GLENCOE MATHEMATICS

Geometry

Chapter 11 Resource Masters

McGraw Hill Glencoe

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Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 11 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Send all inquiries to:
The McGraw-Hill Companies
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Geometry
Chapter 11 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 11 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 11 Resource Masters* includes the core materials needed for Chapter 11. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 11-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 11-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 11 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 632–633. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

11 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 11. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
apothem		
geometric probability		
irregular figure		
irregular polygon		

(continued on the next page)

11 Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
sector of a circle		
segment of a circle		

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11 Learning to Read Mathematics

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 11. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Postulate 11.1		
Postulate 11.2		
Postulate 11.3 *Area Probability Postulate*		

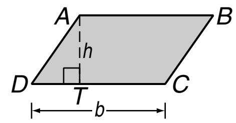
Proof Builder

11-1 Study Guide and Intervention

Areas of Parallelograms

Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

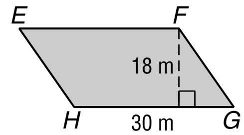
Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram *ABCD* is $CD \cdot AT$.

Lesson 11-1

Example **Find the area of parallelogram *EFGH*.**



$A = bh$ Area of a parallelogram

$= 30(18)$ $b = 30$, $h = 18$

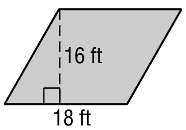
$= 540$ Multiply.

The area is 540 square meters.

Exercises

Find the area of each parallelogram.

1.



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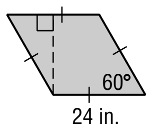

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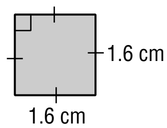
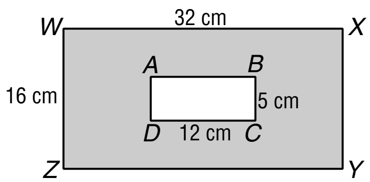
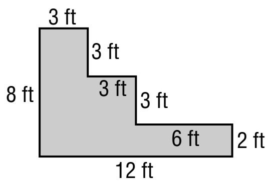


Find the area of each shaded region.

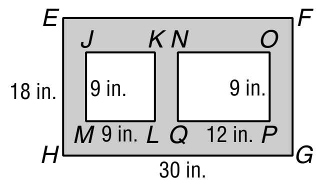
4. *WXYZ* and *ABCD* are rectangles.



5. All angles are right angles.



6. *EFGH* and *NOPQ* are rectangles; *JKLM* is a square.



7. The area of a parallelogram is 3.36 square feet. The base is 2.8 feet. If the measures of the base and height are each doubled, find the area of the resulting parallelogram.

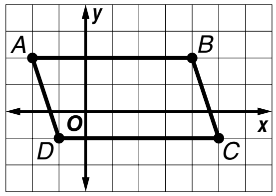
8. A rectangle is 4 meters longer than it is wide. The area of the rectangle is 252 square meters. Find the length.

11-1 Study Guide and Intervention *(continued)*

Areas of Parallelograms

Parallelograms on the Coordinate Plane To find the area of a quadrilateral on the coordinate plane, use the Slope Formula, the Distance Formula, and properties of parallelograms, rectangles, squares, and rhombi.

Example **The vertices of a quadrilateral are $A(-2, 2)$, $B(4, 2)$, $C(5, -1)$, and $D(-1, -1)$.**



a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

Graph the quadrilateral. Then determine the slope of each side.

slope of $\overline{AB} = \frac{2 - 2}{4 - (-2)}$ or 0

slope of $\overline{CD} = \frac{-1 - (-1)}{-1 - 5}$ or 0

slope $\overline{AD} = \frac{2 - (-1)}{-2 - (-1)}$ or -3

slope $\overline{BC} = \frac{-1 - 2}{5 - 4}$ or -3

Opposite sides have the same slope. The slopes of consecutive sides are not negative reciprocals of each other, so consecutive sides are not perpendicular. *ABCD* is a parallelogram; it is not a rectangle or a square.

b. Find the area of *ABCD*.

From the graph, the height of the parallelogram is 3 units and $AB = |4 - (-2)| = 6$.

$A = bh$ Area of a parallelogram

$= 6(3)$ $b = 6, h = 3$

$= 18 \text{ units}^2$ Multiply.

Exercises

Given the coordinates of the vertices of a quadrilateral, determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*. Then find the area.

1. $A(-1, 2)$, $B(3, 2)$, $C(3, -2)$, and $D(-1, -2)$

2. $R(-1, 2)$, $S(5, 0)$, $T(4, -3)$, and $U(-2, -1)$

3. $C(-2, 3)$, $D(3, 3)$, $E(5, 0)$, and $F(0, 0)$

4. $A(-2, -2)$, $B(0, 2)$, $C(4, 0)$, and $D(2, -4)$

5. $M(2, 3)$, $N(4, -1)$, $P(-2, -1)$, and $R(-4, 3)$

6. $D(2, 1)$, $E(2, -4)$, $F(-1, -4)$, and $G(-1, 1)$

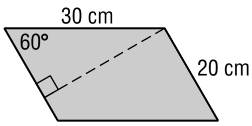
NAME ______________________________ DATE ____________ PERIOD _____

11-1 Skills Practice

Area of Parallelograms

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.



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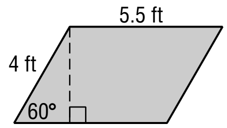

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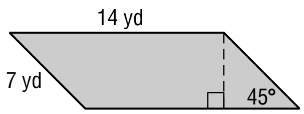

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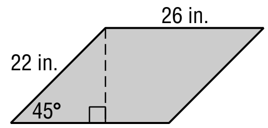

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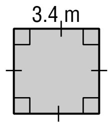
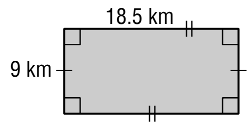
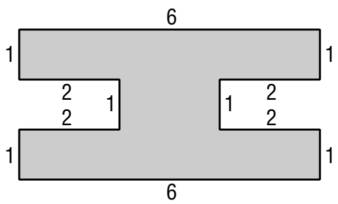


6.

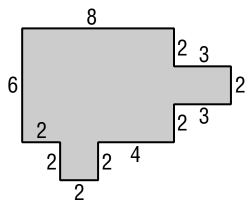


Find the area of each figure.

7.



8.



COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

9. $A(-4, 2)$, $B(-1, 2)$, $C(-1, -1)$, $D(-4, -1)$

10. $P(-3, 3)$, $Q(1, 3)$, $R(1, -3)$, $S(-3, -3)$

11. $D(-5, 1)$, $E(7, 1)$, $F(4, -4)$, $G(-8, -4)$

12. $R(2, 3)$, $S(4, 10)$, $T(12, 10)$, $U(10, 3)$

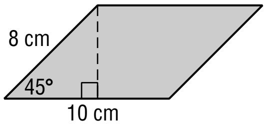
11-1 Practice

Area of Parallelograms

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1.

2.



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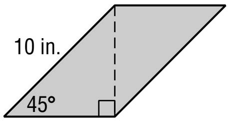
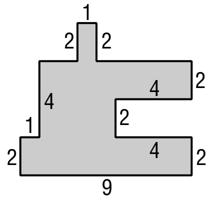


Find the area of each figure.

4.



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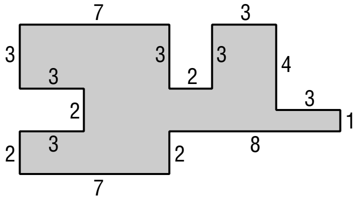

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

6. $C(-4, -1)$, $D(-4, 2)$, $F(1, 2)$, $G(1, -1)$

7. $W(2, 2)$, $X(1, -2)$, $Y(-2, -2)$, $Z(-1, 2)$

8. $M(0, 4)$, $N(4, 6)$, $O(6, 2)$, $P(2, 0)$

9. $P(-5, 2)$, $Q(4, 2)$, $R(5, 5)$, $S(-4, 5)$

FRAMING For Exercises 10–12, use the following information.

A rectangular poster measures 42 inches by 26 inches. A frame shop fitted the poster with a half-inch mat border.

10. Find the area of the poster.

11. Find the area of the mat border.

12. Suppose the wall is marked where the poster will hang. The marked area includes an additional 12-inch space around the poster and frame. Find the total wall area that has been marked for the poster.

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11-1 Reading to Learn Mathematics

Areas of Parallelograms

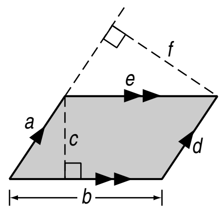
Pre-Activity **How is area related to garden design?**

Read the introduction to Lesson 11-1 at the top of page 595 in your textbook.

How could you describe the pattern you see in the picture of the garden so that someone who doesn't have the picture will know what it looks like?

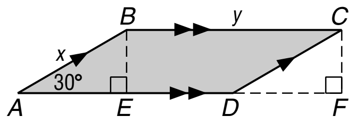
Lesson 11-1

Reading the Lesson



1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.)

A. *ab*	**B.** *cb*	**C.** *ed*
D. *af*	**E.** *ce*	**F.** *cd*
G. *df*	**H.** *bf*	**I.** *cf*



2. Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.

a. $\overline{AB}$ is an altitude of the parallelogram.

b. $\overline{CD}$ is a base of parallelogram *ABCD*.

c. The perimeter of *ABCD* is $(2x + 2y)$ units^2.

d. $BE = CF$

e. $BE = \frac{\sqrt{3}}{2}x$

f. The area of *ABCD* is $2xy$ units^2.

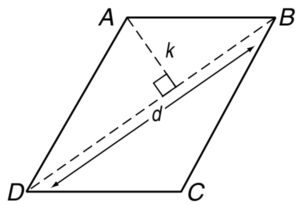
Helping You Remember

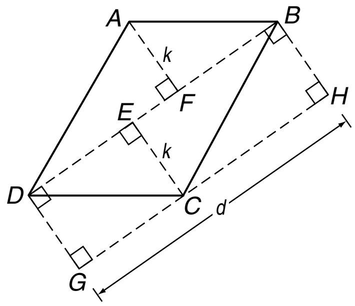
3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?

11-1 Enrichment

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal $\overline{BD}$, and k is the length of the perpendicular segment from A to $\overline{BD}$. Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.





1. What kind of figure is *DBHG*?

2. If you moved △*AFB* to the lower-left end of figure *DBHG*, would it fit perfectly on top of △*DGC*? Explain your answer.

3. Which two triangular pieces of ▱*ABCD* are congruent to △*CBH*?

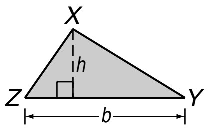
4. The area of ▱*ABCD* is the same as that of figure *DBHG*, since the pieces of ▱*ABCD* can be rearranged to form *DBHG*. Express the area of ▱*ABCD* in terms of the measurements k and d.

11-2 Study Guide and Intervention

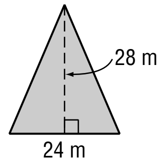
Areas of Triangles, Trapezoids, and Rhombi

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



Example **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(28)$ $b = 24$, $h = 28$

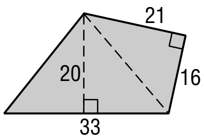
$= 336$ Multiply.

The area is 336 square meters.

Exercises

Find the area of each figure.

1.



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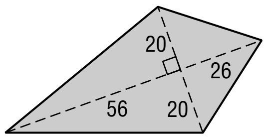

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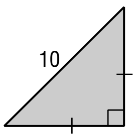

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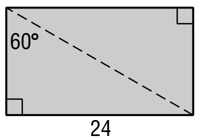

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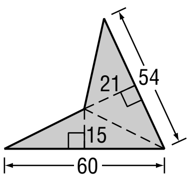
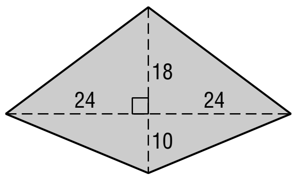


6.



7. The area of a triangle is 72 square inches. If the height is 8 inches, find the length of the base.

8. A right triangle has a perimeter of 36 meters, a hypotenuse of 15 meters, and a leg of 9 meters. Find the area of the triangle.

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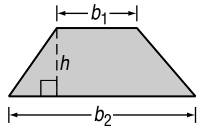
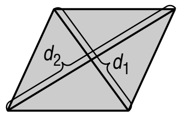
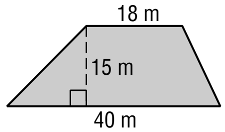
11-2 Study Guide and Intervention *(continued)*

Areas of Triangles, Trapezoids, and Rhombi

Areas of Trapezoids and Rhombi The area of a trapezoid is the product of half the height and the sum of the lengths of the bases. The area of a rhombus is half the product of the diagonals.

If a trapezoid has an area of A square units, bases of b_1 and b_2 units, and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$. 	If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$. 

Example **Find the area of the trapezoid.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

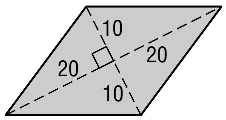
$= \frac{1}{2}(15)(18 + 40)$ $h = 15$, $b_1 = 18$, $b_2 = 40$

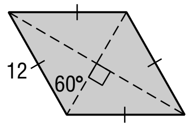
$= 435$ Simplify.

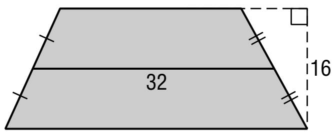
The area is 435 square meters.

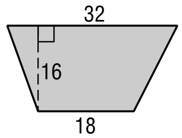
Exercises

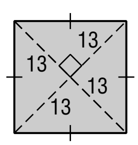
Find the area of each quadrilateral given the coordinates of the vertices.

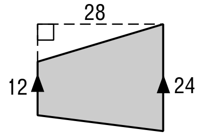
1. 

2. 

3. 

4. 

5. 

6. 

7. The area of a trapezoid is 144 square inches. If the height is 12 inches, find the length of the median.

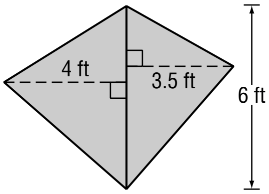
8. A rhombus has a perimeter of 80 meters and the length of one diagonal is 24 meters. Find the area of the rhombus.

11-2 Skills Practice

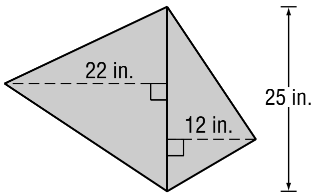
Areas of Triangles, Trapezoids, and Rhombi

Find the area of each figure. Round to the nearest tenth if necessary.

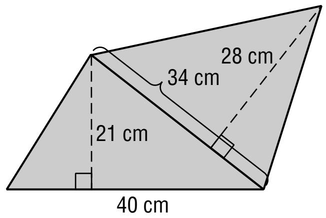
1.



2.



3.



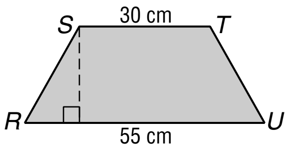
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *WXYZ*
W(−5, 3), *X*(3, 3), *Y*(6, −3), *Z*(−8, −3)

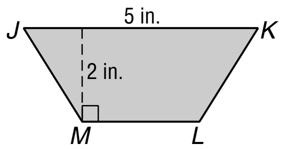
5. rhombus *HIJK*
H(4, −3), *I*(2, −7), *J*(0, −3), *K*(2, 1)

Find the missing measure for each figure.

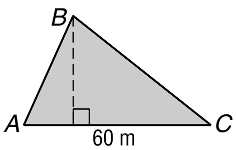
6. Trapezoid *RSTU* has an area of 935 square centimeters. Find the height of *RSTU*.



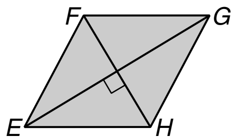
7. Trapezoid *JKLM* has an area of 7.5 square inches. Find *ML*.



8. Triangle *ABC* has an area of 1050 square meters. Find the height of △*ABC*.



9. Rhombus *EFGH* has an area of 750 square feet. If *EG* is 50 feet, find *FH*.



Lesson 11-2

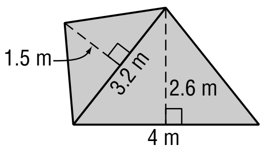
NAME ____ DATE ____________ PERIOD _____

11-2 Practice

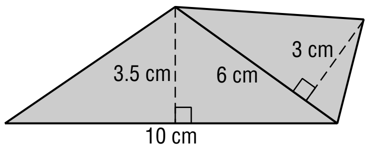
Areas of Triangles, Trapezoids, and Rhombi

Find the area of each figure. Round to the nearest tenth if necessary.

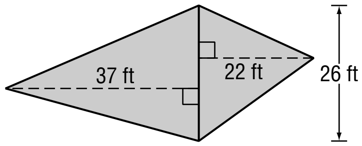
1.



2.



3.



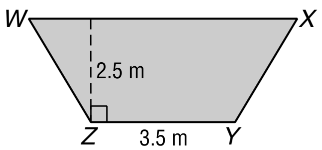
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *ABCD*
$A(-7, 1)$, $B(-4, 4)$, $C(-4, -6)$,
$D(-7, -3)$

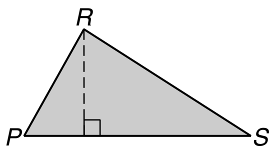
5. rhombus *LMNO*
$L(6, 8)$, $M(14, 4)$, $N(6, 0)$,
$O(-2, 4)$

Find the missing measure for each figure.

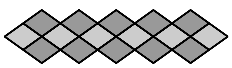
6. Trapezoid *WXYZ* has an area of 13.75 square meters. Find *WX*.



7. Triangle *PRS* has an area of 68 square yards. If the height of △*PRS* is 8 yards, find the base.



DESIGN For Exercises 8 and 9, use the following information.
Mr. Hagarty used 16 congruent rhombi-shaped tiles to design the midsection of the backsplash area above a kitchen sink. The length of the design is 27 inches and the total area is 108 square inches.



8. Find the area of one rhombus.

9. Find the length of each diagonal.

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11-2 Reading to Learn Mathematics

Areas of Triangles, Trapezoids, and Rhombi

Pre-Activity **How is the area of a triangle related to beach umbrellas?**

Read the introduction to Lesson 11-2 at the top of page 601 in your textbook.

Classify the polygons in the panels of the beach umbrella.

Reading the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a. $A = \ell w$	**i.** triangle
b. $A = \frac{1}{2}d_1d_2$	**ii.** parallelogram
c. $A = s^2$	**iii.** trapezoid
d. $A = \frac{1}{2}h(b_1 + b_2)$	**iv.** rhombus
e. $A = \frac{1}{2}bh$	**v.** square
f. $A = bh$	**vi.** rectangle

2. Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning.

a. The area of a square is half the product of its diagonals.

b. The area of a triangle is half the product of two of its sides.

c. You can find the area of a rectangle by multiplying base times height.

d. You can find the area of a rectangle by multiplying the lengths of any two of its sides.

e. The area of a trapezoid is the product of its height and the sum of the bases.

f. The square of the length of a side of a square is equal to half the product of its diagonals.

Helping You Remember

3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember.

