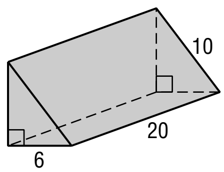


6. ______________

7. Find the lateral area of a triangular prism with a right triangular base with legs that measure 2 feet and 3 feet and a height of 7 feet.

7. ______________

8. Find the surface area of the prism.



8. ______________

For Questions 9 and 10, use a right cylinder with a diameter of 96.4 feet and a height of 58.9 feet. Round to the nearest tenth.

9. Find the lateral area.

9. ______________

10. Find the surface area.

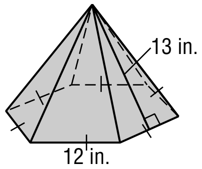
10. ______________

11. If the height of a right cylinder is multiplied by four and the radius remains the same, what happens to the lateral area?

11. ______________

12 Chapter 12 Test, Form 3 *(continued)*

For Questions 12 and 13, use the solid.



12. Find the lateral area. **12.** ____________

13. Find the surface area to the nearest tenth. **13.** ____________

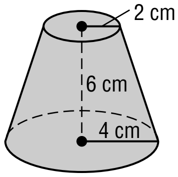
14. If the length of each side of a cube is tripled, what happens to the surface area? **14.** ____________

For Questions 15 and 16, use a right circular cone with a radius of 7 inches and a height of 8 inches. Round to the nearest tenth.

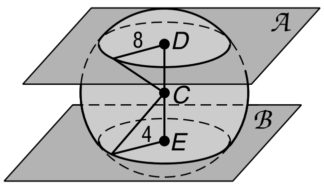
15. Find the lateral area. **15.** ____________

16. Find the surface area. **16.** ____________

17. Find the surface area of this frustum of a cone, to the nearest tenth. **17.** ____________

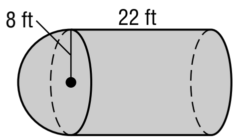


18. Parallel planes $\mathcal{A}$ and $\mathcal{B}$ intersect sphere C in circles D and E and $DC = 6$. Find CE. **18.** ____________

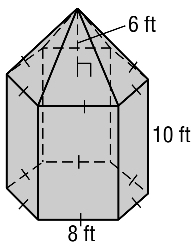


19. Write a formula for the surface area of a hemisphere in terms of π and the radius r. **19.** ____________

20. Find the surface area of the solid to the nearest square foot. **20.** ____________



Bonus Find the surface area of the solid to the nearest square foot. Do not include the area of the base. **B:** ____________



NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Complete this chart.

Figure	No. of Edges (*e*)	No. of Faces (*f*)	No. of Vertices (*v*)	$f + v$
Triangular Pyramid	6	4	4	8
Triangular Prism				
Cube				
Square Pyramid				
Hexagonal Prism				
Hexagonal Pyramid				

b. Write a formula relating the number of edges to the number of faces and vertices.

2. Explain the difference between the lateral area and the surface area of a prism.

3. a. Draw and label a pyramid.

b. Name the base.

c. Name the lateral faces.

d. Draw a net for your pyramid.

4. Draw an oblique cylinder and a right cylinder.

5. a. Draw and label the dimensions of a solid figure composed of three or more different solids studied in this chapter.

b. Find the surface area.

6. Write a practical application problem involving the surface area or lateral area of a solid figure studied in this chapter.

Assessments

NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Vocabulary Test/Review

SCORE ______

axis	great circle	oblique prism	regular prism
bases	hemisphere	orthogonal drawing	regular pyramid
circular cone	lateral area	perspective view	right cone
cone	lateral edges	Platonic solids	right cylinder
corner view	lateral faces	polyhedron	right prism
cross section	net	prism	slant height
cylinder	oblique cone	pyramid	sphere
edges	oblique cylinder	regular polyhedron	surface area
face			

Choose from the terms above to complete each sentence.

1. The height of each lateral face of a regular pyramid is called a(n) __?__. **1.** ____________

2. A(n) __?__ has a circular base and a vertex. **2.** ____________

3. A(n) __?__ is a set of points in space that are a given distance from a given point. **3.** ____________

4. The view of a solid figure from the corner is called a corner view or __?__. **4.** ____________

5. The five types of regular polyhedra are called the __?__. **5.** ____________

6. A(n) __?__ is a polyhedron with two parallel congruent faces called bases. **6.** ____________

7. A polyhedron that has all but one face intersecting at one point is a(n) __?__. **7.** ____________

8. The __?__ is the sum of the areas of all the faces of the solid. **8.** ____________

9. If the axis of a cylinder is also the altitude, then the cylinder is called a(n) __?__. **9.** ____________

10. A sphere is separated by a great circle into two congruent halves, each called a(n) __?__. **10.** ____________

Define each term.

11. lateral area **11.** ____________

12. right prism **12.** ____________

13. great circle **13.** ____________

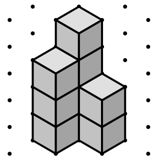
NAME ________________ DATE ________ PERIOD ______

12 Chapter 12 Quiz

(Lessons 12–1 and 12–2)

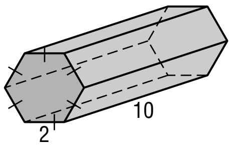
SCORE ______

1. Given the corner view of a figure, draw the left view. **1.** ________



2. Draw a rectangular prism. **2.** ________

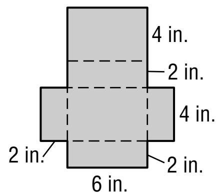
3. Draw a net for this solid. **3.** ________



4. STANDARDIZED TEST PRACTICE Find the surface area of the solid formed by folding this net. **4.** ________

A. 48 in^2 **B.** 64 in^2

C. 72 in^2 **D.** 88 in^2



NAME ________________ DATE ________ PERIOD ______

12 Chapter 12 Quiz

(Lessons 12–3 and 12–4)

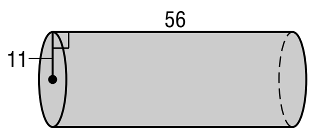
SCORE ______

1. The lateral area of a prism is 70 square inches and the perimeter of its base is 35 inches. Find the height. **1.** ________

2. The surface area of a rectangular prism is 592 square units. The height is 10 units and the width is 8 units. Find the length. **2.** ________

3. Describe the shape of a net that could be used to calculate the lateral area of a cylinder. **3.** ________

For Questions 4 and 5, use the solid figure. Round to the nearest tenth.



4. Find the lateral area. **4.** ________

5. Find the surface area. **5.** ________

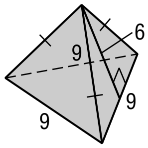
NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Quiz

(Lessons 12–5 and 12–6)

SCORE ______

For Questions 1 and 2, use the solid figure.



1. Find the lateral area. 1. ____________

2. Find the surface area to the nearest tenth. 2. ____________

3. The lateral area of a pyramid is 200 square centimeters and the base is a rectangle with a length measuring 15 centimeters and a width measuring 4 centimeters. Find the surface area of the pyramid. 3. ____________

For Questions 4 and 5, use a right circular cone with a radius of 5 feet and a slant height of 12 feet. Round to the nearest tenth.

4. Find the lateral area. 4. ____________

5. Find the surface area. 5. ____________

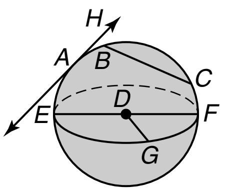
NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Quiz

(Lesson 12–7)

SCORE ______

For Questions 1–3, use the figure.



1. Name a chord of sphere *D* that is not a diameter. 1. ____________

2. Name a tangent to the sphere. 2. ____________

3. Name a great circle. 3. ____________

4. A sphere has a radius that is 18 inches long. Find the surface area to the nearest tenth. 4. ____________

5. The radius of a sphere is doubled. How is the surface area changed? 5. ____________

NAME ______________________ DATE ____________ PERIOD ______

12 Chapter 12 Mid-Chapter Test

SCORE ______

(Lessons 12–1 through 12–3)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Which of these are Platonic solids? **1.** ______

A. cylinder, cone, sphere **B.** prism, pyramid
C. tetrahedron, octahedron, dodecahedron **D.** hexagon, octagon, dodecagon

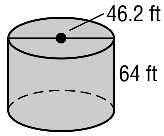
2. Which of the following describes a tetrahedron? **2.** ______

A. triangular pyramid **B.** triangular prism
C. square pyramid **D.** cone

3. The surface area of a prism is 120 square centimeters and the area of each base is 32 square centimeters. Find the lateral area of the prism. **3.** ______

A. 184 cm^2 **B.** 152 cm^2 **C.** 86 cm^2 **D.** 56 cm^2

For Questions 4 and 5, use the solid figure. Round to the nearest tenth.



4. Find the lateral area. **4.** ______

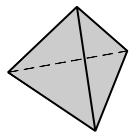
A. 9289.1 ft^2 **B.** 9434.2 ft^2
C. 10,965.4 ft^2 **D.** 12,641.8 ft^2

5. Find the surface area. **5.** ______

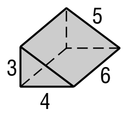
A. 9289.1 ft^2 **B.** 9434.2 ft^2 **C.** 10,965.4 ft^2 **D.** 12,641.8 ft^2

Part II

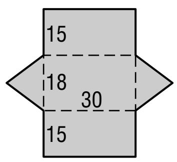
6. Draw the top view of the solid figure. **6.** ______________



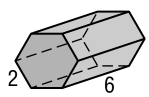
7. Draw a net of the solid. **7.** ______________



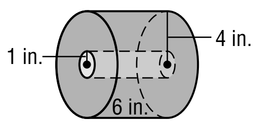
8. Find the surface area of the solid formed by folding this net. **8.** ______________



9. Find the surface area of the regular hexagonal prism to the nearest tenth. **9.** ______________



10. Find the total surface area to the nearest tenth. **10.** ______________



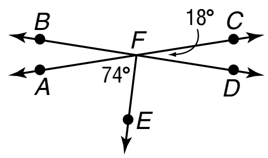
Assessments

12 Chapter 12 Cumulative Review

SCORE ______

(Chapters 1–12)

1. Name two obtuse vertical angles. (Lesson 1-5)



1. ____________

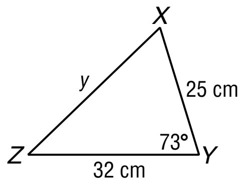
2. Use the statements p: $-8 + 3 = 5$ and q: *A triangle has four sides* to find the truth value of $p \wedge q$. (Lesson 2-2)

2. ____________

3. Find the value of $\frac{1}{x}$, where x initially equals 2. Then, use that value as the next x in the expression. Repeat the process and describe your observations. (Lesson 6-6)

3. ____________

4. Find $m\angle X$ and $m\angle Z$ to the nearest degree and y to the nearest centimeter. (Lesson 7-7)



4. ____________

5. Each side of a rhombus is 18 inches long. One diagonal makes a 40° angle with one side. What is the length of each diagonal to the nearest tenth? (Lesson 8-5)

5. ____________

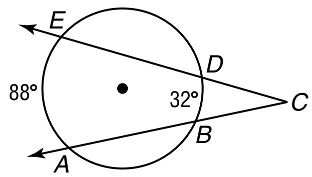
6. Find the magnitude and direction of $\overrightarrow{ST}$ for $S(4, -1)$ and $T(-6, 5)$. Round to the nearest tenth. (Lesson 9-6)

6. ____________

7. A certain figure is the locus of all points in a plane equidistant from point B. What is the shape of this figure? (Lesson 10-1)

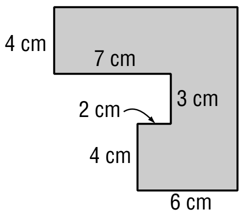
7. ____________

8. Find $m\angle C$. (Lesson 10-6)



8. ____________

9. Find the area and perimeter. (Lesson 11-1)



9. ____________

10. The surface area of a cube is 1261.5 square meters. Find the length of a lateral edge of the cube. (Lesson 12-3)

10. ____________

11. The surface area of a right cylinder is 502.7 square inches and the height is 11 inches. Find the radius of the base to the nearest tenth. (Lesson 12-4)

11. ____________

NAME ______________________ DATE ____________ PERIOD ______

12 Standardized Test Practice

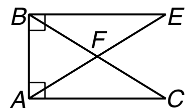
(Chapters 1–12)

SCORE ______

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Which method could you use to prove $\overline{BE} \cong \overline{AC}$ if $AF = BF$? (Lesson 4-5)



A. Show that $\triangle ABE \cong \triangle BAC$ by SSS, then $\overline{BE} \cong \overline{AC}$ by CPCTC.

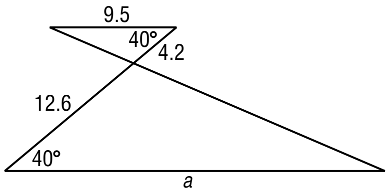
B. Show that $\triangle ABE \cong \triangle BAC$ by ASA, then $\overline{BE} \cong \overline{AC}$ by CPCTC.