Lesson 11-2

11-2 Enrichment

Areas of Similar Triangles

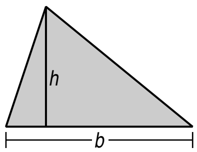
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

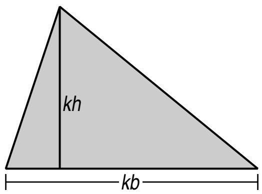
$$\frac{\text{side of } \triangle \text{II}}{\text{side of } \triangle \text{I}} = \frac{kb}{b} \text{ or } \frac{k}{1}$$

$$\frac{\text{area of } \triangle \text{II}}{\text{area of } \triangle \text{I}} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh} \text{ or } \frac{k^2}{1}$$



Triangle I

area $\triangle \text{I} = \frac{1}{2}bh$



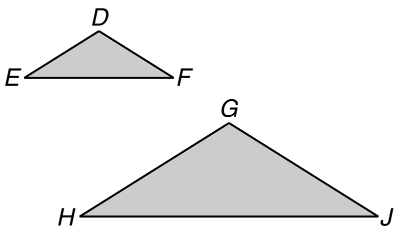
Triangle II

area $\triangle \text{II} = \frac{1}{2}(kb)(kh)$

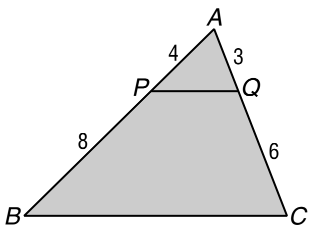
$= \frac{1}{2}k^2bh$

Solve.

1. $\triangle DEF \sim \triangle GHJ$, $HJ = 16$, and $EF = 8$. The area of $\triangle GHJ$ is 40. Find the area of $\triangle DEF$.



2. In the figure below, $\overline{PQ} \parallel \overline{BC}$. The area of $\triangle ABC$ is 72. Find the area of $\triangle APQ$.



3. Two similar triangles have areas of 16 and 36. The length of a side of the smaller triangle is 10 feet. Find the length of the corresponding side of the larger triangle.

4. Find the ratio of the areas of two similar triangles if the lengths of two corresponding sides of the triangles are 3 centimeters and 5 centimeters.

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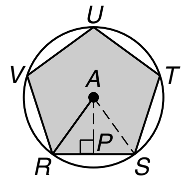
11-3 Study Guide and Intervention

Areas of Regular Polygons and Circles

Areas of Regular Polygons

In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is $A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then $A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon *RSTUV* above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

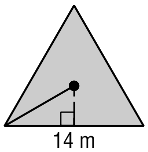
$$\tan \angle RAP = \frac{RP}{AP}$$
$$\tan 36° = \frac{6}{AP}$$
$$AP = \frac{6}{\tan 36°}$$
$$\approx 8.26$$

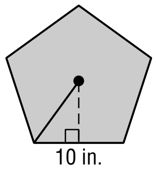
So $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.7.

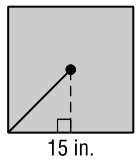
The area is about 248 square centimeters.

Exercises

Find the area of each regular polygon. Round to the nearest tenth.

1.


2.


3.


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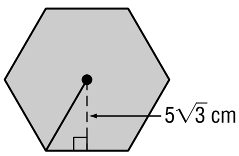
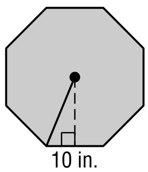


5.


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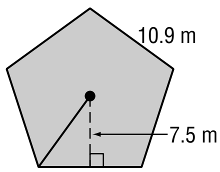
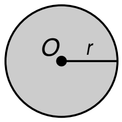


Lesson 11-3

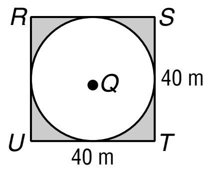
11-3 Study Guide and Intervention *(continued)*

Areas of Regular Polygons and Circles

Areas of Circles As the number of sides of a regular polygon increases, the polygon gets closer and closer to a circle and the area of the polygon gets closer to the area of a circle.

Area of a Circle	If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$. 

Example **Circle Q is inscribed in square $RSTU$. Find the area of the shaded region.**



A side of the square is 40 meters, so the radius of the circle is 20 meters.

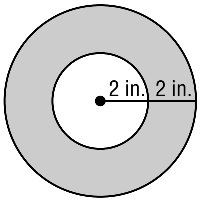
The shaded area is
Area of $RSTU$ − Area of circle Q

$= 40^2 - \pi r^2$
$= 1600 - 400\pi$
$\approx 1600 - 1256.6$
$= 343.4 \text{ m}^2$

Exercises

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

1.



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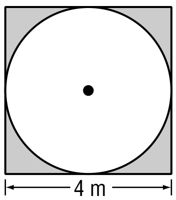
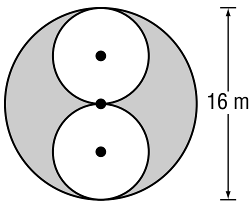


3.



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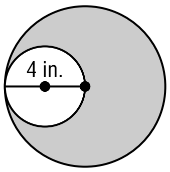

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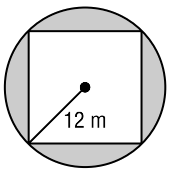
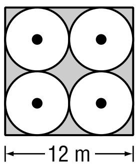


6.



11-3 Skills Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a pentagon with a perimeter of 45 feet

2. a hexagon with a side length of 4 inches

3. a nonagon with a side length of 8 meters

4. a triangle with a perimeter of 54 centimeters

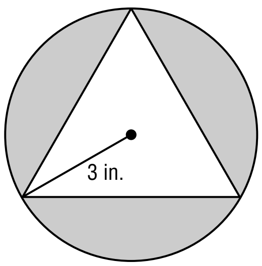
Find the area of each circle. Round to the nearest tenth.

5. a circle with a radius of 6 yards

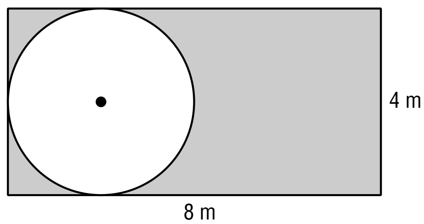
6. a circle with a diameter of 18 millimeters

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

7.



8.



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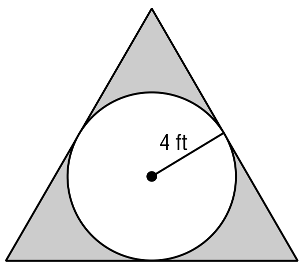

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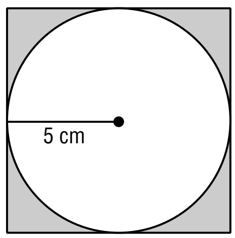


11-3 Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a nonagon with a perimeter of 117 millimeters

2. an octagon with a perimeter of 96 yards

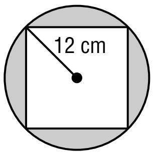
Find the area of each circle. Round to the nearest tenth.

3. a circle with a diameter of 26 feet

4. a circle with a circumference of 88 kilometers

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

5.



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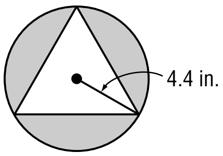

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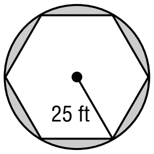

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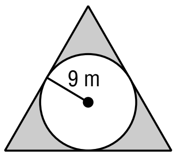

DISPLAYS For Exercises 9 and 10, use the following information.

A display case in a jewelry store has a base in the shape of a regular octagon. The length of each side of the base is 10 inches. The owners of the store plan to cover the base in black velvet.

9. Find the area of the base of the display case.

10. Find the number of square yards of fabric needed to cover the base.

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11-3 Reading to Learn Mathematics

Areas of Regular Polygons and Circles

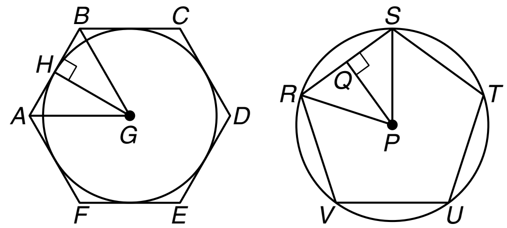
Pre-Activity **How can you find the area of a polygon?**

Read the introduction to Lesson 11-3 at the top of page 610 in your textbook.

How can you find the area of a regular hexagon without a new area formula?

Reading the Lesson

1. *ABCDEF* and *RSTUV* are regular polygons. Name each of the following in one of the figures.



a. a circumscribed polygon

b. an inscribed polygon

c. an apothem of a regular hexagon

d. an isosceles triangle

e. a 30°-60°-90° triangle

f. a central angle with a measure of 72

2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.

a. perimeter of *ABCDEF*	**i.** $\pi(PS)^2$
b. circumference of circle *G*	**ii.** $2\pi(PR)$
c. perimeter of *RSTUV*	**iii.** $\frac{5}{2}(RS)(PQ)$
d. area of circle *G*	**iv.** $3(AB)(HG)$
e. area of *RSTUV*	**v.** $6(CD)$
f. area of *ABCDEF*	**vi.** $\pi(GH)^2$
g. area of circle *P*	**vii.** $5(UV)$
h. circumference of circle *P*	**viii.** $2\pi(GH)$

3. Explain in your own words how to find the area of a circle if you know the circumference.

Helping You Remember

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her?

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11-3 Enrichment

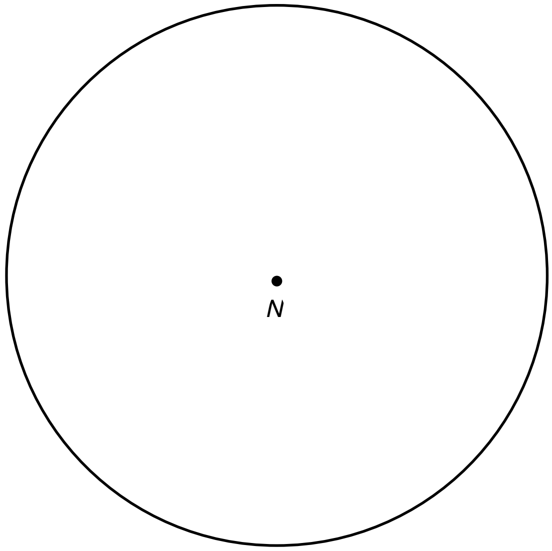
Areas of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙*N*.

Step 1 Find the degree measure of each of the nine congruent arcs.

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon?

2. Measure the distance from the center perpendicular to one of the sides of the nonagon.

3. What is the area of one of the nine triangles formed?

4. What is the area of the nonagon?

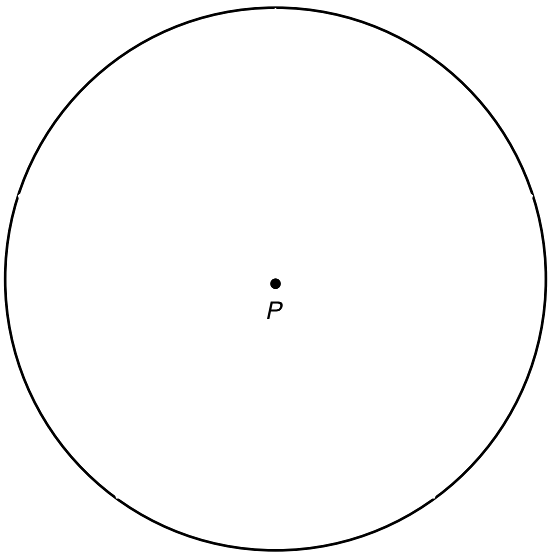
Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in ⊙*P*. Answer each of the following.

5. Use a protractor to inscribe a regular pentagon in ⊙*P*.

6. What is the measure of each of the five congruent arcs?

7. What is the perimeter of the pentagon to the nearest tenth of a centimeter?

8. What is the area of the pentagon to the nearest tenth of a centimeter?



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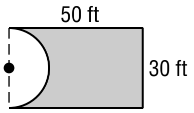
11-4 Study Guide and Intervention

Areas of Irregular Figures

Irregular Figures An **irregular** figure is one that cannot be classified as one of the previously-studied shapes. To find the area of an irregular figure, break it into familiar shapes. Find the area of each shape and add the areas.

Example 1 **Find the area of the irregular figure.**



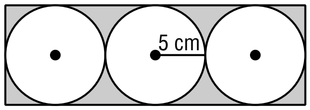
The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$A = lw - \frac{1}{2}\pi r^2$

$\approx 50(30) - 0.5(3.14)(15)^2$

$= 1146.6$ or about 1147 ft^2

Example 2 **Find the area of the shaded region.**



The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

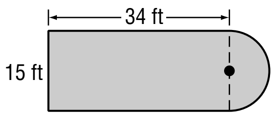
$(10)(30) - 3\pi(5^2) = 300 - 75\pi$

$\approx 64.4\text{ cm}^2$

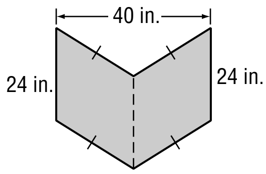
Exercises

Find the area of each figure. Round to the nearest tenth if necessary.

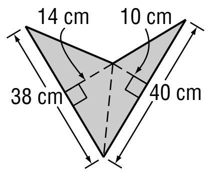
1.



2.



3.



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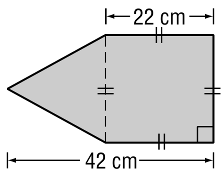
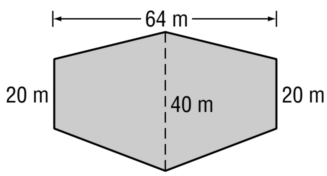
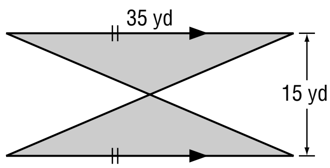


5.



6.



7. Refer to Example 2 above. Draw the largest possible square inside each of the three circles. What is the total area of the three squares?

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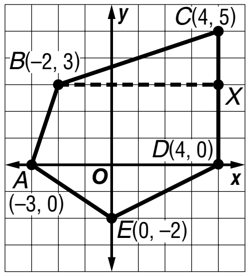
11-4 Study Guide and Intervention *(continued)*

Areas of Irregular Figures

Irregular Figures on the Coordinate Plane To find the area of an irregular figure on the coordinate plane, break up the figure into known figures. You may need to use the Distance Formula to find some of the dimensions.

Example **Find the area of irregular pentagon *ABCDE*.**



Draw $\overline{BX}$ between $B(-2, 3)$ and $X(4, 3)$ and draw $\overline{AD}$. The area of $ABCDE$ is the sum of the areas of $\triangle BCX$, trapezoid $BXDA$, and $\triangle ADE$.

A = area of $\triangle BCX$ + area of $BXDA$ + area of $\triangle ADE$

$$= \frac{1}{2}(2)(6) + \frac{1}{2}(3)(6 + 7) + \frac{1}{2}(2)(7)$$

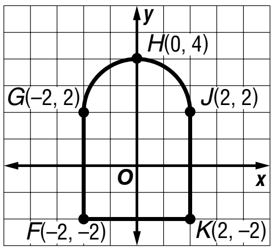
$$= 6 + \frac{39}{2} + 7$$

$= 32.5$ square units

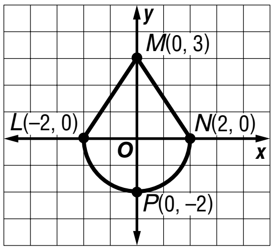
Exercises

Find the area of each figure. Round to the nearest tenth.

1.



2.



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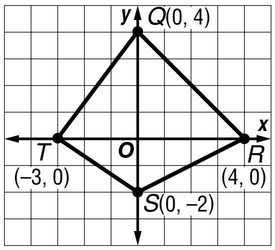
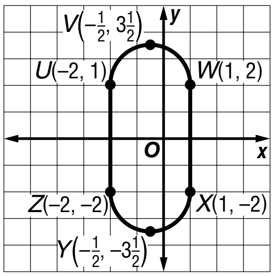


4.



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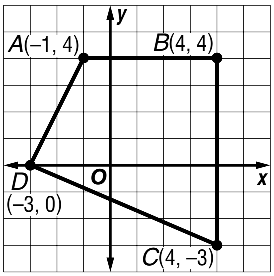

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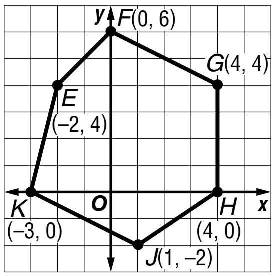
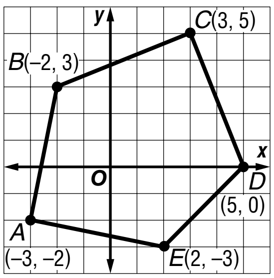
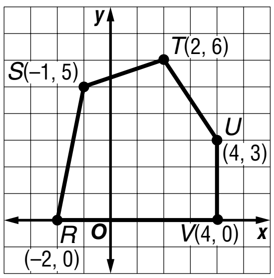


7. pentagon *ABCDE*



8. pentagon *RSTUV*



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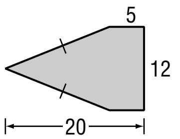
11-4 Skills Practice

Areas of Irregular Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1.



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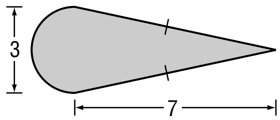
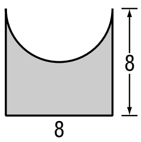


3.



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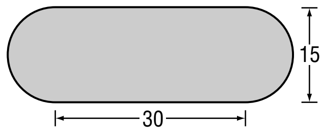

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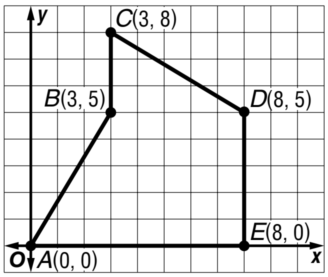
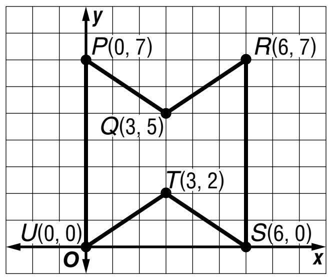


6.



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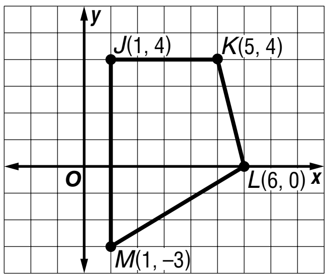

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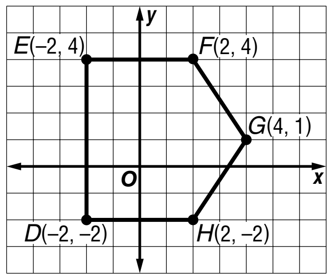
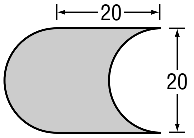


11-4 Practice

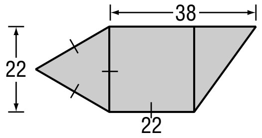
Areas of Irregular Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1.



2.



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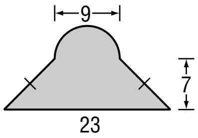

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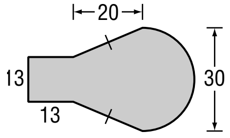

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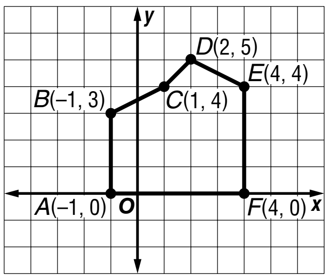
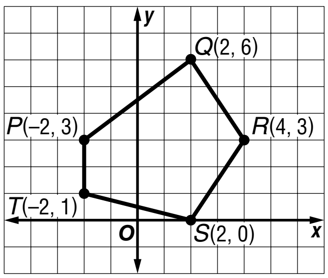
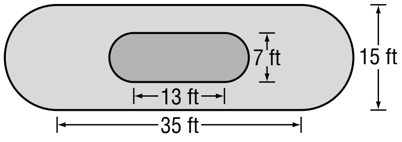


6.



LANDSCAPING For Exercises 7 and 8, use the following information.
One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.



7. Find the area of the pond to the nearest tenth.

8. Find the area of the walkway to the nearest tenth.

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11-4 Reading to Learn Mathematics

Areas of Irregular Figures

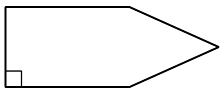
Pre-Activity **How do windsurfers use area?**

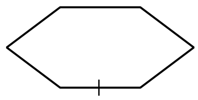
Read the introduction to Lesson 11-4 at the top of page 617 in your textbook.

How do you think the areas of the figures outlined in the picture of the sail are related?

Reading the Lesson

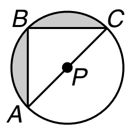
1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other.

a. 

b. 

c. 

2. In the figure, B is the midpoint of $\widehat{ABC}$. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is ________.

$m\angle ABC =$ ________ because

____.

$m\widehat{AB} = m\widehat{BC}$ because

____.

$\overline{AB} \cong \overline{BC}$ because ____

____.

Therefore, $\triangle ABC$ is a(n) ______________________ triangle.

$AC =$ __________, so $AB =$ __________ and $BC =$ __________.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot$ __________ $\cdot$ __________ $=$ __________.

Therefore, the area of the shaded region is given by

$A =$ __________ $-$ __________ $=$ __________.

Helping You Remember

3. Rolando is having trouble remembering when to subtract an area when finding the area of an irregular figure. How can you help him remember?

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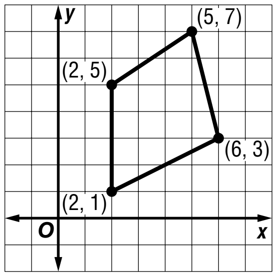
11-4 Enrichment

Aerial Surveyors and Area

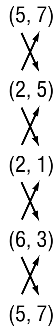
Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).

$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75



Step 3 Find U, the sum of the upward diagonal products (from left to right).

$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3)$
$= 14 + 10 + 6 + 15$ or 45

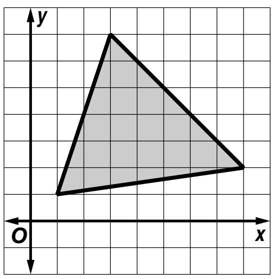
Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

$A = \frac{1}{2}(D - U)$
$= \frac{1}{2}(75 - 45)$
$= \frac{1}{2}(30)$ or 15

The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

Use the coordinate method to find the area of each region in square units.

1.



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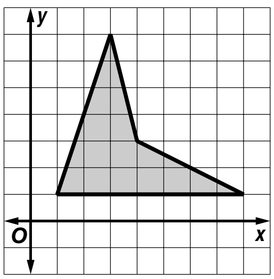

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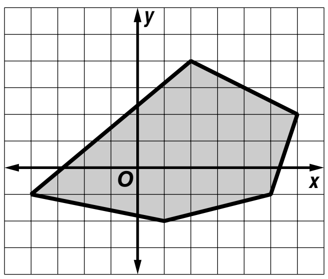

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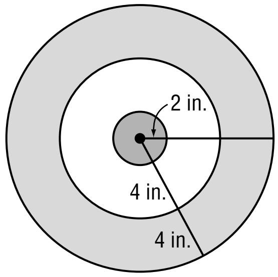
11-5 Study Guide and Intervention

Geometric Probability

Geometric Probability The probability that a point in a figure will lie in a particular part of the figure can be calculated by dividing the area of the part of the figure by the area of the entire figure. The quotient is called the **geometric probability** for the part of the figure.

> If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is
> $P(B) = \frac{\text{area of region } B}{\text{area of region } A}$.

Example **Darts are thrown at a circular dartboard. If a dart hits the board, what is the probability that the dart lands in the bull's-eye?**



Area of bull's-eye: $A = \pi(2)^2$ or 4π

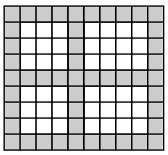
Area of entire dartboard: $A = \pi(10)^2$ or 100π

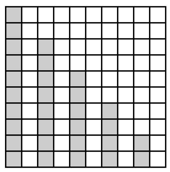
The probability of landing in the bull's-eye is

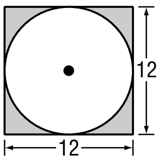
$$\frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4\pi}{100\pi}$$
$$= \frac{1}{25} \text{ or } 0.04.$$

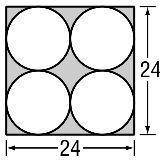
Exercises

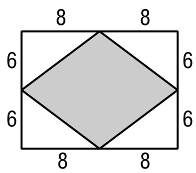
Find the probability that a point chosen at random lies in the shaded region. Round to the nearest hundredth if necessary.

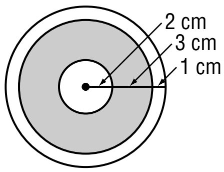
1. 

2. 

3. 

4. 

5. 

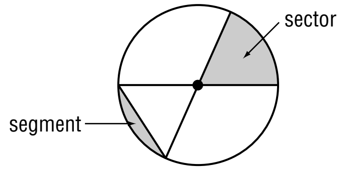
6. 

Lesson 11-5

11-5 Study Guide and Intervention *(continued)*

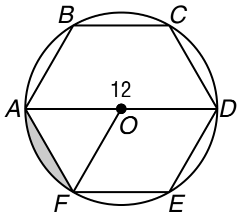
Geometric Probability

Sectors and Segments of Circles A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted arc. A **segment of a circle** is bounded by a chord and its arc. Geometric probability problems sometimes involve sectors or segments of circles.



If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.

Example **A regular hexagon is inscribed in a circle with diameter 12. Find the probability that a point chosen at random in the circle lies in the shaded region.**



The area of the shaded segment is the area of sector AOF − the area of $\triangle AOF$.

$$\text{Area of sector } AOF = \frac{N}{360}\pi r^2 = \frac{60}{360}\pi(6^2) = 6\pi$$

$$\text{Area of } \triangle AOF = \frac{1}{2}bh = \frac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3}$$

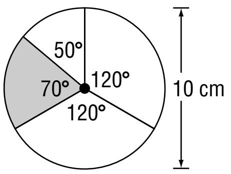
The shaded area is $6\pi - 9\sqrt{3}$ or about 3.26.

The probability is $\frac{\text{area of segment}}{\text{area of circle}} = \frac{3.26}{36\pi}$ or about 0.03.

Exercises

Find the probability that a point in the circle chosen at random lies in the shaded region. Round to the nearest hundredth.

1.



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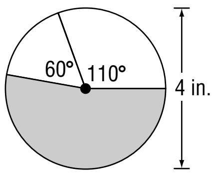
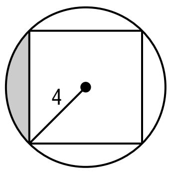


3.



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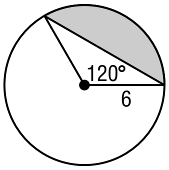

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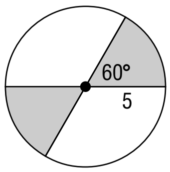
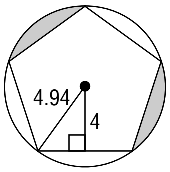


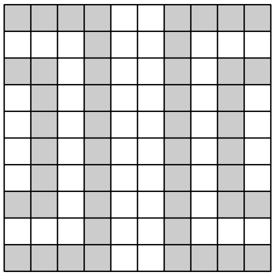
6.

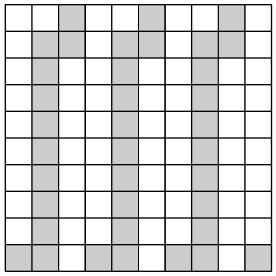


11-5 Skills Practice

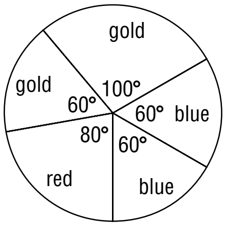
Geometric Probability

Find the probability that a point chosen at random lies in the shaded region.

1. 

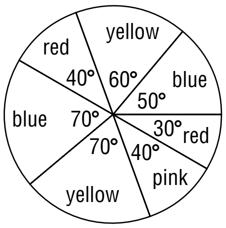
2. 

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 6 inches.



3. red

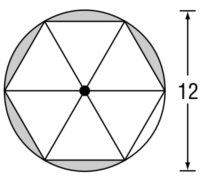
4. gold

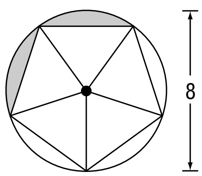


5. blue

6. yellow

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

7. 

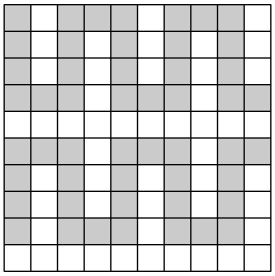
8. 

11-5 Practice

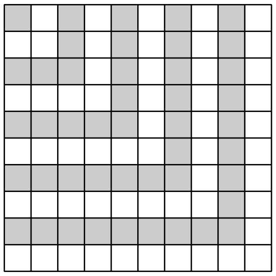
Geometric Probability

Find the probability that a point chosen at random lies in the shaded region.

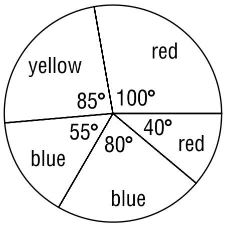
1.



2.



Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of the spinner is 9 meters.



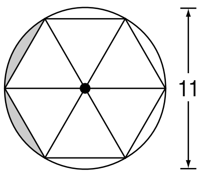
3. red

4. blue

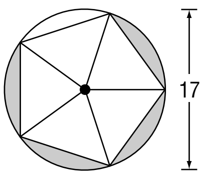
5. yellow

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

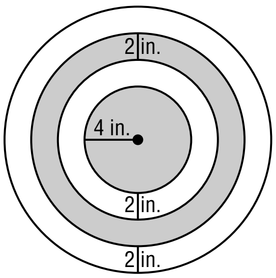
6.



7.



8. ARCHERY A target consists of four concentric rings. The radius of the center circle is 4 inches, and the circles are spaced 2 inches apart. Find the probability that an arrow shot at random by an inexperienced archer will land in a shaded region.



NAME ____ DATE ____________ PERIOD _____

11-5 Reading to Learn Mathematics

Geometric Probability

Pre-Activity **How can geometric probability help you win a game of darts?**

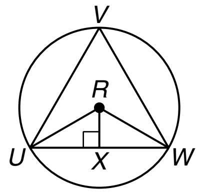
Read the introduction to Lesson 11-5 at the top of page 622 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths?

Reading the Lesson

1. Explain the difference between a sector of a circle and a segment of a circle

2. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point.



The area of circle R is ________.

$\triangle URW$ is a(n) ________________ triangle because $\overline{RU}$ and $\overline{RW}$ are ________________ of the same ________________.

$\angle URW$ is a(n) ________________ angle of the circle, and $m\angle URW =$ ________.

$m\angle RUX =$ ________ and $m\angle RWX =$ ________.

The angle measures in $\triangle RUX$ are ________, ________, and ________.

$\overline{RU}$ is a ________________ of the circle, so $RU =$ ________.

$\overline{RX}$ is the leg of $\triangle RUX$ opposite the ________ angle, so $RX =$ ________.

Also, $\overline{UX}$ is the leg of $\triangle RUX$ opposite the ________ angle, so $UX =$ ________.

$UW =$ ________, so the area of $\triangle URW$ is $\frac{1}{2} \cdot$ ________ $\cdot$ ________ $=$ ________.

Then, the area of $\triangle UVW = 3 \cdot$ ________ $=$ ________.

Therefore, the probability that the dart will fall inside the triangle is the ratio

of ________ to ________, which is approximately ________ (to the nearest thousandth).

Helping You Remember

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula?

Lesson 11-5

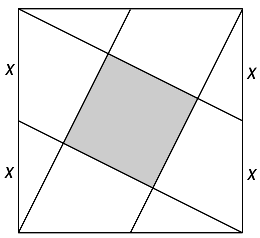
11-5 Enrichment

Polygon Probability

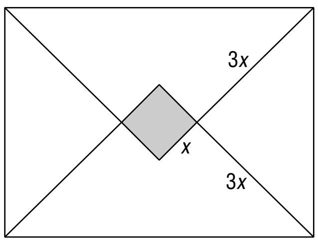
Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

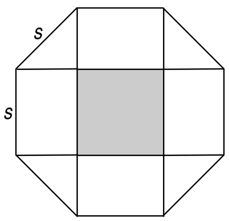
1.



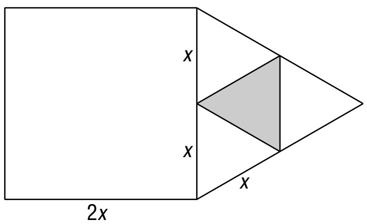
2.



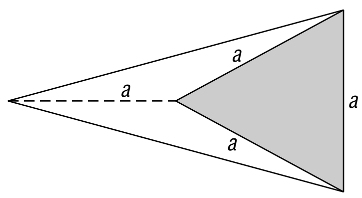
3.



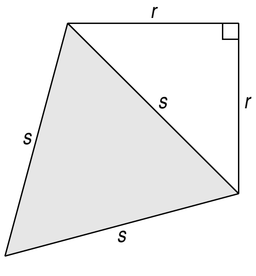
4.



5.



6.



NAME ______________________ DATE ____________ PERIOD ______

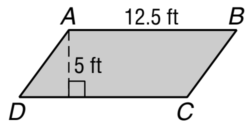
11 Chapter 11 Test, Form 1

SCORE ______

Assessments

Write the letter for the correct answer in the blank at the right of each question.

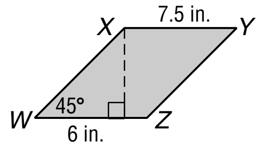
1. Find the area of parallelogram *ABCD* to the nearest tenth. **1.** ______



A. 17.5 ft^2 **B.** 31.25 ft^2

C. 35 ft^2 **D.** 62.5 ft^2

2. Find the area of parallelogram *WXYZ* to the nearest tenth. **2.** ______



A. 27 in^2 **B.** 45 in^2

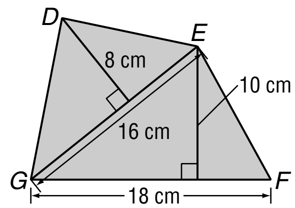
C. 63.6 in^2 **D.** 81 in^2

3. What is the best classification of quadrilateral *PQRS* with vertices *P*(2, 2), *Q*(−1, 2), *R*(−1, −3), and *S*(2, −3)? **3.** ______

A. square **B.** rectangle

C. parallelogram **D.** none of these

4. Find the area of quadrilateral *DEFG*. **4.** ______



A. 154 cm^2 **B.** 218 cm^2

C. 244 cm^2 **D.** 308 cm^2

5. Find the area of trapezoid *ABCD* with vertices *A*(1, −2), *B*(5, −2), *C*(4, 4), and *D*(1, 4). **5.** ______

A. 6.5 $units^2$ **B.** 14 $units^2$

C. 21 $units^2$ **D.** 36 $units^2$

6. Find the area of rhombus *ABCD* with vertices *A*(−1, 3), *B*(3, 0), *C*(−1, −3), and *D*(−5, 0). **6.** ______

A. 8 $units^2$ **B.** 24 $units^2$

C. 26 $units^2$ **D.** 32 $units^2$

7. Find the area of a regular octagon with a perimeter of 96 centimeters. **7.** ______

A. about 695.3 cm^2 **B.** about 576 cm^2

C. about 288 cm^2 **D.** about 119.3 cm^2

8. Find the area of an equilateral triangle with a side length of 14 inches. **8.** ______

A. about 12.1 in^2 **B.** about 42 in^2

C. about 84.9 in^2 **D.** about 254.6 in^2

9. Find the area of a circle with a circumference of 20π. **9.** ______

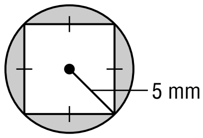
A. 400π **B.** 314π

C. 200π **D.** 100π

Chapter 11 Test, Form 1 *(continued)*

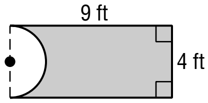
10. Find the area of the shaded region to the nearest tenth. **10.** ______

A. 28.5 mm^2 **B.** 53.5 mm^2
C. 66.3 mm^2 **D.** 72.3 mm^2



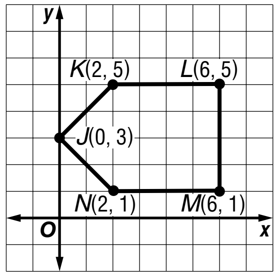
11. Find the area of the figure to the nearest tenth. **11.** ______

A. 23.4 ft^2 **B.** 28.3 ft^2
C. 29.7 ft^2 **D.** 36 ft^2



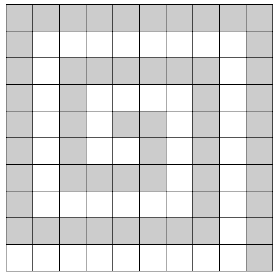
12. Find the area of the figure. **12.** ______

A. 22 $units^2$
B. 20 $units^2$
C. 18 $units^2$
D. 16 $units^2$



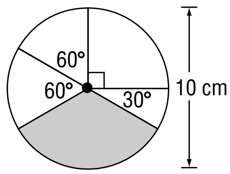
13. Find the probability that a point on the grid selected at random lies in the shaded region. **13.** ______

A. 0.46
B. 0.50
C. 0.55
D. 0.85



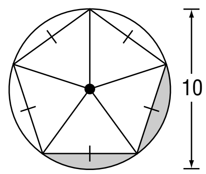
14. Find the probability that a point selected at random lies in the shaded sector. **14.** ______

A. 0.50 **B.** 0.33
C. 0.17 **D.** 0.08



15. Find the area of the shaded segments. **15.** ______

A. about 15.3 $units^2$
B. about 7.6 $units^2$
C. about 3.8 $units^2$
D. about 3.1 $units^2$



Bonus A rhombus has an area of 165 square units. If the length of one of its diagonals is 15 units, find the length of its other diagonal. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ______

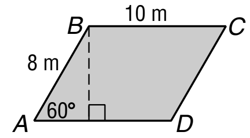
11 Chapter 11 Test, Form 2A

SCORE ______

Assessments

Write the letter for the correct answer in the blank at the right of each question.