C. Show that $\triangle BFE \cong \triangle AFC$ by SAS, then $\overline{BE} \cong \overline{AC}$ by CPCTC.

D. Show that $\triangle ABE \cong \triangle BAC$ by AAS, then $\overline{BE} \cong \overline{AC}$ by CPCTC.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Find a. (Lesson 6-3)

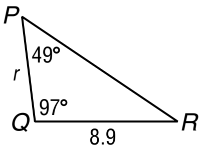


E. 28.5 **F.** 6.3

G. 12.6 **H.** 14

2. Ⓔ Ⓕ Ⓖ Ⓗ

3. Find r. (Lesson 7-6)



A. about 34.0 **B.** about 11.8

C. about 8.9 **D.** about 6.6

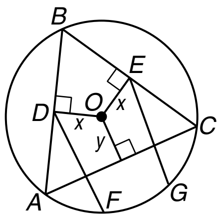
3. Ⓐ Ⓑ Ⓒ Ⓓ

4. A square has side length 18 centimeters. Find the area of the square. (Lesson 8-5)

E. 36 cm^2 **F.** 40 cm^2 **G.** 81 cm^2 **H.** 324 cm^2

4. Ⓔ Ⓕ Ⓖ Ⓗ

5. What can you assume from the figure? (Lesson 10-3)



A. $\triangle ABC$ is isosceles.

B. $\triangle ABC$ is equilateral.

C. $DF = EG$

D. radius of $\odot O = x + y$

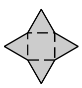
5. Ⓐ Ⓑ Ⓒ Ⓓ

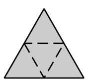
6. Points D, E, and F are on a circle so that $m\widehat{DEF} = 210$. Suppose point G is randomly located on the same circle so that it does not coincide with D, E, or F. What is the probability that $m\angle DGF = 105$? (Lesson 10-4)

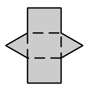
E. $\frac{7}{12}$ **F.** $\frac{5}{12}$ **G.** 1 **H.** $\frac{3}{4}$

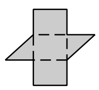
6. Ⓔ Ⓕ Ⓖ Ⓗ

7. Which net could be folded into a triangular prism? (Lesson 12-2)

A. 

B. 

C. 

D. 

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Find the surface area of a square pyramid with a height of 9 centimeters and base with a side measuring 24 centimeters. (Lesson 12-5)

E. 1296 cm^2 **F.** 1806 cm^2 **G.** 2016 cm^2 **H.** 8640 cm^2

8. Ⓔ Ⓕ Ⓖ Ⓗ

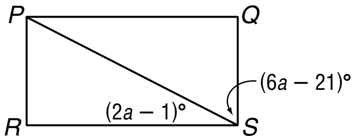
Assessments

12 Standardized Test Practice *(continued)*

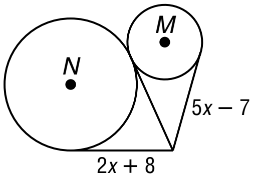
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

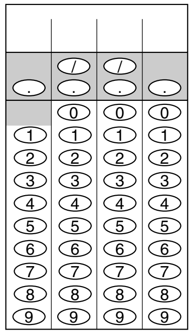
9. Quadrilateral $PQSR$ is a rectangle. Find a. (Lesson 8-4)



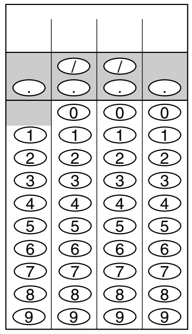
10. Find x. Assume that segments that appear tangent are tangent. (Lesson 10-5)



9.



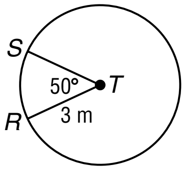
10.



Part 3: Short Response

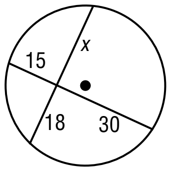
Instructions: Show your work or explain in words how you found your answer.

11. Find the length of $\widehat{SR}$ to the nearest tenth. (Lesson 10-2)



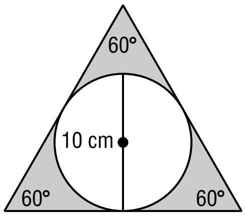
11. ______________

12. Find x. (Lesson 10-7)



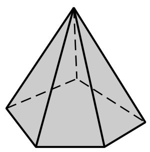
12. ______________

13. Find the area of the shaded region to the nearest tenth. (Lesson 11-3)



13. ______________

14. Identify the solid. (Lesson 12-1)



14. ______________

15. A right circular cone has a slant height of 15 inches and a radius that is 25 inches long. Find the surface area to the nearest tenth. (Lesson 12-6)

15. ______________

16. A ball has a diameter of 26.5 centimeters. Find the surface area of the ball to the nearest tenth. (Lesson 12.7)

16. ______________

NAME ______________________ DATE ____________ PERIOD ______

12

Standardized Test Practice

Student Record Sheet *(Use with pages 684–685 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ	**4** Ⓐ Ⓑ Ⓒ Ⓓ	**7** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**5** Ⓐ Ⓑ Ⓒ Ⓓ	**8** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**6** Ⓐ Ⓑ Ⓒ Ⓓ	**9** Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 ______________ (grid in)

11 ______________ (grid in)

12 ______________ (grid in)

10

11

12

Part 3 *Open-Ended*

Record your answers for Questions 13–14 on the back of this paper.

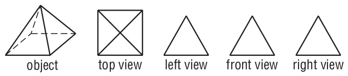
Answers

NAME ______________________ DATE __________ PERIOD _____

12-1 Study Guide and Intervention

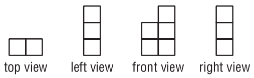
Three-Dimensional Figures

Drawings of Three-Dimensional Figures To work with a three-dimensional object, a useful skill is the ability to make an **orthogonal drawing**, which is a set of two-dimensional drawings of the different sides of the object. For a square pyramid, you would show the top view, the left view, the front view, and the right view.

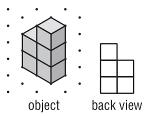


Example **Draw the back view of the figure given the orthogonal drawing.**

- The top view indicates two columns.
- The left view indicates that the height of figure is three blocks.
- The front view indicates that the columns have heights 2 and 3 blocks.
- The right view indicates that the height of the figure is three blocks.



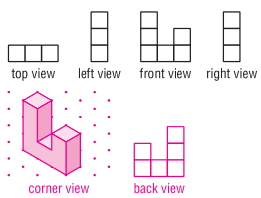
Use blocks to make a model of the object. Then use your model to draw the back view. The back view indicates that the columns have heights 3 and 2 blocks.



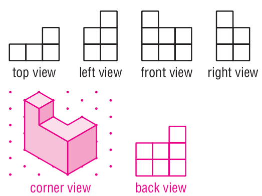
Exercises

Draw the back view and corner view of a figure given each orthogonal drawing.

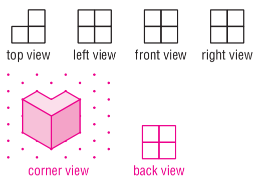
1.



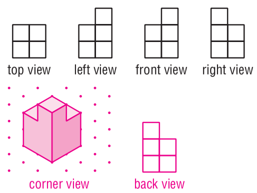
2.



3.



4.

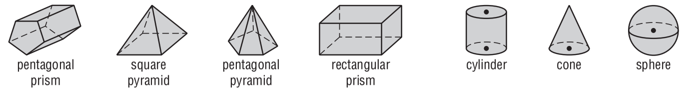


NAME ______________________ DATE __________ PERIOD _____

12-1 Study Guide and Intervention *(continued)*

Three-Dimensional Figures

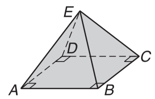
Identify Three-Dimensional Figures A **polyhedron** is a solid with all flat surfaces. Each surface of a polyhedron is called a **face**, and each line segment where faces intersect is called an **edge**. Two special kinds of polyhedra are **prisms**, for which two faces are congruent, parallel **bases**, and **pyramids**, for which one face is a base and all the other faces meet at a point called the **vertex**. Prisms and pyramids are named for the shape of their bases, and a regular polyhedron has a **regular** polygon as its base.



Other solids are a **cylinder**, which has congruent circular bases in parallel planes, a **cone**, which has one circular base and a vertex, and a **sphere**.

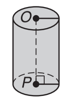
Example **Identify each solid. Name the bases, faces, edges, and vertices.**

a.



The figure is a rectangular pyramid. The base is rectangle $ABCD$, and the four faces $\triangle ABE$, $\triangle BCE$, $\triangle CDE$, and $\triangle ADE$ meet at vertex E. The edges are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$. The vertices are A, B, C, D, and E.

b.

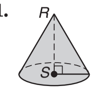


This solid is a cylinder. The two bases are $\odot O$ and $\odot P$.

Exercises

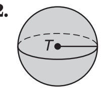
Identify each solid. Name the bases, faces, edges, and vertices.

1.



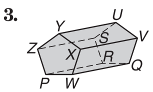
The solid is a cone. It has $\odot S$ for a base and vertex R.

2.



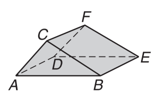
The solid is a sphere with center T.

3.



The figure is a pentagonal prism. The bases are pentagons $PWXYZ$ and $RQVUS$. The faces are pentagons $PWXYZ$ and $RQVUS$ and parallelograms $ZSUY$, $XYUV$, $WXVQ$, $PRQW$, and $ZPRS$. The edges are $\overline{ZS}$, $\overline{YU}$, $\overline{XV}$, $\overline{WQ}$, $\overline{PR}$, $\overline{ZY}$, $\overline{YX}$, $\overline{XW}$, $\overline{WP}$, $\overline{PZ}$, $\overline{SU}$, $\overline{UV}$, $\overline{VQ}$, $\overline{QR}$, and $\overline{RS}$. The vertices are P, W, X, Y, Z, R, Q, V, U, and S.

4.



The solid is a triangular prism. The bases are $\triangle ABC$ and $\triangle DEF$. The faces are $\triangle ABC$ and $\triangle DEF$ and parallelograms $ACFD$, $ABED$, and $BEFC$. The edges are $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{CF}$, $\overline{AD}$, $\overline{BE}$, $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$. The vertices are A, B, C, D, E, and F.

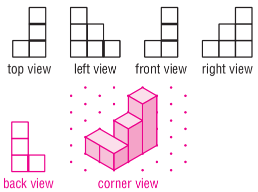
NAME ______________________ DATE __________ PERIOD _____

12-1 Skills Practice

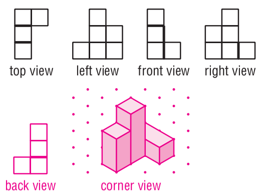
Three-Dimensional Figures

Draw the back view and corner view of a figure given each orthogonal drawing.

1.

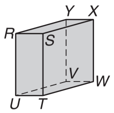


2.



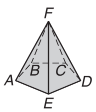
Identify each solid. Name the bases, faces, edges, and vertices.

3.



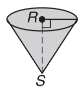
rectangular prism; sample answer: bases: ▱*RSTU*, ▱*VWXY*; faces: ▱*RSTU*, ▱*RSXY*, ▱*VWXY*, ▱*TUVW*, ▱*STWX*, ▱*RUVY*; edges: $\overline{RS}$, $\overline{RU}$, $\overline{ST}$, $\overline{TU}$, $\overline{SX}$, $\overline{RY}$, $\overline{TW}$, $\overline{UV}$, $\overline{VY}$, $\overline{WX}$, $\overline{XY}$, $\overline{VW}$; vertices: *R*, *S*, *T*, *U*, *V*, *W*, *X*, and *Y*

4.



pentagonal pyramid; base: pentagon *ABCDE*; faces: pentagon *ABCDE*, △*AFB*, △*BFC*, △*CFD*, △*DFE*, △*AFE*; edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{AE}$, $\overline{AF}$, $\overline{BF}$, $\overline{CF}$, $\overline{DF}$, $\overline{EF}$; vertices: *A*, *B*, *C*, *D*, *E*, and *F*

5.



cone; base: circle *R*; vertex: *S*

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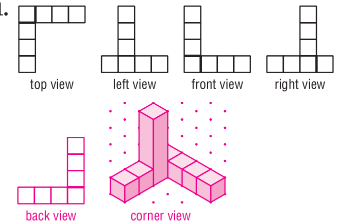
12-1 Practice (Average)

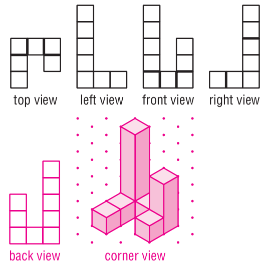
Three-Dimensional Figures

Draw the back view and corner view of a figure given each orthogonal drawing.