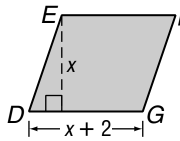
1. Find the area of parallelogram *ABCD* to the nearest tenth. **1.** ______



A. 55.4 m^2 **B.** 60 m^2

C. 69.3 m^2 **D.** 80 m^2

2. The area of parallelogram *DEFG* is 143 square units. Find the lengths of the height and the base to the nearest tenth. **2.** ______



A. 8, 10 **B.** 11, 13

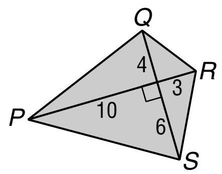
C. 47, 49 **D.** 70.5, 72.5

3. What is the best classification of quadrilateral *WXYZ* with vertices $W(-1, 0)$, $X(4, 0)$, $Y(2, -3)$, and $Z(-3, -3)$? **3.** ______

A. parallelogram **B.** rectangle

C. square **D.** none of these

4. Find the area of quadrilateral *PQRS*. **4.** ______



A. 34.1 $units^2$ **B.** 65 $units^2$

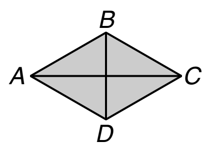
C. 130 $units^2$ **D.** 360 $units^2$

5. Find the area of trapezoid *ABCD* with vertices $A(2, 2)$, $B(4, 6)$, $C(4, -3)$, and $D(2, -1)$. **5.** ______

A. 27 $units^2$ **B.** 22.5 $units^2$

C. 18 $units^2$ **D.** 12 $units^2$

6. Rhombus *ABCD* has an area of 264 square units. If $DB = 12$ units, find *AC*. **6.** ______



A. 44 units **B.** 22 units

C. 18 units **D.** 12 units

7. Find the area of a regular hexagon with side length of 10 centimeters. Round to the nearest tenth. **7.** ______

A. 129.9 cm^2 **B.** 150 cm^2

C. 259.8 cm^2 **D.** 519.6 cm^2

8. Find the area of a nonagon with a perimeter of 126 inches. Round to the nearest tenth. **8.** ______

A. 1289.4 in^2 **B.** 1211.6 in^2

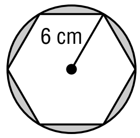
C. 466.2 in^2 **D.** 157.5 in^2

11 Chapter 11 Test, Form 2A *(continued)*

9. Find the area of the shaded region to the nearest tenth. **9.** ______

A. 59.1 cm^2 **B.** 57.5 cm^2

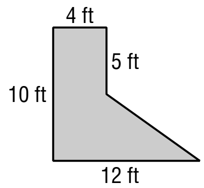
C. 25.7 cm^2 **D.** 19.6 cm^2



10. Find the area of the figure. **10.** ______

A. 31 ft^2 **B.** 40 ft^2

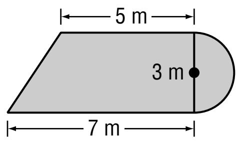
C. 60 ft^2 **D.** 80 ft^2



11. Find the area of the figure to the nearest tenth. **11.** ______

A. 46.3 m^2 **B.** 28.1 m^2

C. 25.1 m^2 **D.** 21.5 m^2



12. Find the area of pentagon *ABCDE* with vertices $A(1, 1)$, $B(4, 1)$, $C(4, -3)$, $D(2.5, -5)$ and $E(1, -3)$. **12.** ______

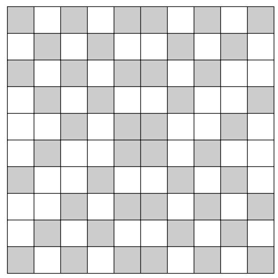
A. 15 $units^2$ **B.** 19 $units^2$

C. 21 $units^2$ **D.** 23 $units^2$

13. Find the probability that a point selected at random lies in the shaded region. **13.** ______

A. about 0.92 **B.** about 0.75

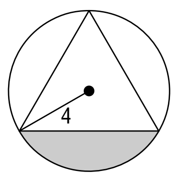
C. about 0.55 **D.** about 0.46



14. Find the probability that a point selected at random lies in the shaded region. **14.** ______

A. about 0.17 **B.** about 0.20

C. about 0.25 **D.** about 0.33



Bonus Find the area of a circle circumscribed about a regular pentagon with a perimeter of 50 inches. Round to the nearest tenth. **B:** ______

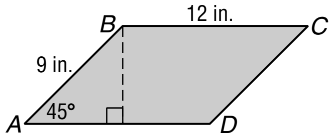
NAME ______________________ DATE ____________ PERIOD ______

11 Chapter 11 Test, Form 2B

SCORE ______

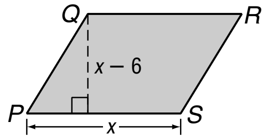
Write the letter for the correct answer in the blank at the right of each question.

1. Find the area of parallelogram *ABCD* to the nearest tenth. **1.** ______



A. 54 in^2 **B.** 76.4 in^2
C. 95.2 in^2 **D.** 152.7 in^2

2. The area of parallelogram *PQRS* is 187 square units. Find the lengths of the height and the base. Round to the nearest tenth. **2.** ______

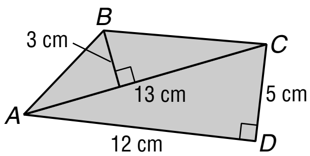


A. 15, 12.5 **B.** 13.5, 7.5
C. 12, 15.6 **D.** 11, 17

3. What is the best classification of quadrilateral *WXYZ* with vertices $W(1, 3)$, $X(2, 1)$, $Y(0, 0)$, and $Z(-1, 2)$? **3.** ______

A. parallelogram **B.** rectangle
C. square **D.** none of these

4. Find the area of quadrilateral *ABCD*. **4.** ______

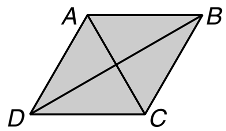


A. 49.5 cm^2 **B.** 52 cm^2
C. 60 cm^2 **D.** 97.5 cm^2

5. Find the area of trapezoid *ABCD* with vertices $A(2, 2)$, $B(4, -2)$, $C(-3, -2)$, and $D(-1, 2)$. **5.** ______

A. 30 units^2 **B.** 25 units^2
C. 24 units^2 **D.** 20 units^2

6. Rhombus *ABCD* has an area of 126 square units. If $DB = 18$ units, find *AC*. **6.** ______



A. 18 units **B.** 14 units
C. 7 units **D.** 3.5 units

7. Find the area of an equilateral triangle with a side length of 12 centimeters. Round to the nearest tenth. **7.** ______

A. 187.1 cm^2 **B.** 93.5 cm^2
C. 62.4 cm^2 **D.** 54 cm^2

8. Find the area of an octagon with a perimeter of 80 inches. Round to the nearest tenth. **8.** ______

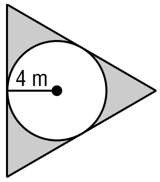
A. 965.7 in^2 **B.** 482.8 in^2
C. 165.7 in^2 **D.** 82.8 in^2

11 Chapter 11 Test, Form 2B *(continued)*

9. Find the area of the shaded region to the nearest tenth. **9.** _____

A. 12.6 m^2 **B.** 24.6 m^2

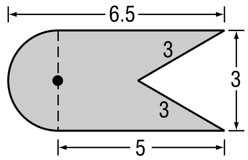
C. 32.9 m^2 **D.** 44.9 m^2



10. Find the area of the figure to the nearest tenth. **10.** _____

A. 14.6 $units^2$ **B.** 15 $units^2$

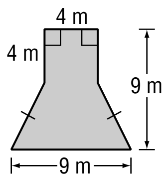
C. 18.2 $units^2$ **D.** 22.4 $units^2$



11. Find the area of the figure to the nearest tenth. **11.** _____

A. 81 m^2 **B.** 65 m^2

C. 58.5 m^2 **D.** 48.5 m^2



12. Find the area of pentagon *ABCDE* with vertices $A(0, 4)$, $B(3, 2)$, $C(3, -1)$, $D(-3, -1)$ and $E(-3, 2)$. **12.** _____

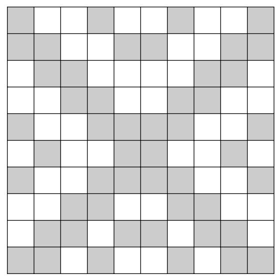
A. 24 $units^2$ **B.** 27 $units^2$

C. 30 $units^2$ **D.** 33 $units^2$

13. Find the probability that a point selected at random lies in the shaded region. **13.** _____

A. 0.47 **B.** 0.50

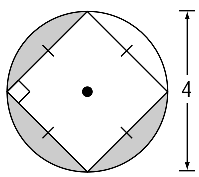
C. 0.53 **D.** 0.75



14. Find the probability that a point selected at random lies in the shaded region. **14.** _____

A. about 0.09 **B.** about 0.27

C. about 0.50 **D.** about 0.75



Bonus Find the area of a circle circumscribed about a regular hexagon with an apothem of 5 inches. Round to the nearest tenth. **B:** ______________

NAME ______________________ DATE ____________ PERIOD ______

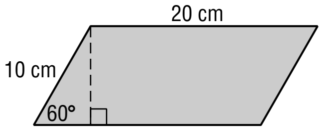
11 Chapter 11 Test, Form 2C

SCORE ______

Assessments

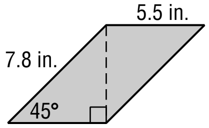
For Questions 1 and 2, find the area of each parallelogram to the nearest tenth.

1.



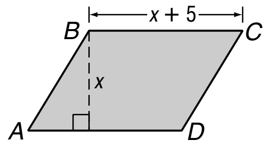
1. ____________

2.



2. ____________

3. The area of parallelogram *ABCD* is 2250 square meters. Find the lengths of the height and base.

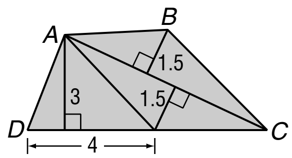


3. ____________

4. Classify quadrilateral *ABCD*, with vertices $A(-1, 4)$, $B(3, 4)$, $C(3, 1)$, and $D(-1, 1)$. List all that apply.

4. ____________

5. Find the area of quadrilateral *ABCD* if $AC = 7$. Round to the nearest tenth.



5. ____________

For Questions 6 and 7, find the area of each quadrilateral given the coordinates of the vertices.

6. trapezoid *GHIJ*; $G(-2, 3)$, $H(1, 3)$, $I(2, -1)$, and $J(-3, -1)$

6. ____________

7. rhombus *KLMN*; $K(4, 4)$, $L(6, 0)$, $M(4, -4)$, and $N(2, 0)$

7. ____________

For Questions 8–10, find the area of each polygon to the nearest tenth.

8. a square with a perimeter of $16\sqrt{2}$ inches

8. ____________

9. a regular hexagon with an apothem length of 4.3 centimeters

9. ____________

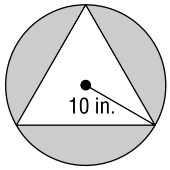
10. an equilateral triangle with a side length of 10.4 meters

10. ____________

NAME ______________________________ DATE ____________ PERIOD ______

11 Chapter 11 Test, Form 2C *(continued)*

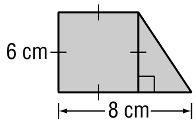
11. Find the area of the shaded region to the nearest tenth. Assume that the triangle is equilateral.



11. ____________________

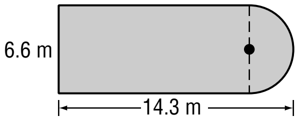
For Questions 12–14, find the area of each figure to the nearest tenth.

12.



12. ____________________

13.

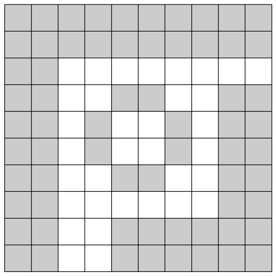


13. ____________________

14. pentagon *RSTUV* with vertices $R(5, 5)$, $S(5, 0)$, $T(2, -3)$, $U(-1,0)$, and $V(-1, 2)$

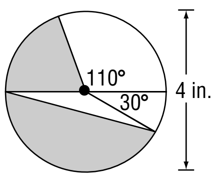
14. ____________________

15. A children's game is won by tossing a coin so that it lands on the white part of this board. If one coin is tossed, what is the probability of winning?



15. ____________________

For Questions 16 and 17, use the figure at the right. Round to the nearest tenth.



16. Find the probability that a point selected at random lies in the shaded sector.

16. ____________________

17. Find the probability that a point selected at random lies in the shaded segment.

17. ____________________

Bonus If the length of the height of a trapezoid is 4 meters, the length of one of its bases is 11 meters, and its area is 62 square meters, then what is the measure of the other base?

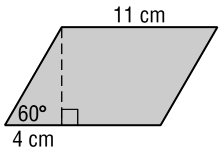
B: ____________________

NAME ______________________ DATE ____________ PERIOD ______

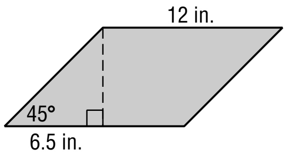
11 Chapter 11 Test, Form 2D

SCORE ______

For Questions 1 and 2, find the area of each parallelogram to the nearest tenth.

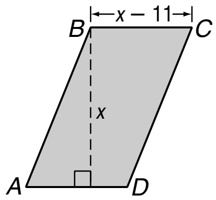
1. 

1. ____________

2. 

2. ____________

3. If the area of parallelogram *ABCD* is 570 square meters, find the lengths of the height and base.

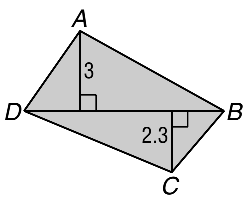


3. ____________

4. Classify quadrilateral *ABCD*, with vertices $A(1, 1)$, $B(1, -3)$, $C(-3, -3)$, and $D(-3, 1)$. List all that apply.

4. ____________

5. Find the area of quadrilateral *ABCD*, if $DB = 7.5$. Round to the nearest tenth.



5. ____________

For Questions 6 and 7, find the area of each quadrilateral given the coordinates of the vertices.

6. trapezoid *GHIJ*; $G(-2, 1)$, $H(2, 3)$, $I(2, -3)$, and $J(-2, -1)$

6. ____________

7. rhombus *KLMN*; $K(-3, 7)$, $L(0, 3)$, $M(-3, -1)$, and $N(-6, 3)$

7. ____________

For Questions 8–10, find the area of each polygon to the nearest tenth.

8. a square with an apothem length of 3 inches

8. ____________

9. a regular hexagon with a side length of 15 centimeters

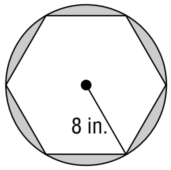
9. ____________

10. an equilateral triangle with a perimeter of 42 meters

10. ____________

11 Chapter 11 Test, Form 2D *(continued)*

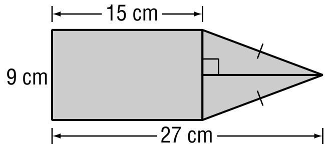
11. Find the area of the shaded region to the nearest tenth. Assume that the hexagon is regular.



11. ____________________

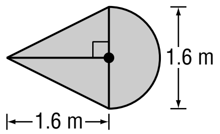
For Questions 12–14, find the area of each figure to the nearest tenth.

12.



12. ____________________

13.

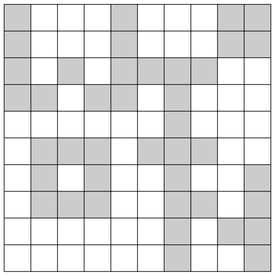


13. ____________________

14. pentagon $RSTUV$ with vertices $R(4, -2)$, $S(0, -1)$, $T(-3, -1)$, $U(0, -4)$, and $V(4, -4)$

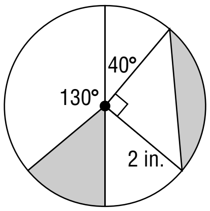
14. ____________________

15. A group of children are tossing a coin on this game board. To win the game the coin must land on a shaded part of the board. What is the probability of winning?



15. ____________________

For Questions 16 and 17, use the figure at the right. Round to the nearest tenth.



16. Find the probability that a point selected at random lies in the shaded sector.

16. ____________________

17. Find the probability that a point selected at random lies in the shaded segment.

17. ____________________

Bonus If one diagonal of a rhombus is 15 meters long and its area is 157.5 square meters, find the measure of the other diagonal.

B: ____________________

NAME ______________________ DATE ____________ PERIOD ______

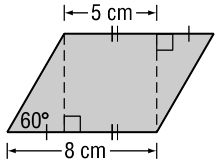
11 Chapter 11 Test, Form 3

SCORE ______

Assessments

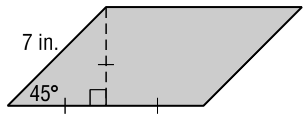
For Questions 1 and 2, find the area of each parallelogram to the nearest tenth.

1.



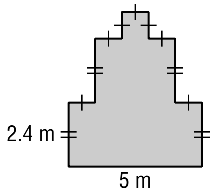
1. ______________

2.



2. ______________

3. Find the area of the figure.

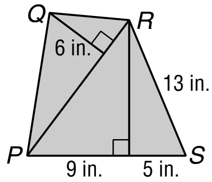


3. ______________

4. Classify quadrilateral $ABCD$, with vertices $A(4, 1)$, $B(8, -2)$, $C(4, -5)$, and $D(0, -2)$. List all that apply.

4. ______________

5. Find the area of quadrilateral $PQRS$ to the nearest tenth.



5. ______________

6. Find the area trapezoid $GHIJ$ with vertices $G(-2, 1)$, $H(8, 7)$, $I(6, -1)$, and $J(1, -4)$.

6. ______________

7. Find the area of a rhombus with a perimeter of 100 meters and one diagonal with a length of 48 meters.

7. ______________

For Questions 8–10, find the area of each polygon to the nearest tenth.

8. a regular octagon with perimeter of 96 meters

8. ______________

9. a regular pentagon with an apothem length of 5 inches

9. ______________

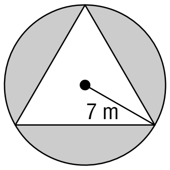
10. a regular nonagon with a side length of 12 centimeters

10. ______________

NAME ______________________ DATE ____________ PERIOD ______

11 Chapter 11 Test, Form 3 *(continued)*

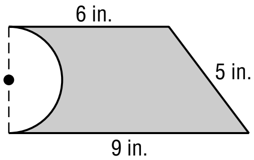
11. Find the area of the shaded region to the nearest tenth. Assume that the triangle is equiangular.



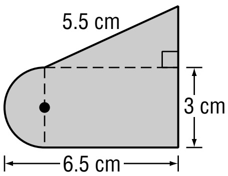
11. ____________

For Questions 12–14, find the area of each figure to the nearest tenth.

12.



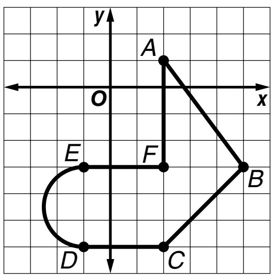
13.



12. ____________

13. ____________

14.

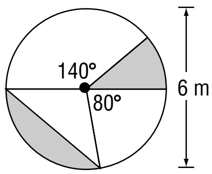


14. ____________

15. To win a certain board game, a tossed token must land on a shaded square on the board. The probability of winning is approximately 23%. If the board has a total area of 130 congruent squares, how many of these squares are shaded?

15. ____________

For Questions 16 and 17, use the figure at the right. Round to the nearest hundredth.



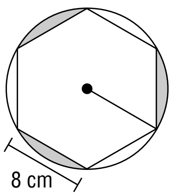
16. What is the probability that a point selected at random lies in the shaded sector?

16. ____________

17. What is the probability that a point selected at random lies in the shaded segment?

17. ____________

Bonus Find the area of the shaded region to the nearest tenth. Assume that the hexagon is regular.



B: ____________

11 Chapter 11 Open-Ended Assessment

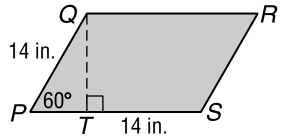
SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. a. Explain how to determine whether quadrilateral *ABCD* with vertices $A(-3, 3)$, $B(0, 0)$, $C(-3, -3)$, and $D(-6, 0)$ is a *square*, a *rectangle* or a *parallelogram*.

b. Find the area of quadrilateral *ABCD*.

2. a. Explain how a 30°-60°-90° triangle is used in finding the area of parallelogram *PQRS*.

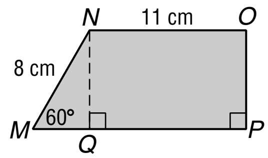


b. Find the area of parallelogram *PQRS* to the nearest tenth.

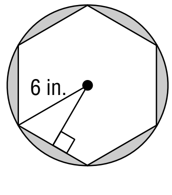
3. Explain how to find the area of each figure described below, then find each area. Round to the nearest tenth.

a. a rhombus with vertices $A(4, 6)$, $B(6, 2)$, $C(4, -2)$, and $D(2, 2)$

b.

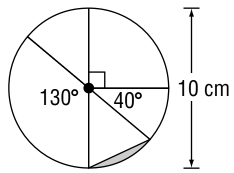


c. the shaded region



d. a pentagon with vertices $A(1, 4)$, $B(5, 4)$, $C(5, -1)$, $D(1, -2)$ and $E(-3, -1)$

4.



a. Explain how to find the probability that a point chosen at random lies in the shaded region.

b. Find the probability that a point chosen at random lies in the shaded region.

NAME ______________________ DATE ____________ PERIOD ______

11 Chapter 11 Vocabulary Test/Review

SCORE ______

geometric probability	height of parallelogram	irregular figure

Underline or circle the correct formula to complete each sentence.

1. The area of a circle can be found by ($A = \frac{1}{2}Pa$ $A = \pi r^2$ $A = bh$).

2. ($A = \frac{1}{2}d_1d_2$ $A = \frac{1}{2}Pa$ $A = \frac{1}{2}bh$) can be used to find the area of a triangle.

3. The formula ($A = bh$ $A = \frac{1}{2}d_1d_2$ $A = \frac{1}{2}h(b_1 + b_2)$) is used to find the area of a trapezoid.

4. The area of all regular polygons can be found using ($A = \frac{1}{2}Pa$ $A = \frac{1}{2}d_1d_2$ $A = \frac{1}{2}bh$).

5. The formula to find the area of a sector is ($A = \frac{N}{360}\pi r^2 - \frac{1}{2}bh$ $A = \frac{N}{360}\pi r^2$ $A = \pi r^2$).

6. To find the area of a parallelogram, the formula ($A = bh$ $A = \frac{1}{2}bh$ $A = \frac{1}{2}d_1d_2$) can be used.

7. ($\frac{1}{2}d_1d_2$ $\frac{\text{area of region } X}{\text{area of region } Y}$ $\frac{1}{2}Pa$) is the ratio used to find the probability that a point chosen at random is in region X, which is in the interior of region Y.

8. The formula to find the area of a rhombus is ($A = \frac{1}{2}bh$ $A = \frac{1}{2}d_1d_2$ $A = \frac{1}{2}h(b_1 + b_2)$).

9. The area of a segment can be found by using ($A = \pi r^2$ $A = \frac{N}{360}\pi r^2$ $A = \frac{N}{360}\pi r^2 - \frac{1}{2}bh$).

Define each term.

10. sector of a circle **10.** ____________________

11. irregular figure **11.** ____________________

12. geometric probability **12.** ____________________

NAME ______________________ DATE __________ PERIOD ______

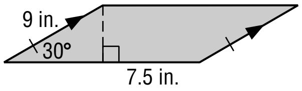
11

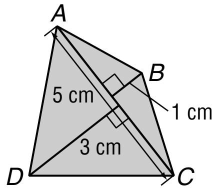
Chapter 11 Quiz

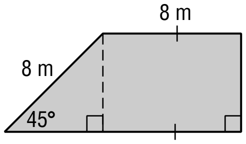
SCORE ______

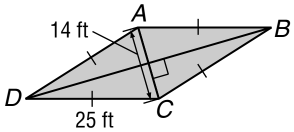
(Lessons 11–1 and 11–2)

Find the area of each figure to the nearest tenth.

1. 

2. 

3. 

4. 

5. Classify quadrilateral *ABCD*, with vertices $A(-1, 3)$, $B(4, 3)$, $C(4, -2)$, and $D(-1, -2)$ as a *square*, a *rectangle*, or a *parallelogram*.

1. ______
2. ______
3. ______
4. ______
5. ______

NAME ______________________ DATE __________ PERIOD ______

11

Chapter 11 Quiz

SCORE ______

(Lesson 11–3)

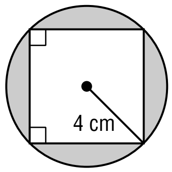
Find the area of each polygon to the nearest tenth.

1. an equilateral triangle with a side length of $8\sqrt{3}$ inches

2. a regular hexagon with apothem 5 centimeters

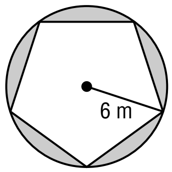
3. a regular octagon with a perimeter of 80 meters

4. Find the area of the shaded region to the nearest tenth. Assume that the polygon is regular.



5. STANDARDIZED TEST PRACTICE Find the area of the shaded region to the nearest tenth. Assume that the polygon is regular.

A. 105.5 m^2 **B.** 113 m^2
C. 70.1 m^2 **D.** 27.5 m^2



1. ______
2. ______
3. ______
4. ______
5. ______

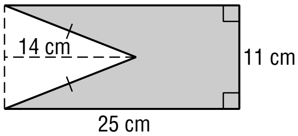
NAME ______________________ DATE ____________ PERIOD ______

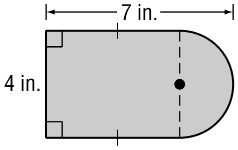
11 Chapter 11 Quiz

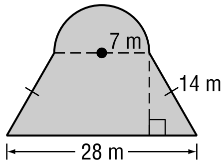
(Lesson 11–4)

SCORE ______

Find the area of each figure to the nearest tenth.

1. 

2. 

3. 

4. quadrilateral $PQRS$ with vertices $P(0, 6)$, $Q(4, 0)$, $R(0, -4)$, and $S(-4, 0)$

5. figure $ABCDEF$ with vertices $A(1, 3)$, $B(4, -1)$, $C(2, -4)$, $D(-4, -4)$, $E(-2, -1)$, and $F(1, -1)$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

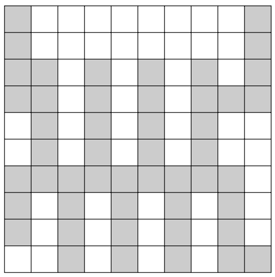
NAME ______________________ DATE ____________ PERIOD ______

11 Chapter 11 Quiz

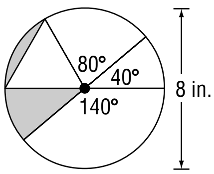
(Lesson 11–5)

SCORE ______

1. Find the probability that a point selected at random lies in the shaded region.



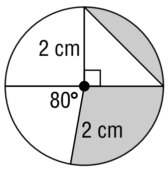
For Questions 2 and 3, use the figure.



2. Find the probability that a point selected at random lies in the shaded sector of the circle.

3. Find the probability that a point selected at random lies in the shaded segment of the circle.

For Questions 4 and 5, use the figure.



4. Find the probability that a point selected at random lies in the shaded sector of the circle.

5. Find the probability that a point selected at random lies in the shaded segment of the circle.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

NAME ______________________ DATE ____________ PERIOD _____

11

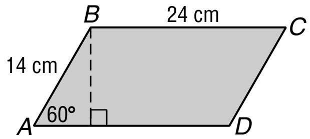
Chapter 11 Mid-Chapter Test

SCORE _____

(Lessons 11–1 through 11–3)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Find the area of parallelogram $ABCD$ to the nearest tenth. **1.** _____



A. 145.5 cm^2 **B.** 168.0 cm^2

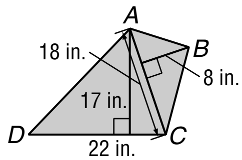
C. 190.5 cm^2 **D.** 291.0 cm^2

2. What is the best classification of quadrilateral $WXYZ$ with vertices $W(1, 1)$, $X(4, 1)$, $Y(4, -2)$, and $Z(1, -2)$? **2.** _____

A. square **B.** rectangle

C. parallelogram **D.** none of these

3. Find the area of quadrilateral $ABCD$ to the nearest tenth. **3.** _____



A. 187 in^2 **B.** 259 in^2

C. 374 in^2 **D.** 518 in^2

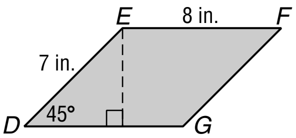
4. Find the area of trapezoid $ABCD$ with vertices $A(-1, 3)$, $B(3, 3)$, $C(4, -2)$, and $D(-4, -2)$. **4.** _____

A. 15 units^2 **B.** 20 units^2

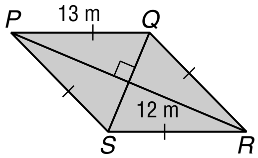
C. 30 units^2 **D.** 60 units^2

Part II

5. Find the area of parallelogram $DEFG$ to the nearest tenth. **5.** ____________



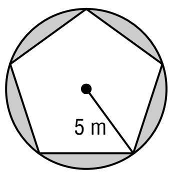
6. Find the area of rhombus $PQRS$ to the nearest tenth. **6.** ____________



7. Find the area of an equilateral triangle with an apothem having a length of 4 feet. Round to the nearest tenth. **7.** ____________

8. Find the area of a regular hexagon with perimeter of 72 inches. Round to the nearest tenth. **8.** ____________

9. Find the area of the shaded region to the nearest tenth. Assume that the pentagon is regular. **9.** ____________



NAME ______________________ DATE ____________ PERIOD ______

11 Chapter 11 Cumulative Review

(Chapters 1–11)

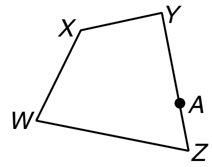
SCORE ______

1. What is the precision for a measurement of 76 millimeters? (Lesson 1-2)

1. ______________

2. Draw the segment that represents the shortest distance from A to $\overline{WX}$. (Lesson 3-6)

2.



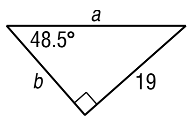
3. If $LM = 3x + 2$, $MN = 7x - 18$, and $NL = 22 - x$, find the length of the sides of equilateral triangle LMN. (Lesson 4-1)

3. ______________

4. Determine whether $A(2, 5)$, $B(-3, 7)$ and $C(-8, 9)$ are the vertices of a triangle. Explain why or why not. (Lesson 5-3)

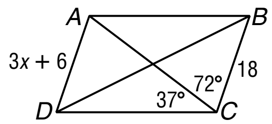
4. ______________

5. Find the values of a and b to the nearest tenth. (Lesson 7-3)



5. ______________

6. If $ABCD$ is a parallelogram, find $m\angle DAB$, $m\angle CDA$, and x. (Lesson 8-3)

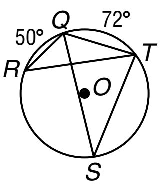


6. ______________

7. A plane is flying due south at 310 miles per hour and the wind is blowing from the east at 40 miles per hour. Find the resultant speed and direction of the plane to the nearest tenth. (Lesson 9-6)

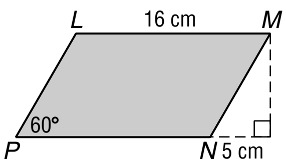
7. ______________

8. Find $m\angle R$, $m\angle S$, $m\angle QTR$, and $m\angle QTS$ if $\widehat{SR} \cong \widehat{TS}$. (Lesson 10-4)



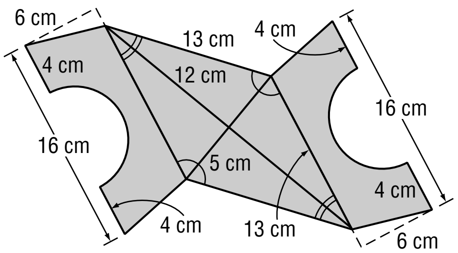
8. ______________

9. What is the area of parallelogram $LMNP$? (Lesson 11-1)



9. ______________

10. Find the area to the nearest tenth. (Lesson 11-4)



10. ______________

NAME ______________________________ DATE ____________ PERIOD ________

11 Standardized Test Practice

(Chapters 1–11)

SCORE ________

Assessments

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Mai knows that if two arcs are congruent, then their corresponding central angles are congruent. She is given $\odot D$ with $\overset{\frown}{BC} \cong \overset{\frown}{GH}$, and she concludes that $\angle BDC \cong \angle GDH$. Which form of reasoning does she use? (Lesson 2-4)

A. Inductive Reasoning **B.** Law of Detachment
C. Law of Syllogism **D.** Formal Proof

1. (A) (B) (C) (D)

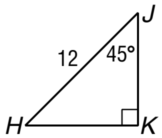
2. The distance between A and B is 17.8 centimeters, and the distance between B and C is 9.5 centimeters. If A, B, and C are noncollinear, which inequality represents the possible distance between A and C? (Lesson 5-4)

E. $9.5 \text{ cm} < AC < 17.8 \text{ cm}$ **F.** $8.3 \text{ cm} < AC < 27.3 \text{ cm}$
G. $10 \text{ cm} < AC < 18 \text{ cm}$ **H.** $8 \text{ cm} < AC < 27 \text{ cm}$

2. (E) (F) (G) (H)

3. Find HK. (Lesson 7-3)

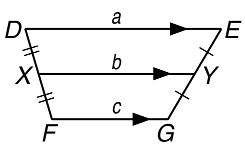
A. $3\sqrt{2}$ **B.** 6
C. $6\sqrt{2}$ **D.** $2\sqrt{3}$



3. (A) (B) (C) (D)

4. Which expression can you use to find a? (Lesson 8-6)

E. $c^2 - b^2$ **F.** $2b - c$
G. $\frac{b + c}{2}$ **H.** $2c - b$



4. (E) (F) (G) (H)

5. If $\overline{ST}$ with endpoints $S(3, -7)$ and $T(-5, -2)$ is reflected in the line $y = x$, find the coordinates of $\overline{S'T'}$. (Lesson 9-1)

A. $S'(-3, -7)$ and $T'(5, -2)$ **B.** $S'(3, 7)$ and $T'(-5, 2)$
C. $S'(-3, 7)$ and $T'(5, 2)$ **D.** $S'(-7, 3)$ and $T'(-2, -5)$

5. (A) (B) (C) (D)

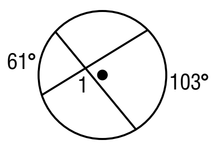
6. Find the circumference of a circle whose radius is 26.5 centimeters. (Lesson 10-1)

E. 26.5π cm **F.** 53π cm **G.** 702.25π cm **H.** 2809π cm

6. (E) (F) (G) (H)

7. Find $m\angle 1$. (Lesson 10-6)

A. 61 **B.** 82
C. 98 **D.** 103



7. (A) (B) (C) (D)

8. Find the area of a regular hexagon with a perimeter of 72 inches to the nearest square inch. (Lesson 11-3)

E. 72 in^2 **F.** 432 in^2 **G.** 374 in^2 **H.** 864 in^2

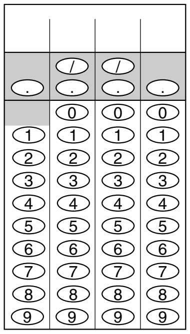
8. (E) (F) (G) (H)

11 Standardized Test Practice *(continued)*

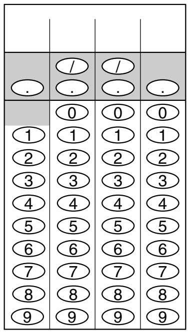
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

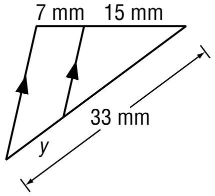
9. Find the distance between $B(-12, 18)$ and $C(-3, 30)$. (Lesson 1-3)

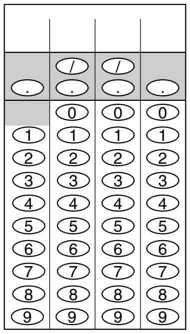
9. 

10. In right triangle PQR, angles P and R are acute angles and $m\angle P = 43.4$. What is $m\angle R$? (Lesson 4-2)

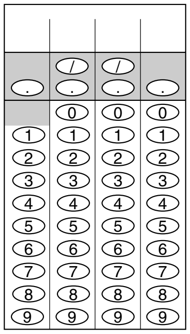
10. 

11. Find y. (Lesson 6-2)

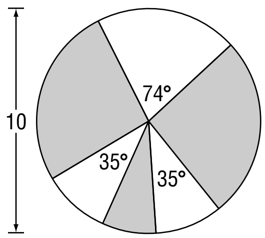


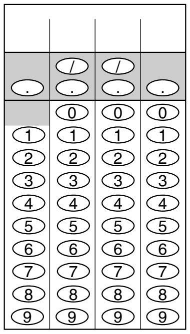
11. 

12. Angles B and G are opposite angles of a parallelogram. Find $m\angle G$ if $m\angle B = 3x + 80$ and $m\angle G = 140 - x$. (Lesson 8-2)

12. 

13. Find the probability as a percent that a point chosen at random lies in the unshaded region. (Lesson 11-5)



13. 