1.

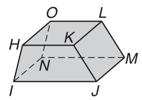


2.



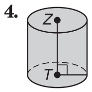
Identify each solid. Name the bases, faces, edges, and vertices.

3.



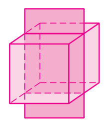
trapezoidal prism; bases: trapezoid *HIJK*, trapezoid *LMNO*; faces: trapezoid *HIJK*, trapezoid *LMNO*, ▱*HKLO*, ▱*HINO*, ▱*IJMN*, ▱*JKLM*; edges: $\overline{HI}$, $\overline{IJ}$, $\overline{JK}$, $\overline{HK}$, $\overline{HO}$, $\overline{ON}$, $\overline{IN}$, $\overline{KL}$, $\overline{LO}$, $\overline{LM}$, $\overline{JM}$, $\overline{MN}$; vertices: *H*, *I*, *J*, *K*, *L*, *M*, *N*, and *O*

4.

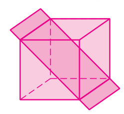


cylinder; bases: circle *T*, circle *Z*

5. MINERALS Pyrite, also known as fool's gold, can form crystals that are perfect cubes. Suppose a gemologist wants to cut a cube of pyrite to get a square and a rectangular face. What cuts should be made to get each of the shapes? Illustrate your answers.



a cut parallel to the bases to get a square



a cut through diagonally opposite top and bottom edges to get a rectangle

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12-1 Reading to Learn Mathematics

Three-Dimensional Figures

Pre-Activity **Why are drawings of three-dimensional structures valuable to archeologists?**

Read the introduction to Lesson 12-1 at the top of page 636 in your textbook.

Why do you think archeologists would want to know the details of the sizes and shapes of ancient three-dimensional structures? **Sample answer: This information might provide clues about construction methods and tools and help archeologists trace advances in construction methods over time.**

Reading the Lesson

1. Match each description from the first column with one of the terms from the second column. (Some of the terms may be used more than once or not at all.)

a. a polyhedron with two parallel congruent bases **v**
b. the set of points in space that are a given distance from a given point **viii**
c. a regular polyhedron with eight faces **i**
d. a polyhedron that has all faces but one intersecting at one point **xi**
e. a line segment where two faces of a polyhedron intersect **iv**
f. a solid with congruent circular bases in a pair of parallel planes **vii**
g. a regular polyhedron whose faces are squares **x**
h. a flat surface of a polyhedron **ii**

i. octahedron
ii. face
iii. icosahedron
iv. edge
v. prism
vi. dodecahedron
vii. cylinder
viii. sphere
ix. cone
x. hexahedron
xi. pyramid
xii. tetrahedron

2. Fill in the missing numbers, words, or phrases to complete each sentence.

a. A triangular prism has __6__ vertices. It has __5__ faces: __2__ bases are congruent __triangles__, and __3__ faces are parallelograms.

b. A regular octahedron has __6__ vertices and __8__ faces. Each face is a(n) __equilateral triangle__.

c. A hexagonal prism has __12__ vertices. It has __8__ faces: __2__ of them are the bases, which are congruent __hexagons__, and the other __6__ faces are parallelograms.

d. An octagonal pyramid has __9__ vertices and __9__ faces. The base is a(n) __octagon__, and the other __8__ faces are __triangles__.

e. There are exactly __5__ types of regular polyhedra. These are called the __Platonic__ solids. A polyhedron whose faces are regular pentagons is a __dodecahedron__, which has __12__ faces.

Helping You Remember

3. A good way to remember the characteristics of geometric solids is to think about how different solids are alike. Name a way in which pyramids and cones are alike. **Sample answer: They just have one base and the surfaces that are not the base meet at one point (the vertex).**

Lesson 12-1

NAME ______________________ DATE ___________ PERIOD _____

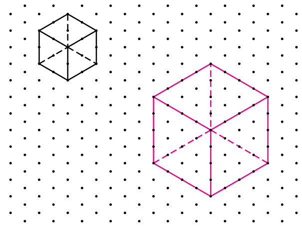
12-1 Enrichment

Drawing Solids on Isometric Dot Paper

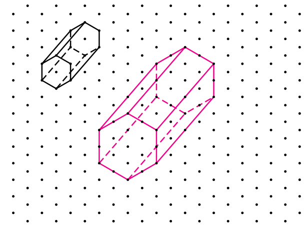
Isometric dot paper is helpful for drawing solids. Remember to use dashed lines for hidden edges.

For each solid shown, draw another solid whose dimensions are twice as large.

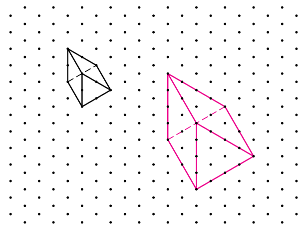
1.



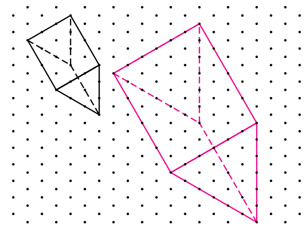
2.



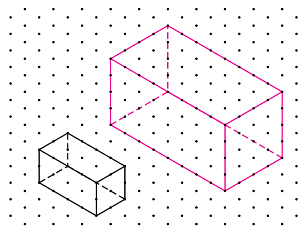
3.



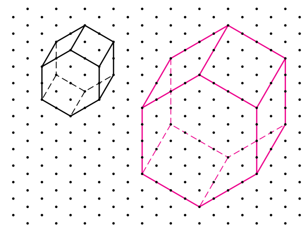
4.



5.



6.



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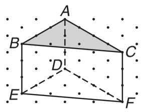
12-2 Study Guide and Intervention

Nets and Surface Area

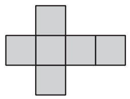
Models for Three-Dimensional Figures One way to relate a three-dimensional figure and a two-dimensional drawing is to use isometric dot paper. Another way is to make a flat pattern, called a *net*, for the surfaces of a solid.

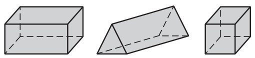
Example 1 **Use isometric dot paper to sketch a triangular prism with 3-4-5 right triangles as bases and with a height of 3 units.**



Step 1 Draw $\overline{AB}$ at 3 units and draw $\overline{AC}$ at 4 units.
Step 2 Draw $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, each at 3 units.
Step 3 Draw $\overline{BC}$ and $\triangle DEF$.

Example 2 **Match the net at the right with one of the solids below.**



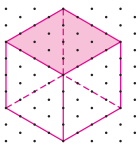


The six squares of the net can be folded into a cube. The net represents solid c.

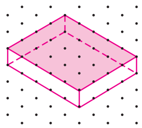
Exercises

Sketch each solid using isometric dot paper.

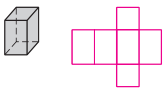
1. cube with edge 4

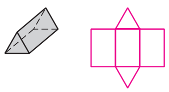


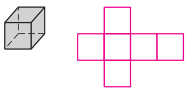
2. rectangular prism 1 unit high, 5 units long, and 4 units wide

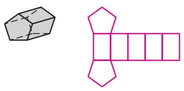


Draw a net for each solid.

3. 

4. 

5. 

6. 

Lesson 12-2

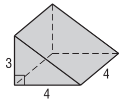
NAME ______________________ DATE __________ PERIOD _____

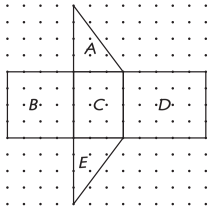
12-2 Study Guide and Intervention *(continued)*

Nets and Surface Area

Surface Area The **surface area** of a solid is the sum of the areas of the faces of the solid. Nets are useful in visualizing each face and calculating the area of the faces.

Example **Find the surface area of the triangular prism.**





First draw a net using rectangular dot paper. Using the Pythagorean Theorem, the hypotenuse of the right triangle is $\sqrt{3^2 + 4^2}$ or 5.

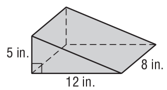
Surface area $= A + B + C + D + E$

$= \frac{1}{2}(4 \cdot 3) + 4 \cdot 4 + 4 \cdot 3 + 4 \cdot 5 + \frac{1}{2}(4 \cdot 3)$

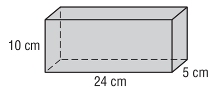
$= 60$ square units

Exercises

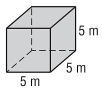
Find the surface area of each solid. Round to the nearest tenth if necessary.

1. 

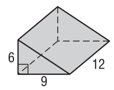
300 in^2

2. 

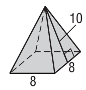
820 cm^2

3. 

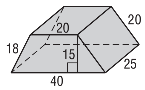
150 m^2

4. 

363.8 units^2

5. 

224 units^2

6. 

3350 units^2

Answers

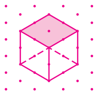
NAME ______________________ DATE __________ PERIOD _____

12-2 Skills Practice

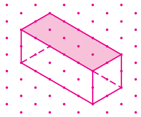
Nets and Surface Area

Sketch each solid using isometric dot paper.

1. cube 2 units on each edge

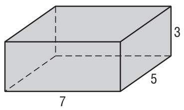


2. rectangular prism 2 units high, 5 units long, and 2 units wide

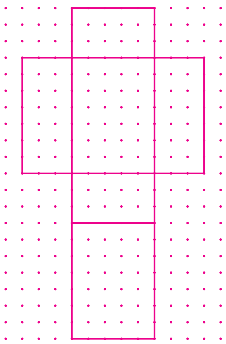


For each solid, draw a net and find the surface area.

3.

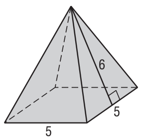


Sample net:

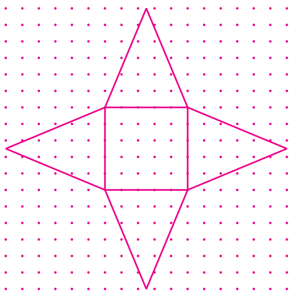


142 units^2

4.



Sample net:



85 units^2

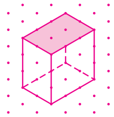
NAME ______________________ DATE __________ PERIOD _____

12-2 Practice (Average)

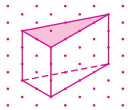
Nets and Surface Area

Sketch each solid using isometric dot paper.

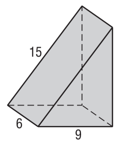
1. rectangular prism 3 units high, 3 units long, and 2 units wide



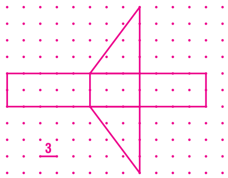
2. triangular prism 3 units high, whose bases are right triangles with legs 2 units and 4 units long



3. For the solid, draw a net and find the surface area.

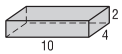


Sample net:

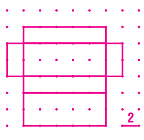


342 units^2

4. SHIPPING Rawanda needs to wrap a package to ship to her aunt. The rectangular package measures 2 inches high, 10 inches long, and 4 inches wide. Draw a net of the package. How much wrapping paper does Rawanda need to cover the package?



Sample net:



136 in^2

NAME ______________ DATE ________ PERIOD _____

12-2 Reading to Learn Mathematics

Nets and Surface Area

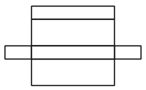
Pre-Activity **Why is surface area important to car manufacturers?**

Read the introduction to Lesson 12-2 at the top of page 643 in your textbook.

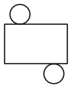
Suppose you are a passenger in a car that is moving at 50 miles per hour on the highway. If you hold your right hand just outside the right window, in what position does your hand encounter the greatest air resistance? **palm of the hand open, with the air hitting the palm at a 90° angle**

Reading the Lesson

1. Name the solid that can be formed by folding each net without any overlap.

a. 