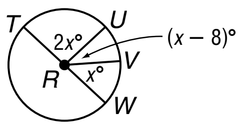
Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

14. If $\triangle XYZ$ has vertices $X(0, 3)$, $Y(4, 7)$, and $Z(-5, 4)$ and is reflected in the x-axis and then the y-axis, find the coordinates of the rotated image. (Lesson 9-3)

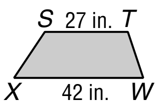
14. ____________________

15. Find $m\angle TRU$, $m\angle URV$, and $m\angle VRW$. (Lesson 10-2)



15. ____________________

16. Trapezoid $STWX$ has an area of 517.5 square inches. Find the height of $STWX$. (Lesson 11-2)



16. ____________________

NAME ______________________________ DATE ____________ PERIOD ______

11 Standardized Test Practice

Student Record Sheet *(Use with pages 632–633 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ	**4** Ⓐ Ⓑ Ⓒ Ⓓ	**7** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**5** Ⓐ Ⓑ Ⓒ Ⓓ	**8** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**6** Ⓐ Ⓑ Ⓒ Ⓓ	

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 10 and 11, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

9 ______________

10 ______________

11 ______________ (grid in)

12 ______________ (grid in)

13 ______________

11

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3 *Open-Ended*

Record your answers for Questions 14–15 on the back of this paper.

Answers

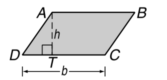
NAME ______________________ DATE ________ PERIOD ____

11-1 Study Guide and Intervention

Areas of Parallelograms

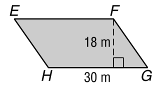
Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram $ABCD$ is $CD \cdot AT$.

Example **Find the area of parallelogram *EFGH*.**

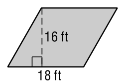


$A = bh$ Area of a parallelogram
$= 30(18)$ $b = 30$, $h = 18$
$= 540$ Multiply.

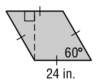
The area is 540 square meters.

Exercises

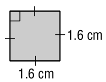
Find the area of each parallelogram.

1. 

288 ft²

2. 

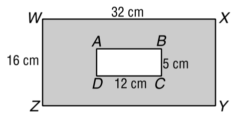
$288\sqrt{3}$ in²

3. 

2.56 cm²

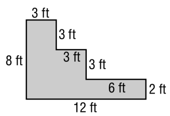
Find the area of each shaded region.

4. *WXYZ* and *ABCD* are rectangles.



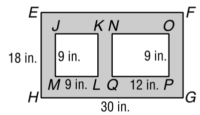
452 cm²

5. All angles are right angles.



51 ft²

6. *EFGH* and *NOPQ* are rectangles; *JKLM* is a square.



351 in²

7. The area of a parallelogram is 3.36 square feet. The base is 2.8 feet. If the measures of the base and height are each doubled, find the area of the resulting parallelogram. **13.44 ft²**

8. A rectangle is 4 meters longer than it is wide. The area of the rectangle is 252 square meters. Find the length. **18 m**

Lesson 11-1

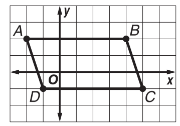
NAME ______________________ DATE ________ PERIOD ____

11-1 Study Guide and Intervention *(continued)*

Areas of Parallelograms

Parallelograms on the Coordinate Plane To find the area of a quadrilateral on the coordinate plane, use the Slope Formula, the Distance Formula, and properties of parallelograms, rectangles, squares, and rhombi.

Example **The vertices of a quadrilateral are $A(-2, 2)$, $B(4, 2)$, $C(5, -1)$, and $D(-1, -1)$.**



a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

Graph the quadrilateral. Then determine the slope of each side.

slope of $\overline{AB} = \frac{2-2}{4-(-2)}$ or 0

slope of $\overline{CD} = \frac{-1-(-1)}{-1-5}$ or 0

slope $\overline{AD} = \frac{2-(-1)}{-2-(-1)}$ or -3

slope $\overline{BC} = \frac{-1-2}{5-4}$ or -3

Opposite sides have the same slope. The slopes of consecutive sides are not negative reciprocals of each other, so consecutive sides are not perpendicular. *ABCD* is a parallelogram; it is not a rectangle or a square.

b. Find the area of *ABCD*.

From the graph, the height of the parallelogram is 3 units and $AB = |4 - (-2)| = 6$.

$A = bh$ Area of a parallelogram
$= 6(3)$ $b = 6$, $h = 3$
$= 18$ units² Multiply.

Exercises

Given the coordinates of the vertices of a quadrilateral, determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*. Then find the area.

1. $A(-1, 2)$, $B(3, 2)$, $C(3, -2)$, and $D(-1, -2)$ **square; 16 units²**

2. $R(-1, 2)$, $S(5, 0)$, $T(4, -3)$, and $U(-2, -1)$ **rectangle; 20 units²**

3. $C(-2, 3)$, $D(3, 3)$, $E(5, 0)$, and $F(0, 0)$ **parallelogram; 15 units²**

4. $A(-2, -2)$, $B(0, 2)$, $C(4, 0)$, and $D(2, -4)$ **square; 20 units²**

5. $M(2, 3)$, $N(4, -1)$, $P(-2, -1)$, and $R(-4, 3)$ **parallelogram; 24 units²**

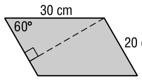
6. $D(2, 1)$, $E(2, -4)$, $F(-1, -4)$, and $G(-1, 1)$ **rectangle; 15 units²**

NAME ______________________ DATE __________ PERIOD _____

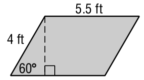
11-1 Skills Practice

Area of Parallelograms

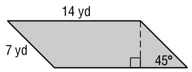
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1. 

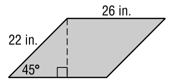
100 cm, 519.6 cm^2

2. 

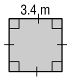
19 ft, 19.1 ft^2

3. 

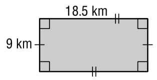
42 yd, 69.3 yd^2

4. 

96 in., 404.5 in^2

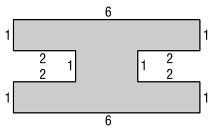
5. 

13.6 m, 11.6 m^2

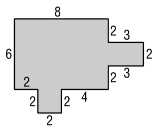
6. 

55 km, 166.5 km^2

Find the area of each figure.

7. 

14 units^2

8. 

58 units^2

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

9. $A(-4, 2), B(-1, 2), C(-1, -1), D(-4, -1)$
square, 9 units^2

10. $P(-3, 3), Q(1, 3), R(1, -3), S(-3, -3)$
rectangle, 24 units^2

11. $D(-5, 1), E(7, 1), F(4, -4), G(-8, -4)$
parallelogram, 60 units^2

12. $R(2, 3), S(4, 10), T(12, 10), U(10, 3)$
parallelogram, 56 units^2

Lesson 11-1

NAME ______________________ DATE __________ PERIOD _____

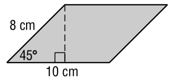
11-1 Practice (Average)

Area of Parallelograms

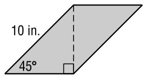
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

1. 

32 m, 47.6 m^2

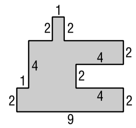
2. 

36 cm, 56.6 cm^2

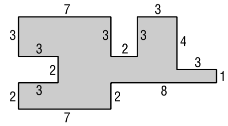
3. 

34.1 in., 50 in^2

Find the area of each figure.

4. 

44 units^2

5. 

65 units^2

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

6. $C(-4, -1), D(-4, 2), F(1, 2), G(1, -1)$
rectangle, 15 units^2

7. $W(2, 2), X(1, -2), Y(-2, -2), Z(-1, 2)$
parallelogram, 12 units^2

8. $M(0, 4), N(4, 6), O(6, 2), P(2, 0)$
square, 20 units^2

9. $P(-5, 2), Q(4, 2), R(5, 5), S(-4, 5)$
parallelogram, 27 units^2

FRAMING For Exercises 10–12, use the following information.

A rectangular poster measures 42 inches by 26 inches. A frame shop fitted the poster with a half-inch mat border.

10. Find the area of the poster. **1092 in^2**

11. Find the area of the mat border. **69 in^2**

12. Suppose the wall is marked where the poster will hang. The marked area includes an additional 12-inch space around the poster and frame. Find the total wall area that has been marked for the poster. **3417 in^2**

Answers

NAME ______________________ DATE ____________ PERIOD _____

11-1 Reading to Learn Mathematics

Areas of Parallelograms

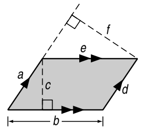
Pre-Activity **How is area related to garden design?**

Read the introduction to Lesson 11-1 at the top of page 595 in your textbook.

How could you describe the pattern you see in the picture of the garden so that someone who doesn't have the picture will know what it looks like?

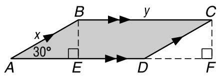
Sample answer: There are squares of granite and squares of moss of the same size placed in a checkerboard design.

Reading the Lesson



1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.) **B, D, E, G**

A. ab	**B.** cb	**C.** ed
D. af	**E.** ce	**F.** cd
G. df	**H.** bf	**I.** cf



2. Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.

a. $\overline{AB}$ is an altitude of the parallelogram. **False; $\overline{AB}$ is not perpendicular to any side of the parallelogram.**

b. $\overline{CD}$ is a base of parallelogram *ABCD*. **true**

c. The perimeter of *ABCD* is $(2x + 2y)$ units^2. **False; perimeter is measured in linear units, not square units. The perimeter is $(2x + 2y)$ units.**

d. $BE = CF$ **true**

e. $BE = \frac{\sqrt{3}}{2}x$ **False; $\overline{BE}$ is opposite the 30° angle in a 30°-60°-90° triangle, so $BE = \frac{1}{2}x$ or $\frac{x}{2}$.**

f. The area of *ABCD* is $2xy$ units^2. **False; since $BE = \frac{x}{2}$, the area of *ABCD* is $\frac{xy}{2}$ units^2.**

Helping You Remember

3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?

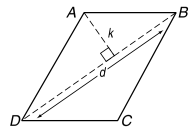
Sample answer: To find the area of a rectangle, you multiply the lengths of two segments that are perpendicular to each other. To find the area of a parallelogram, you do the same thing, but the height is not necessarily one of the sides.

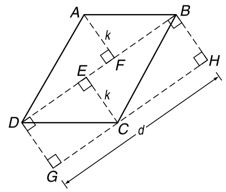
NAME ______________________ DATE ____________ PERIOD _____

11-1 Enrichment

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal $\overline{BD}$, and k is the length of the perpendicular segment from A to $\overline{BD}$. Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.





1. What kind of figure is *DBHG*?

rectangle

2. If you moved $\triangle AFB$ to the lower-left end of figure *DBHG*, would it fit perfectly on top of $\triangle DGC$? Explain your answer.

Yes; $\triangle AFB \cong \triangle CED$ (by HA) and $\triangle CED \cong \triangle DGC$ (since $\overline{DC}$ is a diagonal of rectangle *DECD*). So $\triangle AFB \cong \triangle DGC$.

3. Which two triangular pieces of ▱*ABCD* are congruent to $\triangle CBH$?

$\triangle DAF$ and $\triangle BCE$

4. The area of ▱*ABCD* is the same as that of figure *DBHG*, since the pieces of ▱*ABCD* can be rearranged to form *DBHG*. Express the area of ▱*ABCD* in terms of the measurements k and d.

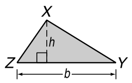
Area of ▱*ABCD* = dk

11-2 Study Guide and Intervention

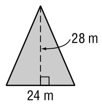
Areas of Triangles, Trapezoids, and Rhombi

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of *A* square units, a base of *b* units, and a corresponding height of *h* units, then $A = \frac{1}{2}bh$.



Example **Find the area of the triangle.**



$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(28)$ $b = 24, h = 28$

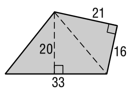
$= 336$ Multiply.

The area is 336 square meters.

Exercises

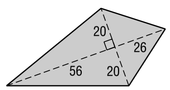
Find the area of each figure.

1.



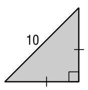
498 units^2

2.



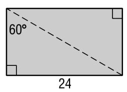
1640 units^2

3.



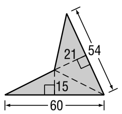
25 units^2

4.



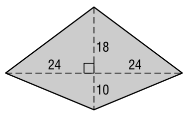
332.6 units^2

5.



1017 units^2

6.



672 units^2

7. The area of a triangle is 72 square inches. If the height is 8 inches, find the length of the base. **18 in.**

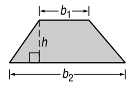
8. A right triangle has a perimeter of 36 meters, a hypotenuse of 15 meters, and a leg of 9 meters. Find the area of the triangle. **54 m^2**

11-2 Study Guide and Intervention *(continued)*

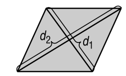
Areas of Triangles, Trapezoids, and Rhombi

Areas of Trapezoids and Rhombi The area of a trapezoid is the product of half the height and the sum of the lengths of the bases. The area of a rhombus is half the product of the diagonals.

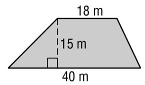
If a trapezoid has an area of *A* square units, bases of b_1 and b_2 units, and a height of *h* units, then $A = \frac{1}{2}h(b_1 + b_2)$.



If a rhombus has an area of *A* square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.



Example **Find the area of the trapezoid.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(15)(18 + 40)$ $h = 15, b_1 = 18, b_2 = 40$

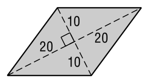
$= 435$ Simplify.

The area is 435 square meters.

Exercises

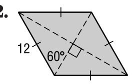
Find the area of each quadrilateral given the coordinates of the vertices.

1.



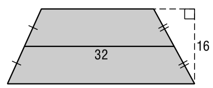
400 units^2

2.



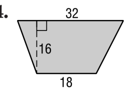
$72\sqrt{3}$ units^2

3.



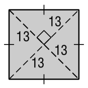
512 units^2

4.



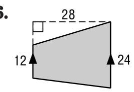
400 units^2

5.



338 units^2

6.



504 units^2

7. The area of a trapezoid is 144 square inches. If the height is 12 inches, find the length of the median. **12 in.**

8. A rhombus has a perimeter of 80 meters and the length of one diagonal is 24 meters. Find the area of the rhombus. **384 m^2**

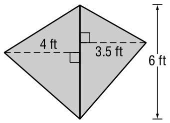
NAME ______________ DATE ________ PERIOD _____

11-2 Skills Practice

Areas of Triangles, Trapezoids, and Rhombi

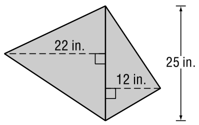
Find the area of each figure. Round to the nearest tenth if necessary.

1.



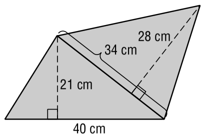
22.5 ft²

2.



425 in²

3.



896 cm²

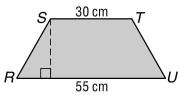
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *WXYZ*
$W(-5, 3), X(3, 3), Y(6, -3), Z(-8, -3)$
66 units²

5. rhombus *HIJK*
$H(4, -3), I(2, -7), J(0, -3), K(2, 1)$
16 units²

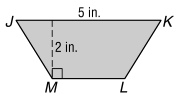
Find the missing measure for each figure.

6. Trapezoid *RSTU* has an area of 935 square centimeters. Find the height of *RSTU*.



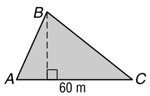
22 cm

7. Trapezoid *JKLM* has an area of 7.5 square inches. Find *ML*.



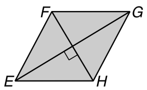
2.5 in.

8. Triangle *ABC* has an area of 1050 square meters. Find the height of △*ABC*.



35 m

9. Rhombus *EFGH* has an area of 750 square feet. If *EG* is 50 feet, find *FH*.



30 ft

Lesson 11-2

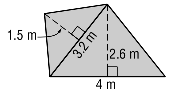
NAME ______________ DATE ________ PERIOD _____

11-2 Practice (Average)

Areas of Triangles, Trapezoids, and Rhombi

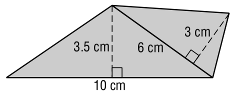
Find the area of each figure. Round to the nearest tenth if necessary.

1.



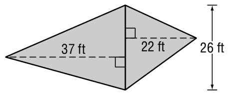
7.6 m²

2.



26.5 cm²

3.



767 ft²

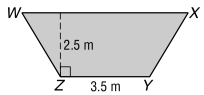
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid *ABCD*
$A(-7, 1), B(-4, 4), C(-4, -6), D(-7, -3)$
21 units²

5. rhombus *LMNO*
$L(6, 8), M(14, 4), N(6, 0), O(-2, 4)$
64 units²

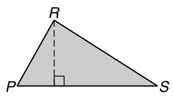
Find the missing measure for each figure.

6. Trapezoid *WXYZ* has an area of 13.75 square meters. Find *WX*.



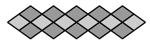
7.5 m

7. Triangle *PRS* has an area of 68 square yards. If the height of △*PRS* is 8 yards, find the base.



17 yd

DESIGN For Exercises 8 and 9, use the following information.
Mr. Hagarty used 16 congruent rhombi-shaped tiles to design the midsection of the backsplash area above a kitchen sink. The length of the design is 27 inches and the total area is 108 square inches.



8. Find the area of one rhombus.
$6\frac{3}{4}$ **in²**

9. Find the length of each diagonal.
$4\frac{1}{2}$ **in., 3 in.**

NAME ______________________ DATE ________ PERIOD ____

11-2 Reading to Learn Mathematics

Areas of Triangles, Trapezoids, and Rhombi

Pre-Activity **How is the area of a triangle related to beach umbrellas?**

Read the introduction to Lesson 11-2 at the top of page 601 in your textbook.

Classify the polygons in the panels of the beach umbrella.
Isosceles triangles and isosceles trapezoids

Reading the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a. $A = \ell w$ **vi**	**i.** triangle
b. $A = \frac{1}{2}d_1d_2$ **iv**	**ii.** parallelogram
c. $A = s^2$ **v**	**iii.** trapezoid
d. $A = \frac{1}{2}h(b_1 + b_2)$ **iii**	**iv.** rhombus
e. $A = \frac{1}{2}bh$ **i**	**v.** square
f. $A = bh$ **ii**	**vi.** rectangle

2. Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning. **For explanations, sample answers are given.**

a. The area of a square is half the product of its diagonals. **Always; a square is a rhombus, so you can use the rhombus formula.**

b. The area of a triangle is half the product of two of its sides. **Sometimes; this is true only for a right triangle.**

c. You can find the area of a rectangle by multiplying base times height. **Always; a rectangle is a parallelogram, so you can use the parallelogram formula. If the length of a rectangle is used as the base, then the width is the height.**

d. You can find the area of a rectangle by multiplying the lengths of any two of its sides. **Sometimes; this is true only for a square. Otherwise, you must use two *consecutive* sides, not any two sides.**

e. The area of a trapezoid is the product of its height and the sum of the bases. **Never; the area is one-half the product of its height and the sum of the bases.**

f. The square of the length of a side of a square is equal to half the product of its diagonals. **Always; a square is a rhombus, so the formulas for a square and a rhombus must give the same answer whenever the rhombus is a square.**

Helping You Remember

3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember. **Sample answer: Average the lengths of the bases and multiply by the height.**

Lesson 11-2

NAME ______________________ DATE ________ PERIOD ____

11-2 Enrichment

Areas of Similar Triangles

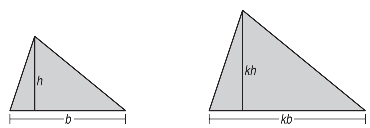
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

$$\frac{\text{side of } \triangle II}{\text{side of } \triangle I} = \frac{kb}{b} \text{ or } \frac{k}{1}$$

$$\frac{\text{area of } \triangle II}{\text{area of } \triangle I} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh} \text{ or } \frac{k^2}{1}$$

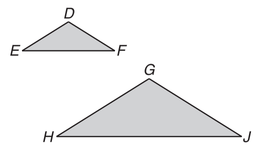


Triangle I
area $\triangle I = \frac{1}{2}bh$

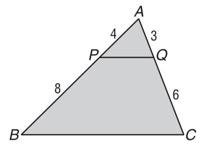
Triangle II
area $\triangle II = \frac{1}{2}(kb)(kh)$
$= \frac{1}{2}k^2bh$

Solve.