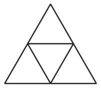
rectangular prism

b. 

cylinder

c. 

square pyramid

d. 

tetrahedron

2. Supply the missing number, word, or phrase to make a true statement. Be as specific as possible.

a. To find the surface area of a cube, add the areas of __**6**__ congruent squares.

b. To find the surface area of a cylinder, add the areas of two congruent __**circles**__ and one __**rectangle**__.

c. To find the surface area of a dodecahedron, add the areas of 12 congruent __**regular**__ __**pentagons**__.

d. To find the surface area of an icosahedron, add the areas of __**20**__ congruent __**equilateral**__ triangles.

Helping You Remember

3. A good way to remember a mathematical concept is to think about how the concept relates to everyday life. How could you explain the concept of *surface area* using an everyday situation?
Sample answer: The surface area of a solid shape tells you how much wrapping paper you would need to wrap a present with that shape and size.

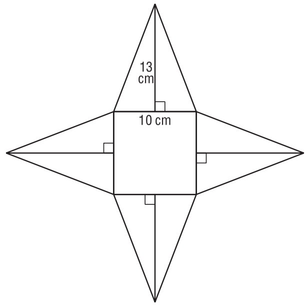
Lesson 12-2

NAME ______________ DATE ________ PERIOD _____

12-2 Enrichment

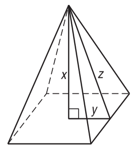
Pyramids

1. On a sheet of paper, draw the figure at the right so that the measurements are accurate. Cut out the shape and fold it to make a pyramid.
See students' work.



2. Measure the height from the highest point straight down (perpendicular) to the base. What is the height of the pyramid?
12 cm

The drawing at the right shows the pyramid you made. Answer each of the following.

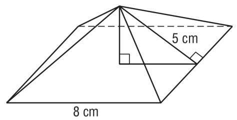


3. Measure the length of y in your pyramid.
5 cm

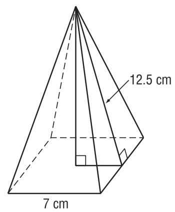
4. Measure the length of z, the height of one of the triangular faces.
13 cm

5. Use the Pythagorean Theorem to find x, the height of the pyramid. Is this equal to the height of your model?
12 cm; yes

Make a paper pattern for each pyramid below. Cut out your pattern and fold it to make a model. Measure the height of the pyramid and use the Pythagorean Theorem to verify this measure.

6. 

3 cm

7. 

12 cm

Answers

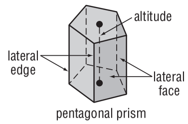
NAME ______________ DATE ________ PERIOD _____

12-3 Study Guide and Intervention

Surface Areas of Prisms

Lateral Areas of Prisms Here are some characteristics of prisms.

- The bases are parallel and congruent.
- The **lateral faces** are the faces that are not bases.
- The lateral faces intersect at **lateral edges**, which are parallel.
- The **altitude** of a prism is a segment that is perpendicular to the bases with an endpoint in each base.
- For a **right prism**, the lateral edges are perpendicular to the bases. Otherwise, the prism is **oblique**.



Lateral Area of a Prism	If a prism has a lateral area of L square units, a height of h units, and each base has a perimeter of P units, then $L = Ph$.

Example **Find the lateral area of the regular pentagonal prism above if each base has a perimeter of 75 centimeters and the altitude is 10 centimeters.**

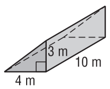
$L = Ph$ — Lateral area of a prism
$= 75(10)$ — $P = 75$, $h = 10$
$= 750$ — Multiply.

The lateral area is 750 square centimeters.

Exercises

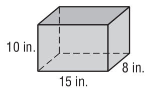
Find the lateral area of each prism.

1.



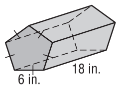
120 m²

2.



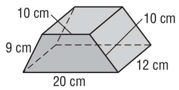
460 in² (8 in. × 15 in. base); 400 in² (10 in. × 15 in. base); 540 in² (10 in. × 8 in. base)

3.



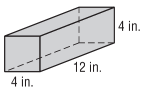
540 in²

4.



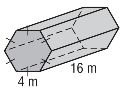
588 cm²

5.



128 in² (rectangular base)
192 in² (square base)

6.



384 m²

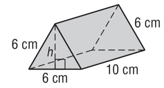
Lesson 12-3

NAME ______________ DATE ________ PERIOD _____

12-3 Study Guide and Intervention *(continued)*

Surface Areas of Prisms

Surface Areas of Prisms The surface area of a prism is the lateral area of the prism plus the areas of the bases.



Surface Area of a Prism	If the total surface area of a prism is T square units, its height is h units, and each base has an area of B square units and a perimeter of P units, then $T = L + 2B$.

Example **Find the surface area of the triangular prism above.**

Find the lateral area of the prism.

$L = Ph$ — Lateral area of a prism
$= (18)(10)$ — $P = 18$, $h = 10$
$= 180 \text{ cm}^2$ — Multiply.

Find the area of each base. Use the Pythagorean Theorem to find the height of the triangular base.

$h^2 + 3^2 = 6^2$ — Pythagorean Theorem
$h^2 = 27$ — Simplify.
$h = 3\sqrt{3}$ — Take the square root of each side.
$B = \frac{1}{2} \times \text{base} \times \text{height}$ — Area of a triangle
$= \frac{1}{2}(6)(3\sqrt{3}) \text{ or } 15.6 \text{ cm}^2$

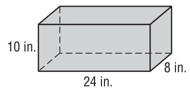
The total area is the lateral area plus the area of the two bases.

$T = 180 + 2(15.6)$ — Substitution
$= 211.2 \text{ cm}^2$ — Simplify.

Exercises

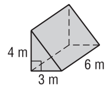
Find the surface area of each prism. Round to the nearest tenth if necessary.

1.



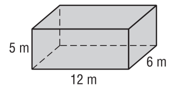
1024 in²

2.



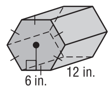
84 m²

3.



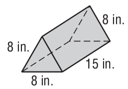
324 m²

4.



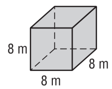
619.1 in²

5.



415.4 in²

6.



384 m²

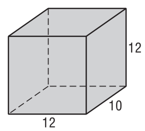
NAME ______________________ DATE __________ PERIOD _____

12-3 Skills Practice

Surface Areas of Prisms

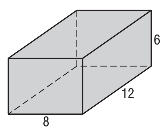
Find the lateral area of each prism.

1.



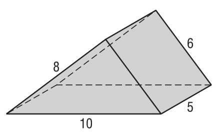
480 units^2 (square base)
528 units^2 (rectangular base)

2.



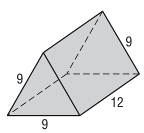
240 units^2 (8 × 12 base)
288 units^2 (12 × 6 base)
336 units^2 (8 × 6 base)

3.



120 units^2

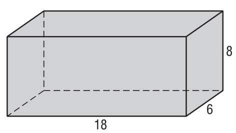
4.



324 units^2

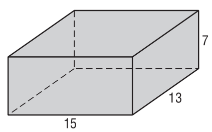
Find the surface area of each prism. Round to the nearest tenth if necessary.

5.



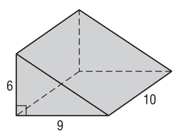
600 units^2

6.



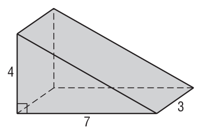
782 units^2

7.



312.2 units^2

8.



85.2 units^2

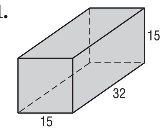
NAME ______________________ DATE __________ PERIOD _____

12-3 Practice (Average)

Surface Areas of Prisms

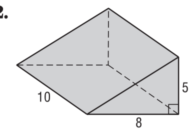
Find the lateral area of each prism. Round to the nearest tenth if necessary.

1.



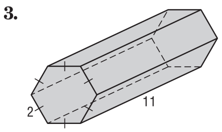
1920 units^2 (square base)
1410 units^2 (rectangular base)

2.



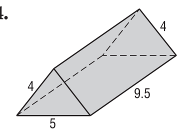
224.3 units^2

3.



132 units^2

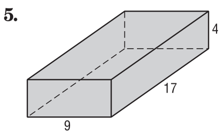
4.



123.5 units^2

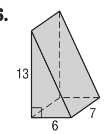
Find the surface area of each prism. Round to the nearest tenth if necessary.

5.



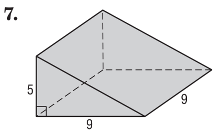
514 units^2

6.



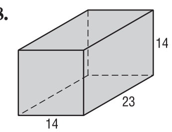
311.2 units^2

7.



263.7 units^2

8.



1680 units^2

9. CRAFTS Becca made a rectangular jewelry box in her art class and plans to cover it in red silk. If the jewelry box is $6\frac{1}{2}$ inches long, $4\frac{1}{2}$ inches wide, and 3 inches high, find the surface area that will be covered. **$124\frac{1}{2}$ in^2**

Lesson 12-3

Answers

NAME ______________ DATE ________ PERIOD ____

12-3 Reading to Learn Mathematics

Surface Areas of Prisms

Pre-Activity How do brick masons know how many bricks to order for a project?

Read the introduction to Lesson 12-3 at the top of page 649 in your textbook.

How could the brick mason figure out approximately how many bricks are needed for the garage? **Sample answer: Find the area of each wall of the garage and add these areas. Divide this total area by the area of one face of a brick, but take into account the amount of space for the mortar between the bricks.**

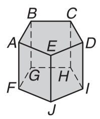
Reading the Lesson

1. Determine whether each sentence is *always*, *sometimes*, or *never* true.

- **a.** A base of a prism is a face of the prism. **always**
- **b.** A face of a prism is a base of the prism. **sometimes**
- **c.** The lateral faces of a prism are rectangles. **sometimes**
- **d.** If a base of a prism has n vertices, then the prism has n faces. **never**
- **e.** If a base of a prism has n vertices, then the prism has n lateral edges. **always**
- **f.** In a right prism, the lateral edges are also altitudes. **always**
- **g.** The bases of a prism are congruent regular polygons. **sometimes**
- **h.** Any two lateral edges of a prism are perpendicular to each other. **never**
- **i.** In a rectangular prism, any pair of opposite faces can be called the bases. **always**
- **j.** All of the lateral faces of a prism are congruent to each other. **sometimes**

2. Explain the difference between the *lateral area* of a prism and the *surface area* of a prism. Your explanation should apply to both right and oblique prisms. Do not use any formulas in your explanation. **Sample answer: The lateral area is the sum of the areas of lateral faces. The surface area is the sum of the areas of all the faces, including both the lateral faces and the bases.**

3. Refer to the figure.



- **a.** Name this solid with as specific a name as possible. **right pentagonal prism**
- **b.** Name the bases of the solid. **pentagons *ABCDE* and *FGHIJ***
- **c.** Name the lateral faces. **rectangles *ABGF*, *BCHG*, *CDIH*, *DEJI*, and *EAFJ***
- **d.** Name the edges. $\overline{AF}$, $\overline{BG}$, $\overline{CH}$, $\overline{DI}$, $\overline{EJ}$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JF}$
- **e.** Name an altitude of the solid. $\overline{AF}$, $\overline{BG}$, $\overline{CH}$, $\overline{DI}$, **or** $\overline{EJ}$
- **f.** If a represents the area, P represents the perimeter of one of the bases, and $x = AF$, write an expression for the surface area of the solid that involves a, P, and x. $Px + 2a$

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to an everyday use of the same word. How can the way the word *lateral* is used in sports help you remember the meaning of the *lateral area* of a solid? **Sample answer: In football, a pass thrown to the *side* is called a *lateral pass*. In geometry, the *lateral area* is the "side area," or the sum of the areas of all the side surfaces of the solid (the surfaces other than the bases).**

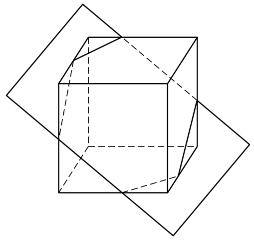
Lesson 12-3

NAME ______________ DATE ________ PERIOD ____

12-3 Enrichment

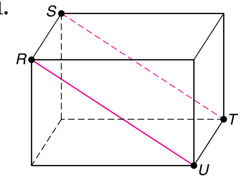
Cross Sections of Prisms

When a plane intersects a solid figure to form a two-dimensional figure, the results is called a **cross section**. The figure at the right shows a plane intersecting a cube. The cross section is a hexagon.



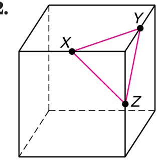
For each right prism, connect the labeled points in alphabetical order to show a cross section. Then identify the polygon.

1.



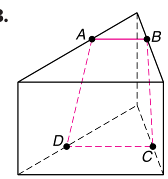
rectangle

2.