1. $\triangle DEF \sim \triangle GHJ$, $HJ = 16$, and $EF = 8$. The area of $\triangle GHJ$ is 40. Find the area of $\triangle DEF$. **10**



2. In the figure below, $\overline{PQ} \parallel \overline{BC}$. The area of $\triangle ABC$ is 72. Find the area of $\triangle APQ$. **8**



3. Two similar triangles have areas of 16 and 36. The length of a side of the smaller triangle is 10 feet. Find the length of the corresponding side of the larger triangle.

15 ft

4. Find the ratio of the areas of two similar triangles if the lengths of two corresponding sides of the triangles are 3 centimeters and 5 centimeters.

$\frac{9}{25}$

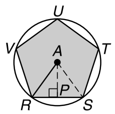
Answers

NAME ______________________ DATE __________ PERIOD _____

11-3 Study Guide and Intervention

Areas of Regular Polygons and Circles

Areas of Regular Polygons In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is

$A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then

$A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon *RSTUV* above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

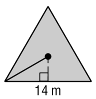
$$\tan \angle RAP = \frac{RP}{AP}$$
$$\tan 36° = \frac{6}{AP}$$
$$AP = \frac{6}{\tan 36°}$$
$$\approx 8.26$$

So $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.7.

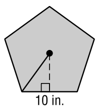
The area is about 248 square centimeters.

Exercises

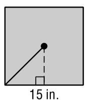
Find the area of each regular polygon. Round to the nearest tenth.

1. 

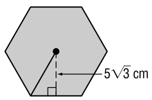
84.9 m²

2. 

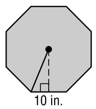
172.0 in²

3. 

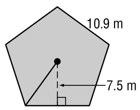
225 in²

4. 

259.8 cm²

5. 

482.8 in²

6. 

204.4 m²

Lesson 11-3

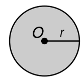
NAME ______________________ DATE __________ PERIOD _____

11-3 Study Guide and Intervention *(continued)*

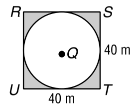
Areas of Regular Polygons and Circles

Areas of Circles As the number of sides of a regular polygon increases, the polygon gets closer and closer to a circle and the area of the polygon gets closer to the area of a circle.

Area of a Circle	If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$.



Example **Circle Q is inscribed in square *RSTU*. Find the area of the shaded region.**



A side of the square is 40 meters, so the radius of the circle is 20 meters.

The shaded area is

Area of $RSTU$ − Area of circle Q

$= 40^2 - \pi r^2$

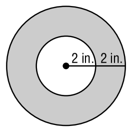
$= 1600 - 400\pi$

$\approx 1600 - 1256.6$

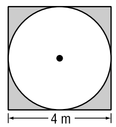
$= 343.4 \text{ m}^2$

Exercises

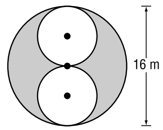
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

1. 

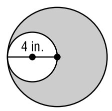
37.7 in²

2. 

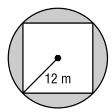
3.4 m²

3. 

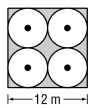
100.5 m²

4. 

37.7 in²

5. 

164.4 in²

6. 

30.9 m²

NAME ______________________ DATE __________ PERIOD _____

11-3 Skills Practice

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a pentagon with a perimeter of 45 feet
139.4 ft^2

2. a hexagon with a side length of 4 inches
41.6 in^2

3. a nonagon with a side length of 8 meters
395.6 m^2

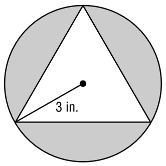
4. a triangle with a perimeter of 54 centimeters
140.3 cm^2

Find the area of each circle. Round to the nearest tenth.

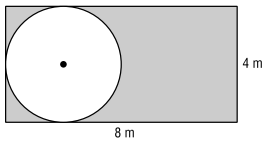
5. a circle with a radius of 6 yards
113.1 yd^2

6. a circle with a diameter of 18 millimeters
254.5 mm^2

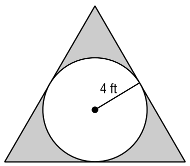
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

7.


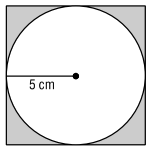
16.6 in^2

8.


19.4 m^2

9.


32.9 ft^2

10.


21.5 cm^2

Lesson 11-3

NAME ______________________ DATE __________ PERIOD _____

11-3 Practice (Average)

Areas of Regular Polygons and Circles

Find the area of each regular polygon. Round to the nearest tenth.

1. a nonagon with a perimeter of 117 millimeters
1044.7 mm^2

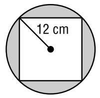
2. an octagon with a perimeter of 96 yards
695.3 yd^2

Find the area of each circle. Round to the nearest tenth.

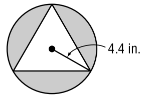
3. a circle with a diameter of 26 feet
530.9 ft^2

4. a circle with a circumference of 88 kilometers
616.2 km^2

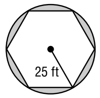
Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.

5.


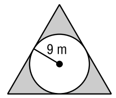
164.4 cm^2

6.


35.7 in^2

7.


339.7 ft^2

8.


166.4 m^2

DISPLAYS For Exercises 9 and 10, use the following information.
A display case in a jewelry store has a base in the shape of a regular octagon. The length of each side of the base is 10 inches. The owners of the store plan to cover the base in black velvet.

9. Find the area of the base of the display case.
about 482.8 in^2

10. Find the number of square yards of fabric needed to cover the base.
about 0.37 yd^2

Answers

NAME ______ DATE ______ PERIOD ______

11-3 Reading to Learn Mathematics

Areas of Regular Polygons and Circles

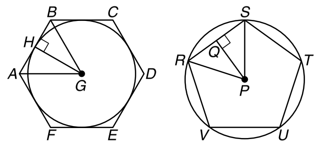
Pre-Activity **How can you find the area of a polygon?**

Read the introduction to Lesson 11-3 at the top of page 610 in your textbook.

How can you find the area of a regular hexagon without a new area formula? **Sample answer: Divide the hexagon into six congruent equilateral triangles. Use properties of the 30°-60°-90° triangle to find the area of one of these triangles and multiply the result by 6.**

Reading the Lesson

1. *ABCDEF* and *RSTUV* are regular polygons. Name each of the following in one of the figures.



a. a circumscribed polygon **hexagon *ABCDEF***

b. an inscribed polygon **pentagon *RSTUV***

c. an apothem of a regular hexagon $\overline{GH}$

d. an isosceles triangle **△*PRS* or △*GAB***

e. a 30°-60°-90° triangle **△*GAH* or △*GBH***

f. a central angle with a measure of 72 **∠*RPS***

2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.

a. perimeter of *ABCDEF* **v**	**i.** $\pi(PS)^2$
b. circumference of circle *G* **viii**	**ii.** $2\pi(PR)$
c. perimeter of *RSTUV* **vii**	**iii.** $\frac{5}{2}(RS)(PQ)$
d. area of circle *G* **vi**	**iv.** $3(AB)(HG)$
e. area of *RSTUV* **iii**	**v.** $6(CD)$
f. area of *ABCDEF* **iv**	**vi.** $\pi(GH)^2$
g. area of circle *P* **i**	**vii.** $5(UV)$
h. circumference of circle *P* **ii**	**viii.** $2\pi(GH)$

3. Explain in your own words how to find the area of a circle if you know the circumference. **Sample answer: Divide the circumference by 2π to find the radius. Then square the radius and multiply by π to find the area.**

Helping You Remember

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her? **Sample answer: Circumference is measured in linear units, while area is measured in square units, so the formula containing r^2 must be the one for area.**

Lesson 11-3

NAME ______ DATE ______ PERIOD ______

11-3 Enrichment

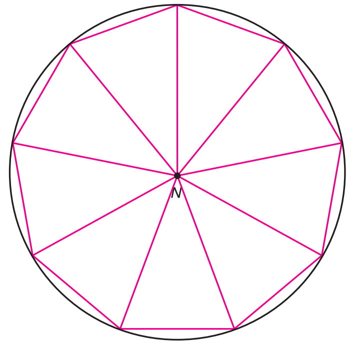
Areas of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙*N*.

Step 1 Find the degree measure of each of the nine congruent arcs. **40**

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon? **2.5 cm, *P* = 22.5 cm**

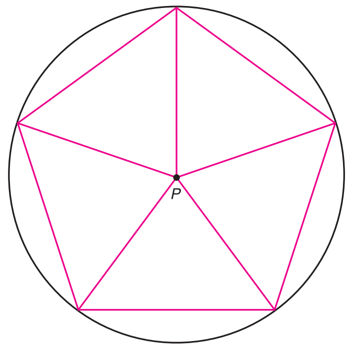
2. Measure the distance from the center perpendicular to one of the sides of the nonagon. **3.3 cm**

3. What is the area of one of the nine triangles formed? **4.125 cm²**

4. What is the area of the nonagon? **37.125 cm²**

Make the appropriate changes in Steps 1–3 above to inscribe a regular pentagon in ⊙*P*. Answer each of the following.

5. Use a protractor to inscribe a regular pentagon in ⊙*P*.



6. What is the measure of each of the five congruent arcs? **72**

7. What is the perimeter of the pentagon to the nearest tenth of a centimeter? **21 cm**

8. What is the area of the pentagon to the nearest tenth of a centimeter? **30.45 cm²**

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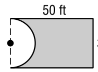
11-4 Study Guide and Intervention

Areas of Irregular Figures

Irregular Figures An **irregular** figure is one that cannot be classified as one of the previously-studied shapes. To find the area of an irregular figure, break it into familiar shapes. Find the area of each shape and add the areas.

Example 1 **Find the area of the irregular figure.**



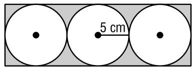
The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$A = lw - \frac{1}{2}\pi r^2$

$\approx 50(30) - 0.5(3.14)(15)^2$

$= 1146.6$ or about 1147 ft^2

Example 2 **Find the area of the shaded region.**



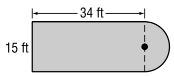
The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

$(10)(30) - 3\pi(5^2) = 300 - 75\pi$

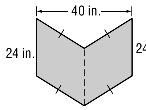
≈ 64.4 cm^2

Exercises

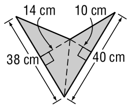
Find the area of each figure. Round to the nearest tenth if necessary.

1.


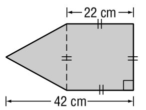
598.4 ft²

2.


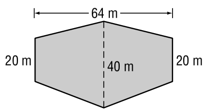
960 in²

3.


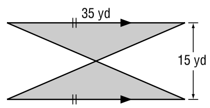
466 cm²

4.


704 cm²

5.


1920 m²

6.


262.5 yd²

7. Refer to Example 2 above. Draw the largest possible square inside each of the three circles. What is the total area of the three squares? **150 cm²**

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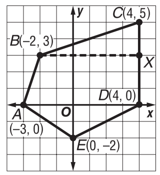
11-4 Study Guide and Intervention *(continued)*

Areas of Irregular Figures

Irregular Figures on the Coordinate Plane To find the area of an irregular figure on the coordinate plane, break up the figure into known figures. You may need to use the Distance Formula to find some of the dimensions.

Example **Find the area of irregular pentagon *ABCDE*.**



Draw $\overline{BX}$ between $B(-2, 3)$ and $X(4, 3)$ and draw $\overline{AD}$. The area of $ABCDE$ is the sum of the areas of $\triangle BCX$, trapezoid $BXDA$, and $\triangle ADE$.

A = area of $\triangle BCX$ + area of $BXDA$ + area of $\triangle ADE$

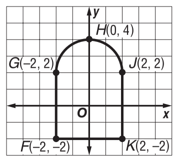
$= \frac{1}{2}(2)(6) + \frac{1}{2}(3)(6 + 7) + \frac{1}{2}(2)(7)$

$= 6 + \frac{39}{2} + 7$

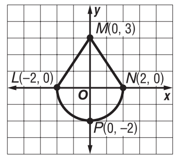
$= 32.5$ square units

Exercises

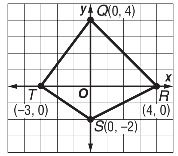
Find the area of each figure. Round to the nearest tenth.

1.


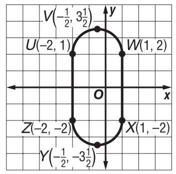
22.3 units²

2.


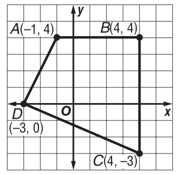
12.3 units²

3.


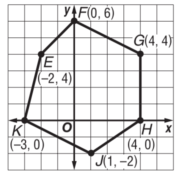
21 units²

4.


19.1 units²

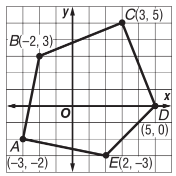
5.


34.5 units²

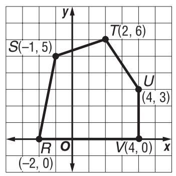
6.


39 units²

7. pentagon *ABCDE* **42.5 units²**



8. pentagon *RSTUV* **28 units²**



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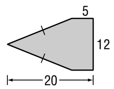
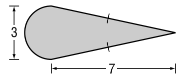
11-4 Skills Practice

Areas of Irregular Figures

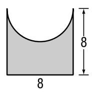
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

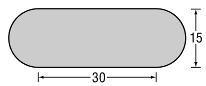
150 units^2

2. 

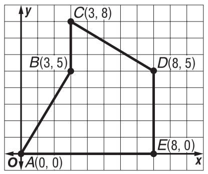
14.0 units^2

3. 

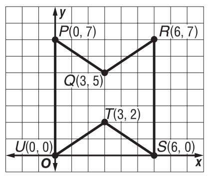
38.9 units^2

4. 

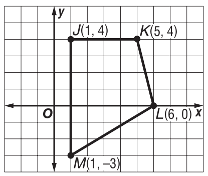
626.7 units^2

5. 

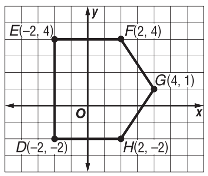
40 units^2

6. 

30 units^2

7. 

25.5 units^2

8. 

30 units^2

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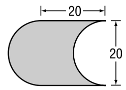
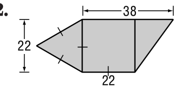
11-4 Practice (Average)

Areas of Irregular Figures

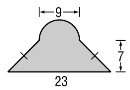
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

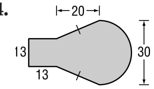
400 units^2

2. 

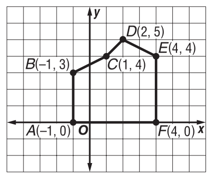
869.6 units^2

3. 

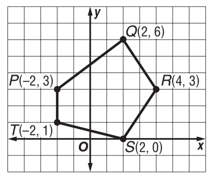
143.8 units^2

4. 

952.4 units^2

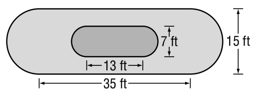
5. 

20.5 units^2

6. 

22 units^2

LANDSCAPING For Exercises 7 and 8, use the following information.
One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.



7. Find the area of the pond to the nearest tenth.
129.5 ft^2

8. Find the area of the walkway to the nearest tenth.
572.2 ft^2

Lesson 11-4

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11-4 Reading to Learn Mathematics

Areas of Irregular Figures

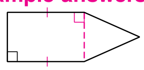
Pre-Activity **How do windsurfers use area?**

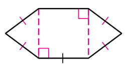
Read the introduction to Lesson 11-4 at the top of page 617 in your textbook.

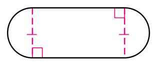
How do you think the areas of the figures outlined in the picture of the sail are related? **Sample answer: The areas get smaller as you move further up the sail. The area of the triangle is smaller than the area of any of the trapezoids.**

Reading the Lesson

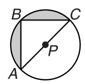
1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other.
Sample answers are given.

a. 
rectangle and isosceles triangle

b. 
square and two congruent isosceles triangles

c. 
rectangle and two congruent semicircles

2. In the figure, B is the midpoint of $\widehat{ABC}$. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is **πr^2**.

$m\angle ABC =$ **90** because **Sample answer: It is an inscribed angle that intercepts a semicircle**.

$m\widehat{AB} = m\widehat{BC}$ because **Sample answer: B is the midpoint of $\widehat{ABC}$ (definition of midpoint)**.

$\overline{AB} \cong \overline{BC}$ because **Sample answer: If two minor arcs of a circle are congruent, their corresponding chords are congruent**.

Therefore, $\triangle ABC$ is a(n) **isosceles right or 45°-45°-90°** triangle.

$AC =$ **$2r$**, so $AB =$ **$\frac{2r}{\sqrt{2}}$ or $r\sqrt{2}$** and $BC =$ **$\frac{2r}{\sqrt{2}}$ or $r\sqrt{2}$**.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot$ **$r\sqrt{2}$** $\cdot$ **$r\sqrt{2}$** $=$ **r^2**.

Therefore, the area of the shaded region is given by

$A =$ **$\frac{1}{2}\pi r^2$** $-$ **r^2** $=$ **$\left(\frac{\pi}{2} - 1\right)r^2$**.

Helping You Remember

3. Rolando is having trouble remembering when to subtract an area when finding the area of an irregular figure. How can you help him remember? **Sample answer: Subtract when there is an indentation, or a hole in the figure.**

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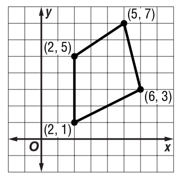
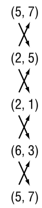
11-4 Enrichment

Aerial Surveyors and Area

Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.



Step 2 Find D, the sum of the downward diagonal products (from left to right).

$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75

Step 3 Find U, the sum of the upward diagonal products (from left to right).

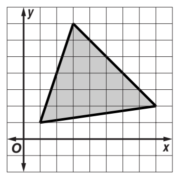
$U = (2 \cdot 7) + (2 \cdot 5) + (6 \cdot 1) + (5 \cdot 3)$
$= 14 + 10 + 6 + 15$ or 45

Step 4 Use the formula $A = \frac{1}{2}(D - U)$ to find the area.