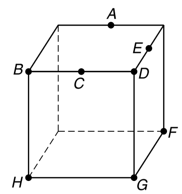
triangle

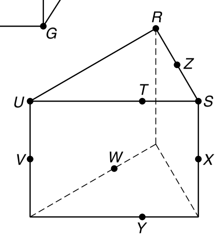
3.



trapezoid

Refer to the right prisms shown at the right. In the rectangular prism, *A* and *C* are midpoints. Identify the cross-section polygon formed by a plane containing the given points.





4. A, C, H **rectangle**

5. C, E, G **triangle**

6. H, C, E, F **trapezoid**

7. H, A, E **pentagon**

8. B, D, F **rectangle**

9. V, X, R **triangle**

10. R, T, Y **rectangle**

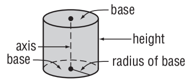
11. R, S, W **trapezoid**

NAME ______________________ DATE __________ PERIOD _____

12-4 Study Guide and Intervention

Surface Areas of Cylinders

Lateral Areas of Cylinders A **cylinder** is a solid whose bases are congruent circles that lie in parallel planes. The **axis** of a cylinder is the segment whose endpoints are the centers of these circles. For a **right cylinder**, the axis and the altitude of the cylinder are equal. The lateral area of a right cylinder is the circumference of the cylinder multiplied by the height.



Lateral Area of a Cylinder	If a cylinder has a lateral area of L square units, a height of h units, and the bases have radii of r units, then $L = 2\pi rh$.

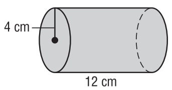
Example **Find the lateral area of the cylinder above if the radius of the base is 6 centimeters and the height is 14 centimeters.**

$L = 2\pi rh$ Lateral area of a cylinder
$= 2\pi(6)(14)$ Substitution
≈ 527.8 Simplify.

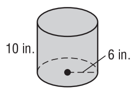
The lateral area is about 527.8 square centimeters.

Exercises

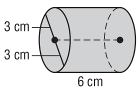
Find the lateral area of each cylinder. Round to the nearest tenth.

1. 

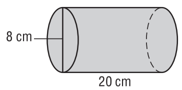
$301.6\ cm^2$

2. 

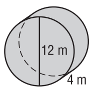
$377.0\ in^2$

3. 

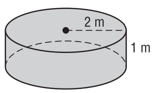
$113.1\ cm^2$

4. 

$502.7\ cm^2$

5. 

$150.8\ m^2$

6. 

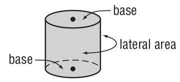
$12.6\ m^2$

NAME ______________________ DATE __________ PERIOD _____

12-4 Study Guide and Intervention *(continued)*

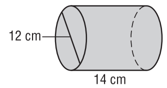
Surface Areas of Cylinders

Surface Areas of Cylinders The surface area of a cylinder is the lateral area of the cylinder plus the areas of the bases.



Surface Area of a Cylinder	If a cylinder has a surface area of T square units, a height of h units, and the bases have radii of r units, then $T = 2\pi rh + 2\pi r^2$.

Example **Find the surface area of the cylinder.**



Find the lateral area of the cylinder. If the diameter is 12 centimeters, then the radius is 6 centimeters.

$L = Ph$ Lateral area of a cylinder
$= (2\pi r)h$ $P = 2\pi r$
$= 2\pi(6)(14)$ $r = 6$, $h = 14$
≈ 527.8 Simplify.

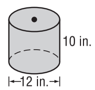
Find the area of each base.

$B = \pi r^2$ Area of a circle
$= \pi(6)^2$ $r = 6$
≈ 113.1 Simplify.

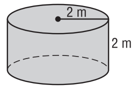
The total area is the lateral area plus the area of the two bases. $T = 527.8 + 113.1 + 113.1$ or 754 square centimeters.

Exercises

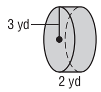
Find the surface area of each cylinder. Round to the nearest tenth.

1. 

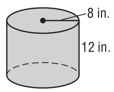
$603.2\ in^2$

2. 

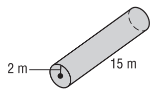
$50.3\ m^2$

3. 

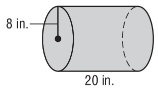
$94.2\ yd^2$

4. 

$1005.3\ in^2$

5. 

$213.6\ m^2$

6. 

$1407.4\ in^2$

NAME ______________________ DATE __________ PERIOD _____

12-4 Skills Practice

Surface Areas of Cylinders

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. $r = 10$ in., $h = 12$ in.
1382.3 in^2

2. $r = 8$ cm, $h = 15$ cm
1156.1 cm^2

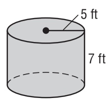
3. $r = 5$ ft, $h = 20$ ft
785.4 ft^2

4. $d = 20$ yd, $h = 5$ yd
942.5 yd^2

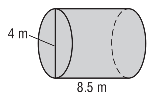
5. $d = 8$ m, $h = 7$ m
276.5 m^2

6. $d = 24$ mm, $h = 20$ mm
2412.7 mm^2

Find the surface area of each cylinder. Round to the nearest tenth.

7.


377.0 ft^2

8.


131.9 cm^2

Find the radius of the base of each cylinder.

9. The surface area is 603.2 square meters, and the height is 10 meters.
6 m

10. The surface area is 100.5 square inches, and the height is 6 inches.
2 in.

11. The surface area is 226.2 square centimeters, and the height is 5 centimeters.
4 cm

12. The surface area is 1520.5 square yards, and the height is 14.2 yards.
10 yd

Lesson 12-4

NAME ______________________ DATE __________ PERIOD _____

12-4 Practice (Average)

Surface Areas of Cylinders

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. $r = 8$ cm, $h = 9$ cm
854.5 cm^2

2. $r = 12$ in., $h = 14$ in.
1960.4 in^2

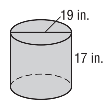
3. $d = 14$ mm, $h = 32$ mm
1715.3 mm^2

4. $d = 6$ yd, $h = 12$ yd
282.7 yd^2

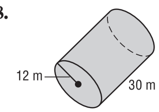
5. $r = 2.5$ ft, $h = 7$ ft
149.2 ft^2

6. $d = 13$ m, $h = 20$ m
1082.3 m^2

Find the surface area of each cylinder. Round to the nearest tenth.

7.


1581.8 in^2

8.


3166.7 m^2

Find the radius of the base of each right cylinder.

9. The surface area is 628.3 square millimeters, and the height is 15 millimeters.
5 mm

10. The surface area is 892.2 square feet, and the height is 4.2 feet.
10 ft

11. The surface area is 158.3 square inches, and the height is 5.4 inches.
3 in.

12. KALEIDOSCOPES Nathan built a kaleidoscope with a 20-centimeter barrel and a 5-centimeter diameter. He plans to cover the barrel with embossed paper of his own design. How many square centimeters of paper will it take to cover the barrel of the kaleidoscope?
about 314.2 cm^2

NAME ______________ DATE ________ PERIOD _____

12-4 Reading to Learn Mathematics

Surface Areas of Cylinders

Pre-Activity **How are cylinders used in extreme sports?**

Read the introduction to Lesson 12-4 at the top of page 655 in your textbook.

Why is the surface area of a half-pipe more than half the surface area of a complete pipe?
Sample answer: The surface area of the half-pipe includes an added flat section in the middle.

Reading the Lesson

1. Underline the correct word or phrase to form a true statement.
   - **a.** The bases of a cylinder are (rectangles/regular polygons/circles).
   - **b.** The (axis/radius/diameter) of a cylinder is the segment whose endpoints are the centers of the bases.
   - **c.** The net of a cylinder is composed of two congruent (rectangles/circles) and one (rectangle/semicircle).
   - **d.** In a right cylinder, the axis of the cylinder is also a(n) (base/lateral edge/altitude).
   - **e.** A cylinder that is not a right cylinder is called an (acute/obtuse/oblique) cylinder.

2. Match each description from the first column with an expression from the second column that represents its value.
   - **a.** the lateral area of a right cylinder in which the radius of each base is x cm and the length of the axis is y cm **v**
   - **b.** the surface area of a right prism with square bases in which the length of a side of a base is x cm and the length of a lateral edge is y cm **i**
   - **c.** the surface area of a right cylinder in which the radius of a base is x cm and the height is y cm **ii**
   - **d.** the surface area of regular hexahedron (cube) in which the length of each edge is x cm **iv**
   - **e.** the lateral area of a triangular prism in which the bases are equilateral triangles with side length x cm and the height is y cm **iii**
   - **f.** the surface area of a right cylinder in which the diameter of the base is x cm and the length of the axis is y cm **vi**

   - **i.** $(2x^2 + 4xy)$ cm^2
   - **ii.** $(2\pi xy + 2\pi x^2)$ cm^2
   - **iii.** $3xy$ cm^2
   - **iv.** $6x^2$ cm^2
   - **v.** $2\pi xy$ cm^2
   - **vi.** $\left(\frac{\pi x^2}{2} + \pi xy\right)$ cm^2

Helping You Remember

3. Often the best way to remember a mathematical formula is to think about where the different parts of the formula come from. How can you use this approach to remember the formula for the surface area of a cylinder? **Sample answer: Think about the net for a cylinder. The two bases are circles, each with radius r, so the sum of their areas is $2\pi r^2$. The lateral surface is a rectangle whose dimensions are the circumference $2\pi r$ of the circular base and the height h of the cylinder, so the lateral area is $2\pi rh$. Add the lateral area and the area of the two bases to get the surface area of the cylinder: $2\pi rh + 2\pi r^2$.**

Lesson 12-4

NAME ______________ DATE ________ PERIOD _____

12-4 Enrichment

Can-didly Speaking

Beverage companies have mathematically determined the ideal shape for a 12-ounce soft-drink can. Why won't any shape do? Some shapes are more economical than others. In order to hold 12 ounces of soft drink, an extremely skinny can would have to be very tall. The total amount of aluminum used for such a shape would be greater than the amount used for the conventional shape, thus costing the company more to make the skinny can. The radius r chosen determines the height h needed to hold 12 ounces of liquid. This also determines the amount of aluminum needed to make the can. Companies also have to keep in mind that the top of the can is three times thicker than the bottom and sides. Why? So you won't tear off the entire top when you open the can! The following formulas can be used to find the height and amount of aluminum a needed for a 12-ounce soft-drink can.

$$h = \frac{17.89}{\pi r^2}$$

$$a = 0.02\pi\left[\frac{r^2 + 35.78}{r}\right]$$ The values of r and h are measured in inches.

Find the height needed for a 12-ounce can for each radius. Round to the nearest tenth.

1. $2\frac{2}{3}$ in. **0.8 in.**
2. 1 in. **5.7 in.**
3. 3 in. **0.6 in.**
4. $1\frac{3}{4}$ in. **1.9 in.**

Sketch the shape of each can in Exercises 1–4.

5. Exercise 1
6. Exercise 2
7. Exercise 3
8. Exercise 4

9. **a.** Measure the radius of a soft-drink can. **$1\frac{1}{8}$ in.**
   b. Use the formula to find the height of the can. **$4\frac{1}{2}$ in.**
   c. Measure the height of a soft-drink can. **$\approx 4\frac{3}{4}$ in.**
   d. How does this measure compare to your findings in part a? **See students' work.**

10. Find the amount of aluminum used in making a soft-drink can. **≈ 2.3 oz**

Answers

NAME ______________________ DATE __________ PERIOD _____

12-5 Study Guide and Intervention

Surface Areas of Pyramids

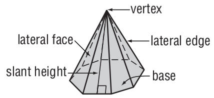
Lateral Areas of Regular Pyramids Here are some properties of pyramids.

- The base is a polygon.
- All of the faces, except the base, intersect in a common point known as the **vertex**.
- The faces that intersect at the vertex, which are called **lateral faces**, are triangles.

For a **regular pyramid**, the base is a regular polygon and the **slant height** is the height of each lateral face.

Lateral Area of a Regular Pyramid	If a regular pyramid has a lateral area of L square units, a slant height of ℓ units, and its base has a perimeter of P units, then $L = \frac{1}{2}P\ell$.

Example **The roof of a barn is a regular octagonal pyramid. The base of the pyramid has sides of 12 feet, and the slant height of the roof is 15 feet. Find the lateral area of the roof.**



The perimeter of the base is 8(12) or 96 feet.

$L = \frac{1}{2}P\ell$ — Lateral area of a pyramid

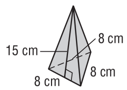
$= \frac{1}{2}(96)(15)$ — $P = 96$, $\ell = 15$

$= 720$ — Multiply.

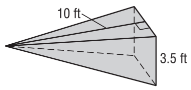
The lateral area is 720 square feet.

Exercises

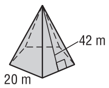
Find the lateral area of each regular pyramid. Round to the nearest tenth if necessary.

1.


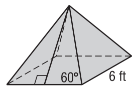
180 cm²

2.


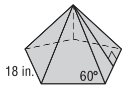
70 ft²

3.


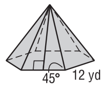
2100 m²

4.


62.4 ft²

5.


701.5 in²

6.


216 yd²

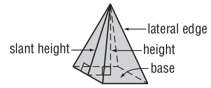
Lesson 12-5

NAME ______________________ DATE __________ PERIOD _____

12-5 Study Guide and Intervention *(continued)*

Surface Areas of Pyramids

Surface Areas of Regular Pyramids The surface area of a regular pyramid is the lateral area plus the area of the base.



Surface Area of a Regular Pyramid	If a regular pyramid has a surface area of T square units, a slant height of ℓ units, and its base has a perimeter of P units and an area of B square units, then $T = \frac{1}{2}P\ell + B$.

Example **For the regular square pyramid above, find the surface area to the nearest tenth if each side of the base is 12 centimeters and the height of the pyramid is 8 centimeters.**

Look at the pyramid above. The slant height is the hypotenuse of a right triangle. One leg of that triangle is the height of the pyramid, and the other leg is half the length of a side of the base. Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = 6^2 + 8^2$ — Pythagorean Theorem

$= 100$ — Simplify.

$\ell = 10$ — Take the square root of each side.

$T = \frac{1}{2}P\ell + B$ — Surface area of a pyramid

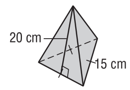
$= \frac{1}{2}(4)(12)(10) + 12^2$ — $P = (4)(12)$, $\ell = 10$, $B = 12^2$

$= 384$ — Simplify.

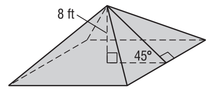
The surface area is 384 square centimeters.

Exercises

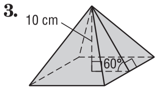
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.


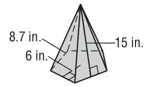
547.4 cm²

2.


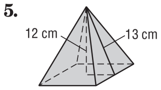
618.0 ft²

3.


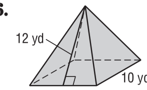
400 cm²

4.


456.8 in²

5.


360 cm²

6.


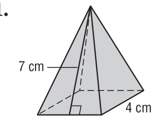
340 yd²

NAME ______________ DATE ______ PERIOD ____

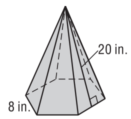
12-5 Skills Practice

Surface Area of Pyramids

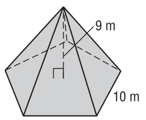
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.


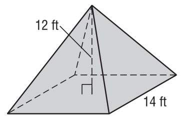
72 cm²

2.


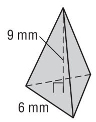
646.3 in²

3.


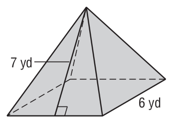
455.3 m²

4.


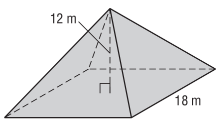
585.0 ft²

5.


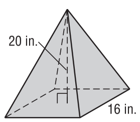
99.9 mm²

6.


120 yd²

7.


864 m²

8.


945.3 in²

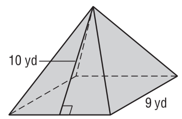
Lesson 12-5

NAME ______________ DATE ______ PERIOD ____

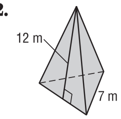
12-5 Practice (Average)

Surface Area of Pyramids

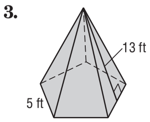
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1.


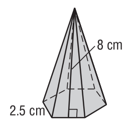
261 yd²

2.


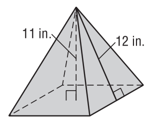
147.2 m²

3.


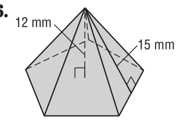
205.5 ft²

4.


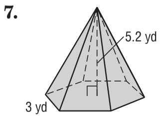
76.2 cm²

5.


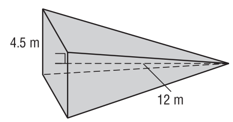
322.2 in²

6.


784.7 mm²

7.


75.7 yd²

8.


130.1 m²

9. GAZEBOS The roof of a gazebo is a regular octagonal pyramid. If the base of the pyramid has sides of 0.5 meters and the slant height of the roof is 1.9 meters, find the area of the roof.
3.8 m²

Answers

NAME ______________________ DATE ____________ PERIOD _____

12-5 Reading to Learn Mathematics

Surface Areas of Pyramids

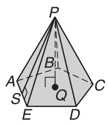
Pre-Activity **How are pyramids used in architecture?**

Read the introduction to Lesson 12-5 at the top of page 660 in your textbook.

Why do you think that the architect for the new entrance to the Louvre decided to use a pyramid rather than a rectangular prism?

Sample answer: The pyramid, with its sharp point at the vertex and its sloping sides, is more unusual and dramatic than a rectangular prism.

Reading the Lesson



1. In the figure, *ABCDE* has congruent sides and congruent angles.

a. Describe this pyramid with as specific a name as possible. **regular pentagonal pyramid**

b. Use the figure to name the base of this pyramid. ***ABCDE***

c. Describe the base of the pyramid. **regular pentagon**

d. Name the vertex of the pyramid. ***P***

e. Name the lateral faces of the pyramid. **$\triangle PAB$, $\triangle PBC$, $\triangle PCD$, $\triangle PDE$, and $\triangle PEA$**

f. Describe the lateral faces. **five congruent isosceles triangles**

g. Name the lateral edges of the pyramid. **$\overline{PA}$, $\overline{PB}$, $\overline{PC}$, $\overline{PD}$, and $\overline{PE}$**

h. Name the altitude of the pyramid. **$\overline{PQ}$**

i. Write an expression for the height of the pyramid. ***PQ***

j. Write an expression for the slant height of the pyramid. ***PS***

2. In a regular square pyramid, let s represent the side length of the base, h represent the height, a represent the apothem, and ℓ represent the slant height. Also, let L represent the lateral area and let T represent the surface area. Which of the following relationships are correct? **A, D, E, F**

A. $s = 2a$ **B.** $a^2 + \ell^2 = h^2$ **C.** $L = 4\ell s$

D. $h = \sqrt{\ell^2 - a^2}$ **E.** $\left(\frac{s}{2}\right)^2 + h^2 = \ell^2$ **F.** $T = s^2 + 2\ell s$

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that one of your classmates is having trouble remembering the difference between the *height* and the *slant height* of a regular pyramid. How can you explain this concept?

Sample answer: The *height* of the pyramid is the length of the perpendicular segment from the vertex to the center of the base. The *slant height* is the length of the perpendicular segment from the vertex of the pyramid to the midpoint of one of the sides of the base of the pyramid.

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12-5 Enrichment

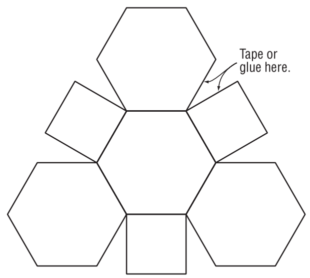
Two Truncated Solids

To create a truncated solid, you could start with an ordinary solid and then cut off the corners. Another way to make such a shape is to use the patterns on this page.

The Truncated Octahedron

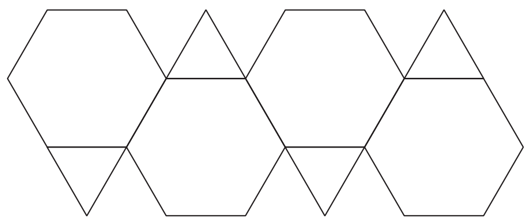
1. Two copies of the pattern at the right can be used to make a *truncated octahedron*, a solid with 6 square faces and 8 regular hexagonal faces.

Each pattern makes half of the truncated octahedron. Attach adjacent faces using glue or tape to make a cup-shaped figure.



The Truncated Tetrahedron

2. The pattern below will make a *truncated tetrahedron*, a solid with 8 polygonal faces: 4 hexagons and 4 equilateral triangles.



Solve.

3. Find the surface area of the truncated octahedron if each polygon in the pattern has sides of 3 inches.

241.1 in^2

4. Find the surface area of the truncated tetrahedron if each polygon in the pattern has sides of 3 inches.

109.1 in^2

Area Formulas for Regular Polygons (s is the length of one side)	
triangle	$A = \frac{s^2}{4}\sqrt{3}$
hexagon	$A = \frac{3s^2}{2}\sqrt{3}$
octagon	$A = 2s^2(\sqrt{2} + 1)$

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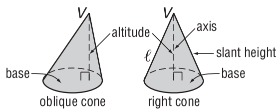
12-6 Study Guide and Intervention

Surface Areas of Cones

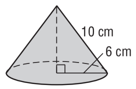
Lateral Areas of Cones Cones have the following properties.

- A cone has one circular base and one vertex.
- The segment whose endpoints are the vertex and the center of the base is the **axis** of the cone.
- The segment that has one endpoint at the vertex, is perpendicular to the base, and has its other endpoint on the base is the **altitude** of the cone.
- For a **right cone** the axis is also the altitude, and any segment from the circumference of the base to the vertex is the **slant height** ℓ. If a cone is not a right cone, it is oblique.



Lateral Area of a Cone	If a cone has a lateral area of L square units, a slant height of ℓ units, and the radius of the base is r units, then $L = \pi r\ell$.

Example **Find the lateral area of a cone with slant height of 10 centimeters and a base with a radius of 6 centimeters.**

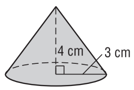


$L = \pi r\ell$ — Lateral area of a cone
$= \pi(6)(10)$ — $r = 6, \ell = 10$
≈ 188.5 — Simplify.

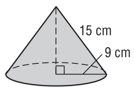
The lateral area is about 188.5 square centimeters.

Exercises

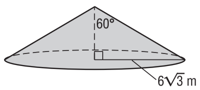
Find lateral area of each circular cone. Round to the nearest tenth.

1.


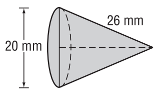
47.1 cm^2

2.


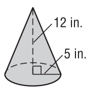
424.1 cm^2

3.


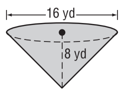
391.8 m^2

4.


816.8 mm^2

5.


204.2 in^2

6.


284.3 yd^2

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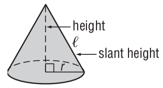
12-6 Study Guide and Intervention *(continued)*

Surface Areas of Cones

Surface Areas of Cones The surface area of a cone is the lateral area of the cone plus the area of the base.

Surface Area of a Right Cone	If a cone has a surface area of T square units, a slant height of ℓ units, and the radius of the base is r units, then $T = \pi r\ell + \pi r^2$.



Example **For the cone above, find the surface area to the nearest tenth if the radius is 6 centimeters and the height is 8 centimeters.**

The slant height is the hypotenuse of a right triangle with legs of length 6 and 8. Use the Pythagorean Theorem.

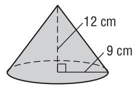
$\ell^2 = 6^2 + 8^2$ — Pythagorean Theorem
$\ell^2 = 100$ — Simplify.
$\ell = 10$ — Take the square root of each side.

$T = \pi r\ell + \pi r^2$ — Surface area of a cone
$= \pi(6)(10) + \pi \cdot 6^2$ — $r = 6, \ell = 10$
≈ 301.6 — Simplify.

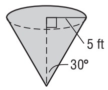
The surface area is about 301.6 square centimeters.

Exercises

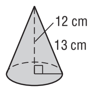
Find the surface area of each cone. Round to the nearest tenth.

1.


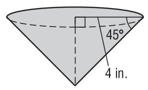
678.6 cm^2

2.


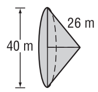
235.6 ft^2

3.


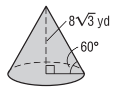
282.7 cm^2

4.


121.4 in^2

5.


2890.3 m^2

6.


603.2 yd^2

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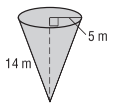
12-6 Skills Practice

Surface Areas of Cones

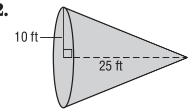
Find the surface area of each cone. Round to the nearest tenth if necessary.

1.



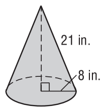
298.5 m^2

2.



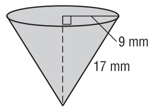
1160.1 ft^2

3.