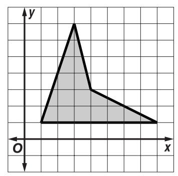
$A = \frac{1}{2}(D - U)$
$= \frac{1}{2}(75 - 45)$
$= \frac{1}{2}(30)$ or 15

The area is 15 square units. Count the number of square units enclosed by the polygon. Does this result seem reasonable?

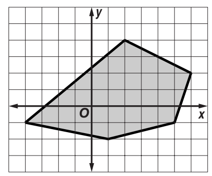
Use the coordinate method to find the area of each region in square units.

1. 

20 units^2

2. 

14 units^2

3. 

34 units^2

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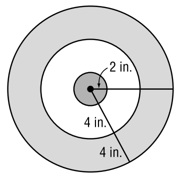
11-5 Study Guide and Intervention

Geometric Probability

Geometric Probability The probability that a point in a figure will lie in a particular part of the figure can be calculated by dividing the area of the part of the figure by the area of the entire figure. The quotient is called the **geometric probability** for the part of the figure.

> If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is
> $P(B) = \frac{\text{area of region } B}{\text{area of region } A}$.

Example **Darts are thrown at a circular dartboard. If a dart hits the board, what is the probability that the dart lands in the bull's-eye?**



Area of bull's-eye: $A = \pi(2)^2$ or 4π

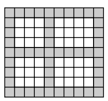
Area of entire dartboard: $A = \pi(10)^2$ or 100π

The probability of landing in the bull's-eye is

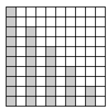
$$\frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4\pi}{100\pi} = \frac{1}{25} \text{ or } 0.04.$$

Exercises

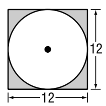
Find the probability that a point chosen at random lies in the shaded region. Round to the nearest hundredth if necessary.

1.


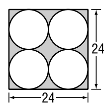
0.53

2.


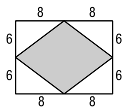
0.3

3.


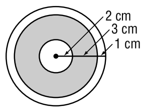
0.21

4.


0.21

5.


0.5

6.


0.58

Lesson 11-5

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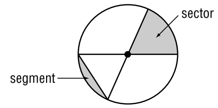
11-5 Study Guide and Intervention *(continued)*

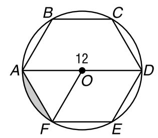
Geometric Probability

Sectors and Segments of Circles A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted arc. A **segment of a circle** is bounded by a chord and its arc. Geometric probability problems sometimes involve sectors or segments of circles.



> If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.

Example **A regular hexagon is inscribed in a circle with diameter 12. Find the probability that a point chosen at random in the circle lies in the shaded region.**



The area of the shaded segment is the area of sector AOF − the area of $\triangle AOF$.

$$\text{Area of sector } AOF = \frac{N}{360}\pi r^2 = \frac{60}{360}\pi(6^2) = 6\pi$$

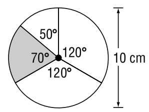
$$\text{Area of } \triangle AOF = \frac{1}{2}bh = \frac{1}{2}(6)(3\sqrt{3}) = 9\sqrt{3}$$

The shaded area is $6\pi - 9\sqrt{3}$ or about 3.26.

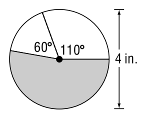
The probability is $\frac{\text{area of segment}}{\text{area of circle}} = \frac{3.26}{36\pi}$ or about 0.03.

Exercises

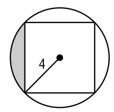
Find the probability that a point in the circle chosen at random lies in the shaded region. Round to the nearest hundredth.

1.


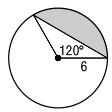
0.19

2.


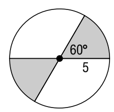
0.53

3.


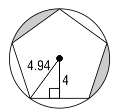
0.09

4.


0.20

5.


0.33

6.


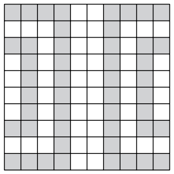
0.10

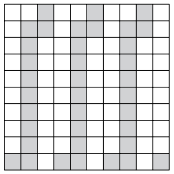
NAME ______________________ DATE __________ PERIOD _____

11-5 Skills Practice

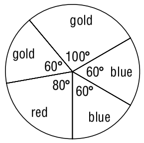
Geometric Probability

Find the probability that a point chosen at random lies in the shaded region.

1.  0.48

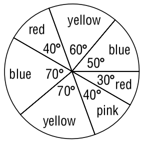
2.  0.37

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 6 inches.



3. red 6.3 in^2, $0.\overline{2}$

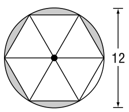
4. gold 12.6 in^2, $0.\overline{4}$



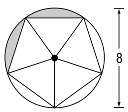
5. blue 9.4 in^2, $0.\overline{3}$

6. yellow 10.2 in^2, $0.3\overline{61}$

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

7. 

16.3 units^2, 0.14

8. 

4.9 units^2, 0.097

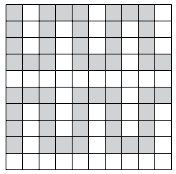
Lesson 11-5

NAME ______________________ DATE __________ PERIOD _____

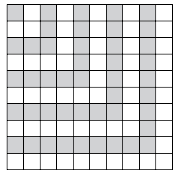
11-5 Practice (Average)

Geometric Probability

Find the probability that a point chosen at random lies in the shaded region.

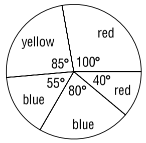
1. 

0.50

2. 

0.45

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of the spinner is 9 meters.

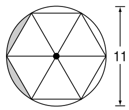


3. red 24.7 m^2, 0.39

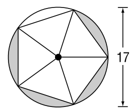
4. blue 23.9 m^2, 0.38

5. yellow 15.0 m^2, 0.24

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

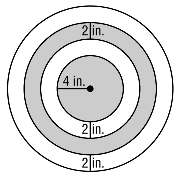
6. 

5.5 units^2, 0.058

7. 

44.2 units^2, 0.195

8. ARCHERY A target consists of four concentric rings. The radius of the center circle is 4 inches, and the circles are spaced 2 inches apart. Find the probability that an arrow shot at random by an inexperienced archer will land in a shaded region. 0.44



Answers

NAME ______________ DATE ________ PERIOD ____

11-5 Reading to Learn Mathematics

Geometric Probability

Pre-Activity **How can geometric probability help you win a game of darts?**

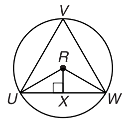
Read the introduction to Lesson 11-5 at the top of page 622 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths? **areas**

Reading the Lesson

1. Explain the difference between a sector of a circle and a segment of a circle **Sample answer: A sector of a circle is bounded by a central angle and its intercept arc, while a segment is bounded by an arc and a chord.**

2. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point.



The area of circle R is **π**.

$\triangle URW$ is a(n) **isosceles** triangle because $\overline{RU}$ and $\overline{RW}$ are **radii** of the same **circle**.

$\angle URW$ is a(n) **central** angle of the circle, and $m\angle URW =$ **120**.

$m\angle RUX =$ **30** and $m\angle RWX =$ **30**.

The angle measures in $\triangle RUX$ are **30**, **60**, and **90**.

$\overline{RU}$ is a **radius** of the circle, so $RU =$ **1**.

$\overline{RX}$ is the leg of $\triangle RUX$ opposite the **30°** angle, so $RX =$ **$\frac{1}{2}$**.

Also, $\overline{UX}$ is the leg of $\triangle RUX$ opposite the **60°** angle, so $UX =$ **$\frac{\sqrt{3}}{2}$**.

$UW =$ **$\sqrt{3}$**, so the area of $\triangle URW$ is $\frac{1}{2} \cdot$ **$\sqrt{3}$** $\cdot$ **$\frac{1}{2}$** $=$ **$\frac{\sqrt{3}}{4}$**.

Then, the area of $\triangle UVW = 3 \cdot$ **$\frac{\sqrt{3}}{4}$** $=$ **$\frac{3\sqrt{3}}{4}$**.

Therefore, the probability that the dart will fall inside the triangle is the ratio of **$\frac{3\sqrt{3}}{4}$** to **π**, which is approximately **0.413** (to the nearest thousandth).

Helping You Remember

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula? **Sample answer: First use $A = \pi r^2$ to find the area of the circle. Then use the measure of the central angle to find out what fraction of the circle the sector is. Multiply the area of the circle by this fraction and you will have the area of the sector.**

Lesson 11-5

NAME ______________ DATE ________ PERIOD ____

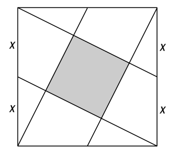
11-5 Enrichment

Polygon Probability

Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

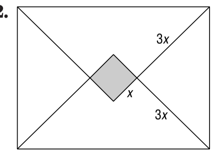
Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

1.



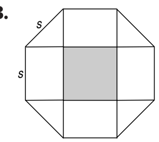
$\frac{1}{5}$ or 0.20

2.



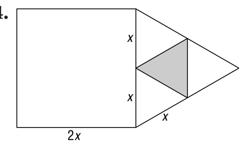
$\frac{1}{24} \approx 0.04$

3.



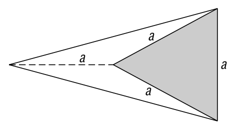
$\frac{1}{2 + 2\sqrt{2}} \approx 0.21$

4.



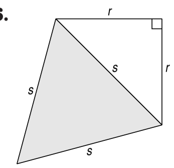
$\frac{\sqrt{3}}{16 + 4\sqrt{3}} \approx 0.08$

5.



$\frac{\sqrt{3}}{2 + \sqrt{3}} \approx 0.46$

6.



$\frac{\sqrt{3}}{\sqrt{3} + 1} \approx 0.63$

Chapter 11 Assessment Answer Key

Form 1
Page 641

1. D
2. B
3. B
4. A
5. C
6. B
7. A
8. C
9. D

Page 642

10. A
11. C
12. B
13. C
14. B
15. B

B: 22 units

Form 2A
Page 643

1. C
2. B
3. A
4. B
5. D
6. A
7. C
8. B

(continued on the next page)

Answers

Chapter 11 Assessment Answer Key

Form 2A *(continued)*

Page 644

9. D

10. C

11. D

12. A

13. D

14. B

B: about 227.3 in^2

Form 2B

Page 645

1. B

2. D

3. C

4. A

5. D

6. B

7. C

8. B

Page 646

9. C

10. A

11. D

12. A

13. B

14. B

B: 104.7 in^2

Chapter 11 Assessment Answer Key

Form 2C

Page 647

1. 173.2 cm^2
2. 30.3 in^2
3. 45 m and 50 m
4. quadrilateral, parallelogram, rectangle
5. 16.5 $units^2$
6. 16 $units^2$
7. 16 $units^2$
8. 32 in^2
9. 64.1 cm^2
10. 46.8 m^2

Page 648

11. 184.3 in^2
12. 42 cm^2
13. 89.7 m^2
14. 30 $units^2$
15. 0.34
16. 0.19
17. 0.34

B: 20 m

Answers

Chapter 11 Assessment Answer Key

Form 2D

Page 649

1. 76.2 cm^2
2. 78 in^2
3. 30 m and 19 m
4. quadrilateral, parallelogram, rectangle, and square
5. 19.9 units^2
6. 16 units^2
7. 24 units^2
8. 36 in^2
9. 584.6 cm^2
10. 84.9 m^2

Page 650

11. 34.8 in^2
12. 189 cm^2
13. 2.3 m^2
14. 14.5 units^2
15. 0.41
16. 0.14
17. 0.09

B: 21 m

Chapter 11 Assessment Answer Key

Form 3

Page 651

1. $41.6\ cm^2$

2. $49\ in^2$

3. $20.2\ in^2$

4. quadrilateral, parallelogram, and rhombus

5. $129\ in^2$

6. $51\ units^2$

7. $336\ m^2$

8. $695.3\ m^2$

9. $90.8\ in^2$

10. $890.2\ cm^2$

Page 652

11. $90.3\ m^2$

12. $23.7\ in^2$

13. $24.3\ cm^2$

14. $23.0\ units^2$

15. 30 squares

16. 0.11

17. 0.12

B: $17.4\ cm^2$

Answers

Chapter 11 Assessment Answer Key

Page 653, Open-Ended Assessment Scoring Rubric

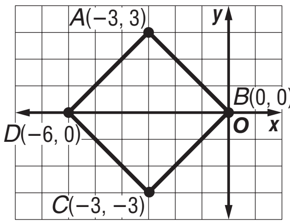
Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and\or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Uses appropriate strategies to solve problems. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and\or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of all problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows partial understanding of most of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and\or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of *using formulas to find the areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, circles, irregular figures, figures graphed on a coordinate plane, segments and sectors of circles, using the slope and\or distance formulas to determine whether a quadrilateral is a parallelogram, rhombus, square, or rectangle, and solving problems involving geometric probability, segments, and sectors*. • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy the requirements of the problems. • No answer may be given.

Chapter 11 Assessment Answer Key

Page 653, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A22, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1. a.



Slope of $\overline{AB} = -1$; slope of $\overline{CD} = -1$; $AB = 3\sqrt{2}$; $CD = 3\sqrt{2}$; slope of $\overline{BC} = 1$; slope of $\overline{DA} = 1$; $BC = 3\sqrt{2}$; and $DA = 3\sqrt{2}$.

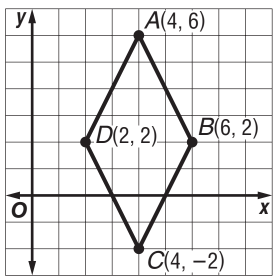
Since the slopes of opposite sides are the same, the figure is a parallelogram. Since the slopes of adjacent sides are opposite reciprocals, indicating perpendicular lines, the figure is either a square or rectangle. Since the lengths of all the sides are congruent, this figure is a square.

b. $(3\sqrt{2})(3\sqrt{2}) = 18$ units^2

2. a. A 30°-60°-90° triangle can be used to find the altitude, $x\sqrt{3}$, and $\overline{PT}$, x. The hypotenuse of ΔPQT is $2x$ or 14. So, $x = 7$, the altitude is $7\sqrt{3}$, and base $PT = 7$. Since the height and base of parallelogram $PQRS$ are known, the area can be calculated.

b. The base is 7 + 14 or 21 and the height is $7\sqrt{3}$. $A = (21)(7\sqrt{3}) \approx 254.6$ in^2.

3. a.

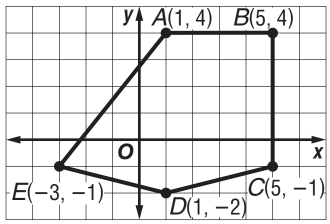


$DB = |2 - 6| = 4$ and $AC = |6 - (-2)| = 8$. $DB = d_1$ and $AC = d_2$, so $A = \frac{1}{2}d_1d_2$, or $\frac{1}{2}(4)(8) = 16$ units^2.

b. By using the ratios of a 30°-60°-90° triangle, the altitude of the trapezoid is $4\sqrt{3}$ cm and the base of ΔMNQ is 4 cm. The longer base of the trapezoid is 4 + 11 or 15 cm. Since $A = \frac{1}{2}h(b_1 + b_2)$, the area is $\frac{1}{2}(4\sqrt{3})(11 + 15)$ or 90.1 cm^2.

c. The area of the circle is 36π. The area of the hexagon is $A = \frac{1}{2}aP$. Since a regular hexagon can be divided into six equilateral triangles, use the 30°-60°-90° triangle ratios to find a and the length of one side of the hexagon. So, $a = 3\sqrt{3}$ in., each side is 6 in., and $P = 36$ in. By substitution, the area of the hexagon is 93.5 in^2. The area of the shaded region is $36\pi - 93.5 \approx 19.6$ in^2.

d.



The shape of this figure contains trapezoid $ABCD$ and $\triangle ADE$. The area of the trapezoid is $(4)\left(\frac{11}{2}\right)$ or 22 units^2. The area of the triangle is $\frac{1}{2}(4)(6)$ or 12 units^2. The total area is 22 + 12 or 34 units^2.

4. a. The area of this segment is $\frac{50}{360} \cdot \pi \cdot 5^2 - \frac{1}{2}(10 \sin 25)(5 \cos 25) \approx$ 1.3 cm^2. The probability that a point chosen at random lies in the shaded region is $\frac{\text{area of the segment}}{\text{area of the circle}}$.

b. 0.02

Answers

Chapter 11 Assessment Answer Key

Vocabulary Test/Review
Page 654

1. $A = \pi r^2$
2. $A = \frac{1}{2}bh$
3. $A = \frac{1}{2}h(b_1 + b_2)$
4. $A = \frac{1}{2}Pa$
5. $A = \frac{N}{360}\pi r^2$
6. $A = bh$
7. $\frac{\text{area of region } X}{\text{area of region } Y}$
8. $A = \frac{1}{2}d_1 d_2$
9. $A = \frac{N}{360}\pi r^2 - \frac{1}{2}bh$
10. a region of a ⊙ bounded by a cent. ∠ and its intercepted arc
11. a figure that cannot be classified into a specific shape such as a square or rectangle
12. the ratio of the area of a specified part of the figure to the total area

Quiz 1
Page 655

1. 68.8 in^2
2. 10 cm^2
3. 61.3 m^2
4. 336 ft^2
5. square

Quiz 2
Page 655

1. 83.1 in^2
2. 86.6 cm^2
3. 482.8 m^2
4. 18.3 cm^2
5. D

Quiz 3
Page 656

1. 198 cm^2
2. 26.3 in^2
3. 331.6 m^2
4. 40 $units^2$
5. 24 $units^2$

Quiz 4
Page 656

1. 0.49
2. 0.11
3. 0.03
4. 0.28
5. 0.09

Chapter 11 Assessment Answer Key

Mid-Chapter Test
Page 657

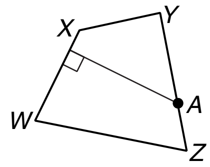
Part I

1. D
2. A
3. B
4. C

Part II

5. 39.6 in^2
6. 120 m^2
7. 83.1 ft^2
8. 374.1 in^2
9. 19.1 m^2

Cumulative Review
Page 658

1. 0.5mm; 75.5 mm to 76.5 mm
2. 

3. 17
4. no; $AB + BC = AC$
5. 25.4, 16.8
6. 109, 71, 4
7. 312.6 mph, 7.4° west of due south
8. 36, 36, 25, 84.5
9. $80\sqrt{3}$ cm^2 or about 138.6 cm^2
10. 243.7 cm^2

Answers

Chapter 11 Assessment Answer Key

Standardized Test Practice

Page 659

1. A **B** C D

2. E **F** G H

3. A B **C** D

4. E **F** G H

5. A B C **D**

6. E **F** G H

7. A B **C** D

8. E F **G** H

Page 660

9. 15

10. 46.6

11. 10.5

12. 125

13. 40

14. $X'(0, -3)$, $Y'(-4, -7)$, $Z'(5, -4)$

15. $m\angle TRU = 94$, $m\angle URV = 39$, $m\angle VRW = 47$

16. 15 in.