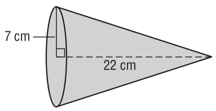
728.8 in^2

4.



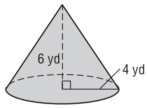
735.1 mm^2

5.



661.6 cm^2

6.



140.9 yd^2

7. Find the surface area of a cone if the height is 12 inches and the slant height is 15 inches.
678.6 in^2

8. Find the surface area of a cone if the height is 9 centimeters and the slant height is 12 centimeters.
497.1 cm^2

9. Find the surface area of a cone if the height is 10 meters and the slant height is 14 meters.
732.5 m^2

10. Find the surface area of a cone if the height is 5 feet and the slant height is 7 feet.
183.1 ft^2

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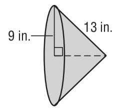
12-6 Practice (Average)

Surface Areas of Cones

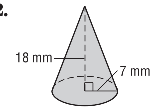
Find the surface area of each cone. Round to the nearest tenth if necessary.

1.



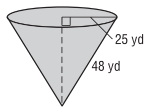
622.0 in^2

2.



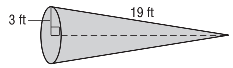
578.7 mm^2

3.



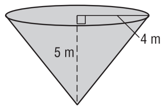
5733.4 yd^2

4.



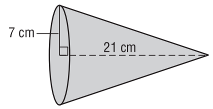
207.3 ft^2

5.



130.7 m^2

6.



640.7 cm^2

7. Find the surface area of a cone if the height is 8 feet and the slant height is 10 feet.
301.6 ft^2

8. Find the surface area of a cone if the height is 14 centimeters and the slant height is 16.4 centimeters.
669.3 cm^2

9. Find the surface area of a cone if the height is 12 inches and the diameter is 27 inches.
1338.6 in^2

10. HATS Cuong bought a conical hat on a recent trip to central Vietnam. The basic frame of the hat is 16 hoops of bamboo that gradually diminish in size. The hat is covered in palm leaves. If the hat has a diameter of 50 centimeters and a slant height of 32 centimeters, what is the lateral area of the conical hat?
about 2513.3 cm^2

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12-6 Reading to Learn Mathematics

Surface Areas of Cones

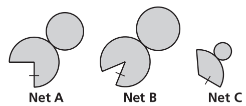
Pre-Activity **How is the lateral area of a cone used to cover tepees?**

Read the introduction to Lesson 12-6 at the top of page 666 in your textbook.

If you wanted to build a tepee of a certain size, how would it help you to know the formula for the lateral area of a cone?
Sample answer: The formula would help you estimate how many animal skins you would need for the tepee.

Reading the Lesson

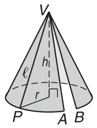
1.



a. Which net will give the cone with the greatest lateral area? **Net B**

b. Which net will give the tallest cone? **Net C**

2. Refer to the figure at the right. Suppose you have removed the circular base of the cone and cut from V to A so that you can unroll the lateral surface onto a flat table.



a. How can you be sure that the flattened-out piece is a sector of a circle?
Sample answer: Before you unroll the lateral surface, all points on the bottom rim of the cone are ℓ units from V. After you flatten out the surface, those points are still ℓ units from V. This means that they will lie on a circle with center V and radius ℓ.

b. How do you know that the flattened-out piece is not a full circle?
Sample answer: $r < \ell$

3. Suppose you have a right cone with radius r, diameter d, height h, and slant height ℓ. Which of the following relationships involving these lengths are correct? **C, E**

A. $r = 2d$ **B.** $r + h = \ell$ **C.** $r^2 + h^2 = \ell^2$

D. $r^2 + \ell^2 = h^2$ **E.** $r = \sqrt{\ell^2 - h^2}$ **F.** $h = \pm\sqrt{\ell^2 - r^2}$

Helping You Remember

4. One way to remember a new formula is to relate it to a formula you already know. Explain how the formulas for the lateral areas of a pyramid and a cone are similar.
Sample answer: Both formulas say that the lateral area includes the distance around the base multiplied by the slant height.

Lesson 12-6

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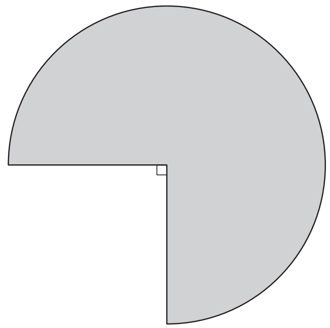
12-6 Enrichment

Cone Patterns

The pattern at the right is made from a circle. It can be folded to make a cone.



1. Measure the radius of the circle to the nearest centimeter. **4 cm**

2. The pattern is what fraction of the complete circle? $\frac{3}{4}$

3. What is the circumference of the complete circle? **8π cm**

4. How long is the circular arc that is the outside of the pattern? **6π cm**

5. Cut out the pattern and tape it together to form a cone. **See students' work.**

6. Measure the diameter of the circular base of the cone. **6 cm**

7. What is the circumference of the base of the cone? **6π cm**

8. What is the slant height of the cone? **4 cm**

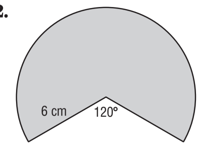
9. Use the Pythagorean Theorem to calculate the height of the cone. Use a decimal approximation. Check your calculation by measuring the height with a metric ruler. **2.65 cm**

10. Find the lateral area. **12π cm²**

11. Find the total surface area. **21π cm²**

Make a paper pattern for each cone with the given measurements. Then cut the pattern out and make the cone. Find the measurements.

12.

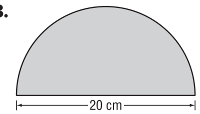


diameter of base = **8 cm**

lateral area = **24π cm²**

height of cone = **4.5 cm**
(to nearest tenth of a centimeter)

13.



diameter of base = **10 cm**

lateral area = **50π cm²**

height of cone = **8.7 cm**
(to nearest tenth of a centimeter)

Answers

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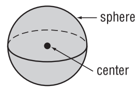
12-7 Study Guide and Intervention

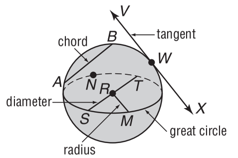
Surface Areas of Spheres

Properties of Spheres A **sphere** is the locus of all points that are a given distance from a given point called its **center**.



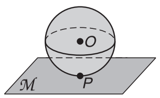
Here are some terms associated with a sphere.

- A **radius** is a segment whose endpoints are the center of the sphere and a point on the sphere.
- A **chord** is a segment whose endpoints are points on the sphere.
- A **diameter** is a chord that contains the sphere's center.
- A **tangent** is a line that intersects the sphere in exactly one point.
- A **great circle** is the intersection of a sphere and a plane that contains the center of the sphere.
- A **hemisphere** is one-half of a sphere. Each great circle of a sphere determines two hemispheres.

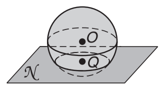


$\overline{RS}$ is a radius. $\overline{AB}$ is a chord. $\overline{ST}$ is a diameter. $\overleftrightarrow{VX}$ is a tangent. The circle that contains points S, M, T, and N is a great circle; it determines two hemispheres.

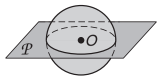
Example **Determine the shapes you get when you intersect a plane with a sphere.**



The intersection of plane $\mathcal{M}$ and sphere O is point P.



The intersection of plane $\mathcal{N}$ and sphere O is circle Q.



The intersection of plane $\mathcal{P}$ and sphere O is circle O.

A plane can intersect a sphere in a point, in a circle, or in a great circle.

Exercises

Describe each object as a model of a *circle*, a *sphere*, a *hemisphere*, or *none of these*.

1. a baseball
sphere

2. a pancake
circle

3. the Earth
sphere

4. a kettle grill cover
hemisphere

5. a basketball rim
circle

6. cola can
none of these

Determine whether each statement is *true* or *false*.

7. All lines intersecting a sphere are tangent to the sphere. **false**

8. Every plane that intersects a sphere makes a great circle. **false**

9. The eastern hemisphere of Earth is congruent to the western hemisphere. **true**

10. The diameter of a sphere is congruent to the diameter of a great circle. **true**

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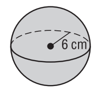
12-7 Study Guide and Intervention *(continued)*

Surface Areas of Spheres

Surface Areas of Spheres You can think of the surface area of a sphere as the total area of all of the nonoverlapping strips it would take to cover the sphere. If r is the radius of the sphere, then the area of a great circle of the sphere is πr^2. The total surface area of the sphere is four times the area of a great circle.

Surface Area of a Sphere	If a sphere has a surface area of T square units and a radius of r units, then $T = 4\pi r^2$.

Example **Find the surface area of a sphere to the nearest tenth if the radius of the sphere is 6 centimeters.**



$T = 4\pi r^2$ Surface area of a sphere
$= 4\pi \cdot 6^2$ $r = 6$
≈ 452.4 Simplify.

The surface area is 452.4 square centimeters.

Exercises

Find the surface area of each sphere with the given radius or diameter to the nearest tenth.

1. $r = 8$ cm
804.2 cm²

2. $r = 2\sqrt{2}$ ft
100.5 ft²

3. $r = \pi$ cm
124.0 cm²

4. $d = 10$ in.
314.2 in²

5. $d = 6\pi$ m
1116.2 m²

6. $d = 16$ yd
804.2 yd²

7. Find the surface area of a hemisphere with radius 12 centimeters.
1357.2 cm²

8. Find the surface area of a hemisphere with diameter π centimeters.
23.3 cm²

9. Find the radius of a sphere if the surface area of a hemisphere is 192π square centimeters.
8 cm

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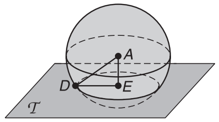
12-7 Skills Practice

Surface Areas of Spheres

In the figure, *A* is the center of the sphere, and plane $\mathcal{T}$ intersects the sphere in circle *E*. Round to the nearest tenth if necessary.



1. If $AE = 5$ and $DE = 12$, find AD.
13

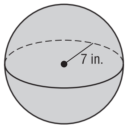
2. If $AE = 7$ and $DE = 15$, find AD.
16.6

3. If the radius of the sphere is 18 units and the radius of $\odot E$ is 17 units, find AE.
5.9

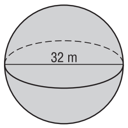
4. If the radius of the sphere is 10 units and the radius of $\odot E$ is 9 units, find AE.
4.4

5. If M is a point on $\odot E$ and $AD = 23$, find AM.
23

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

6.


615.8 in^2

7.


3217.0 m^2

8. a hemisphere with a radius of the great circle 8 yards
603.2 yd^2

9. a hemisphere with a radius of the great circle 2.5 millimeters
58.9 mm^2

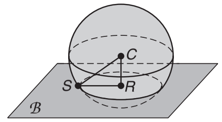
10. a sphere with the area of a great circle 28.6 inches
114.4 in^2

NAME ______________________ DATE ________ PERIOD _____

12-7 Practice (Average)

Surface Areas of Spheres

In the figure, *C* is the center of the sphere, and plane $\mathcal{B}$ intersects the sphere in circle *R*. Round to the nearest tenth if necessary.



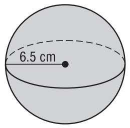
1. If $CR = 4$ and $SR = 14$, find CS.
14.6

2. If $CR = 7$ and $SR = 24$, find CS.
25

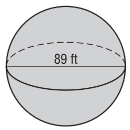
3. If the radius of the sphere is 28 units and the radius of $\odot R$ is 26 units, find CR.
10.4

4. If J is a point on $\odot R$ and $CS = 7.3$, find CJ.
7.3

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

5.


530.9 cm^2

6.


24,884.6 ft^2

7. a sphere with the area of a great circle 29.8 meters
119.2 m^2

8. a hemisphere with a radius of the great circle 8.4 inches
665.0 in^2

9. a hemisphere with the circumference of a great circle 18 millimeters
77.3 mm^2

10. SPORTS A standard size 5 soccer ball for ages 13 and older has a circumference of 27–28 inches. Suppose Breck is on a team that plays with a 28-inch soccer ball. Find the surface area of the ball.
about 249.6 in^2

NAME ______________________ DATE __________ PERIOD _____

12-7 Reading to Learn Mathematics

Surface Areas of Spheres

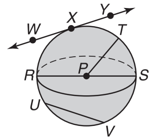
Pre-Activity **How do manufacturers of sports equipment use the surface areas of spheres?**

Read the introduction to Lesson 12-7 at the top of page 671 in your textbook.

How would knowing the formula for the surface area of a sphere help manufacturers of beach balls? **Sample answer: The surface area of the ball tells the manufacturer how much material is needed to make each ball.**

Reading the Lesson

1. In the figure, P is the center of the sphere. Name each of the following in the figure.



   a. three radii of the sphere **$\overline{PT}$, $\overline{PR}$, and $\overline{PS}$**
   b. a diameter of the sphere **$\overline{RS}$**
   c. two chords of the sphere **$\overline{RS}$ and $\overline{UV}$**
   d. a great circle of the sphere **$\odot P$**
   e. a tangent to the sphere **$\overleftrightarrow{WY}$**
   f. the point of tangency **X**

2. Determine whether each sentence is *sometimes*, *always*, or *never* true.
   a. If a sphere and a plane intersect in more than one point, their intersection will be a great circle. **sometimes**
   b. A great circle has the same center as the sphere. **always**
   c. The endpoints of a radius of a sphere are two points on the sphere. **never**
   d. A chord of a sphere is a diameter of the sphere. **sometimes**
   e. A radius of a great circle is also a radius of the sphere. **always**

3. Match each surface area formula with the name of the appropriate solid.

a. $T = \pi r\ell + \pi r^2$ **vi**	**i.** regular pyramid
b. $T = Ph + 2B$ **iv**	**ii.** hemisphere
c. $T = 4\pi r^2$ **v**	**iii.** cylinder
d. $T = \frac{1}{2}P\ell + B$ **i**	**iv.** prism
e. $T = 2\pi rh + 2\pi r^2$ **iii**	**v.** sphere
f. $T = 3\pi r^2$ **ii**	**vi.** cone

Helping You Remember

4. Many students have trouble remembering all of the formulas they have learned in this chapter. What is an easy way to remember the formula for the surface area of a sphere? **Sample answer: A sphere doesn't have any lateral faces or bases, so the expression in the formula for its surface area has just one term, $4\pi r^2$, rather than being the sum of the expressions for the lateral area and the area of the bases like the others.**

Lesson 12-7

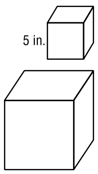
NAME ______________________ DATE __________ PERIOD _____

12-7 Enrichment

Doubling Sizes

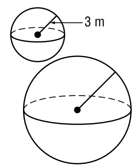
Consider what happens to surface area when the sides of a figure are doubled.

The sides of the large cube are twice the size of the sides of the small cube.



1. How long are the edges of the large cube? **6 in.**
2. What is the surface area of the small cube? **54 in^2**
3. What is the surface area of the large cube? **216 in^2**
4. The surface area of the large cube is how many times greater than that of the small cube? **4 times**

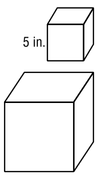
The radius of the large sphere at the right is twice the radius of the small sphere.



5. What is the surface area of the small sphere? **400π m^2**
6. What is the surface area of the large sphere? **1600π m^2**
7. The surface area of the large sphere is how many times greater than the surface area of the small sphere? **4 times**
8. It appears that if the dimensions of a solid are doubled, the surface area is multiplied by ______. **4**

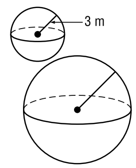
Now consider how doubling the dimensions affects the volume of a cube.

The sides of the large cube are twice the size of the small cube.



9. How long are the edges of the large cube? **10 in.**
10. What is the volume of the small cube? **125 in^3**
11. What is the volume of the large cube? **1000 in^3**
12. The volume of the large cube is how many times greater than that of the small cube? **8 times**

The large sphere at the right has twice the radius of the small sphere.



13. What is the volume of the small sphere? **36π m^3**
14. What is the volume of the large sphere? **288π m^3**
15. The volume of the large sphere is how many times greater than the volume of the small sphere? **8 times**
16. It appears that if the dimensions of a solid are doubled, the volume is multiplied by ______. **8**

Chapter 12 Assessment Answer Key

Form 1

Page 703

1. C
2. A
3. B
4. D
5. B
6. D
7. B
8. D
9. C
10. D

Page 704

11. C
12. A
13. B
14. A
15. B
16. C
17. C
18. B
19. D
20. A

B: $1020\ ft^2$

Form 2A

Page 705

1. D
2. C
3. D
4. C
5. A
6. C
7. B
8. B
9. C
10. D

(continued on the next page)

Answers

Chapter 12 Assessment Answer Key

Form 2A *(continued)*
Page 706

11. A
12. C
13. D
14. A
15. B
16. B
17. C
18. B
19. A
20. A

B: 391.6 ft^2

Form 2B
Page 707

1. B
2. D
3. A
4. B
5. D
6. C
7. B
8. B
9. D
10. B

Page 708

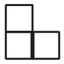
11. C
12. A
13. C
14. C
15. D
16. A
17. B
18. B
19. D
20. C

B: 53 ft^2

Chapter 12 Assessment Answer Key

Form 2C

Page 709

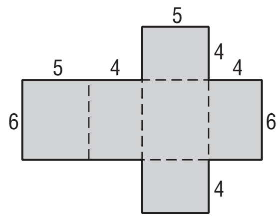
1. 

2. ▱*PQRS*, △*PQT*, △*QTR*, △*RTS*, △*PTS*

3. hexagonal pyramid

4. 12

5. Sample answer:



6. 240 in^2

7. 120 cm^2

8. 3536 $units^2$

9. 7.2 in^2

10. 7 gal

Page 710

11. 524.1 yd^2

12. 6 in.

13. 270 in^2

14. $270 + 150\sqrt{3}$ in^2

15. 144 $units^2$

16. 54.5 cm

17. 62.8 ft^2

18. 113.1 ft^2

19. 17 m

20. 235.6 cm^2

B: 38.5 in^2

Answers

Chapter 12 Assessment Answer Key

Form 2D

Page 711

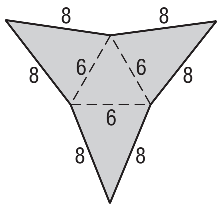
1. 

2. $\overline{GJ}$, $\overline{HJ}$, $\overline{IJ}$, $\overline{GH}$, $\overline{HI}$, $\overline{GI}$

3. pentagonal prism

4. 12

5. Sample answer:



6. 840 cm^2

7. 750 in^2

8. 258 ft^2

9. 3 gal

10. 180 in^2

Page 712

11. 845.2 m^2

12. 7 ft

13. 540 ft^2

14. 931.0 ft^2

15. 156.9 $units^2$

16. 22.4 ft

17. 204.2 cm^2

18. 282.7 cm^2

19. 5 m

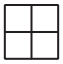
20. 1140.4 in^2

B: 113.1 in^2

Chapter 12 Assessment Answer Key

Form 3

Page 713

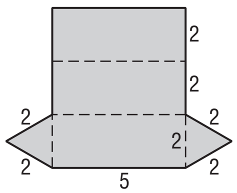
1. 

2. $\triangle XYZ$, $\triangle UVW$

3. octahedron

4. 20

5. Sample answer:



6. $128 + 4\sqrt{21}$ units^2

7. $35 + 7\sqrt{13}$ ft^2

8. 528 units^2

9. 17,837.8 ft^2

10. 32,435.2 ft^2

11. It is multiplied by 4.

Page 714

12. 468 in^2

13. 842.1 in^2

14. It is multiplied by 9.

15. 233.8 in^2

16. 387.7 in^2

17. 182.0 cm^2

18. $2\sqrt{21}$

19. $3\pi r^2$

20. 1709 ft^2

B: 700 ft^2

Chapter 12 Assessment Answer Key

Page 715, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *pyramids*, *prisms*, *cylinders*, *cones*, *spheres*, *surface area*, *lateral area*, *nets*, *and properties of solid figures*. • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Figures and drawings are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *pyramids*, *prisms*, *cylinders*, *cones*, *spheres*, *surface area*, *lateral area*, *nets*, *and properties of solid figures*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Figures and drawings are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *pyramids*, *prisms*, *cylinders*, *cones*, *spheres*, *surface area*, *lateral area*, *nets*, *and properties of solid figures*. • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Figures and drawings are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work shown to substantiate the final computation. • Figures and drawings may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *pyramids*, *prisms*, *cylinders*, *cones*, *spheres*, *surface area*, *lateral area*, *nets*, *and properties of solid figures*. • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Figures and drawings are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer given.

Chapter 12 Assessment Answer Key

Page 715, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A28, the following sample answers may be used as guidance in evaluating open-ended assessment items.

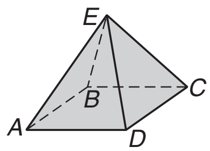
1. a.

Figure	No. of Edges (e)	No. of Faces (f)	No. of Vertices (v)	$f + v$
Triangular Pyramid	6	4	4	8
Triangular Prism	9	5	6	11
Cube	12	6	8	14
Square Pyramid	8	5	5	10
Hexagonal Prism	18	8	12	20
Hexagonal Pyramid	12	7	7	14

b. $e = f + v - 2$

2. The lateral area is the area of the lateral faces. The surface area includes the area of the lateral faces plus the areas of the two bases.

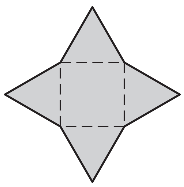
3. a.



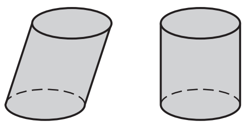
b. $\square ABCD$

c. $\triangle ABE$, $\triangle BCE$, $\triangle CDE$, $\triangle ADE$

d.

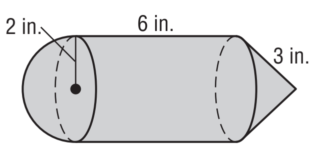


4.



Oblique **Right**

5. a.



b. 38π in^2

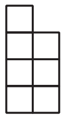
6. Sample answer: Sam is painting the walls of a room. The room is 12 feet long, 10 feet wide, and 8 feet high. A gallon of paint covers 400 square feet and costs $16 per gallon. Find the cost to paint the room.

Chapter 12 Assessment Answer Key

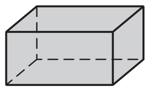
Vocabulary Test/Review
Page 716

1. slant height
2. cone
3. sphere
4. perspective view
5. Platonic Solids
6. prism
7. pyramid
8. surface area
9. right cylinder
10. hemisphere
11. the sum of the areas of the lateral faces
12. a prism whose lateral edges are also altitudes
13. the intersection of a sphere and a plane containing the center of the sphere

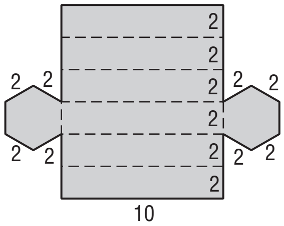
Quiz 1
Page 717

1. 

2. Sample answer:



3. Sample answer:



4. D

Quiz 2
Page 717

1. 2 in.
2. 12 units
3. rectangle
4. 3870.4 units^2
5. 4630.7 units^2

Quiz 3
Page 718

1. 81 units^2
2. 116.1 units^2
3. 260 cm^2
4. 188.5 ft^2
5. 267.0 ft^2

Quiz 4
Page 718

1. $\overline{BC}$
2. $\overleftrightarrow{AH}$
3. $\odot D$
4. 4071.5 in^2
5. It is multiplied by 4.

Chapter 12 Assessment Answer Key

Mid-Chapter Test
Page 719

Part I

1. C

2. A

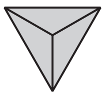
3. D

4. A

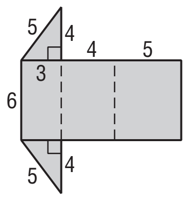
5. D

Part II

6.



7. Sample answer:



8. 1656 $units^2$

9. 92.8 $units^2$

10. 282.7 in^2

Cumulative Review
Page 720

1. $\angle AFD$ and $\angle BFC$

2. false

3. It alternates between $\frac{1}{2}$ and 2.

4. $m\angle X \approx 63$, $m\angle Z \approx 44$, $y \approx 34$ cm

5. 27.6 in. and 23.1 in.

6. magnitude: 11.7, direction: 149.0°

7. circle

8. 28

9. 80 cm^2; 48 cm

10. 14.5 m

11. 5.0 in.

Answers

Chapter 12 Assessment Answer Key

Standardized Test Practice

Page 721

1. A **B** C D

2. **E** F G H

3. A B C **D**

4. E F G **H**

5. **A** B C D

6. E **F** G H

7. A B **C** D

8. **E** F G H

Page 722

9. | 1 | 4 | | |
|---|---|---|---|

10. | 5 | | | |
|---|---|---|---|

11. 2.6 m

12. 25

13. 51.4 cm^2

14. pentagonal pyramid

15. 3141.6 in^2

16. 2206.2 cm^2