

**GLENCOE
MATHEMATICS**

Geometry

Chapter 10 Resource Masters

**Mc
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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

<i>Study Guide and Intervention Workbook</i>	0-07-860191-6
<i>Skills Practice Workbook</i>	0-07-860192-4
<i>Practice Workbook</i>	0-07-860193-2
<i>Reading to Learn Mathematics Workbook</i>	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 10 of these workbooks can be found in the back of this Chapter Resource Masters booklet.



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Geometry
Chapter 10 Resource Masters

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Teacher's Guide to Using the Chapter 10 Resource Masters

The **Fast File** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 10 Resource Masters* includes the core materials needed for Chapter 10. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 10-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 10-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention

Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 10 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

All of the above tests include a free-response Bonus question.

- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.

- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 588–589. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

Reading to Learn Mathematics***Vocabulary Builder***

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 10. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
center		
central angle		
chord		
circle		
circumference		
circumscribed		
diameter		
inscribed		

(continued on the next page)

Reading to Learn Mathematics**Vocabulary Builder** *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
intercepted		
major arc		
minor arc		
pi (π)		
point of tangency		
radius		
secants		
semicircle		
tangent		

10

Learning to Read Mathematics***Proof Builder***

This is a list of key theorems and postulates you will learn in Chapter 10. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 10.1		
Theorem 10.2		
Theorem 10.3		
Theorem 10.4		
Theorem 10.5		
Theorem 10.6		
Theorem 10.7		

(continued on the next page)

Learning to Read Mathematics***Proof Builder*** (continued)

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 10.8		
Theorem 10.9		
Theorem 10.11		
Theorem 10.12		
Theorem 10.13		
Theorem 10.14		
Theorem 10.15		

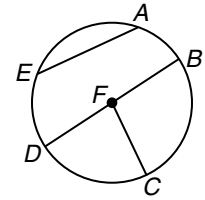
10-1 Study Guide and Intervention

Circles and Circumference

Parts of Circles A **circle** consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.

A segment or line can intersect a circle in several ways.

- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.



chord: \overline{AE} , \overline{BD}
 radius: \overline{FB} , \overline{FC} , \overline{FD}
 diameter: \overline{BD}

Example

- a. **Name the circle.**

The name of the circle is $\odot O$.

- b. **Name radii of the circle.**

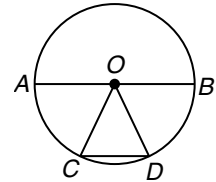
\overline{AO} , \overline{BO} , \overline{CO} , and \overline{DO} are radii.

- c. **Name chords of the circle.**

\overline{AB} and \overline{CD} are chords.

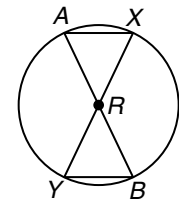
- d. **Name a diameter of the circle.**

\overline{AB} is a diameter.



Exercises

1. Name the circle.
2. Name radii of the circle.
3. Name chords of the circle.
4. Name diameters of the circle.
5. Find AR if AB is 18 millimeters.
6. Find AR and AB if RY is 10 inches.
7. Is $\overline{AB} \cong \overline{XY}$? Explain.



10-1 Study Guide and Intervention *(continued)*

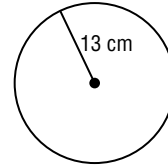
Circles and Circumference

Circumference The **circumference** of a circle is the distance around the circle.

Circumference	For a circumference of C units and a diameter of d units or a radius of r units, $C = \pi d$ or $C = 2\pi r$.
----------------------	---

Example Find the circumference of the circle to the nearest hundredth.

$$\begin{aligned}
 C &= 2\pi r && \text{Circumference formula} \\
 &= 2\pi(13) && r = 13 \\
 &\approx 81.68 && \text{Use a calculator.}
 \end{aligned}$$



The circumference is about 81.68 centimeters.

Exercises

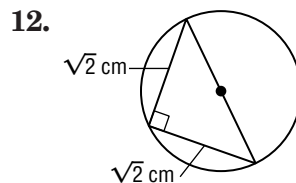
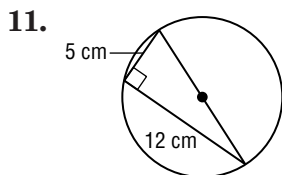
Find the circumference of a circle with the given radius or diameter. Round to the nearest hundredth.

- | | |
|------------------------|-----------------------|
| 1. $r = 8$ cm | 2. $r = 3\sqrt{2}$ ft |
| 3. $r = 4.1$ cm | 4. $d = 10$ in. |
| 5. $d = \frac{1}{3}$ m | 6. $d = 18$ yd |

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- | | |
|--|--|
| 7. $r = 4$ cm
$d =$ _____, $C =$ _____ | 8. $d = 6$ ft
$r =$ _____, $C =$ _____ |
| 9. $r = 12$ cm
$d =$ _____, $C =$ _____ | 10. $d = 15$ in.
$r =$ _____, $C =$ _____ |

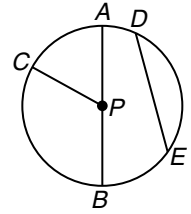
Find the exact circumference of each circle.



10-1 Skills Practice

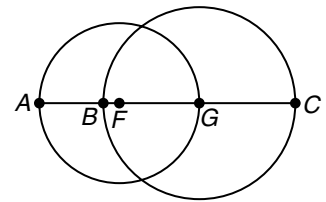
Circles and Circumference

For Exercises 1–5, refer to the circle.



- Name the circle.
- Name a radius.
- Name a chord.
- Name a diameter.
- Name a radius not drawn as part of a diameter.
- Suppose the diameter of the circle is 16 centimeters. Find the radius.
- If $PC = 11$ inches, find AB .

The diameters of $\odot F$ and $\odot G$ are 5 and 6 units, respectively. Find each measure.

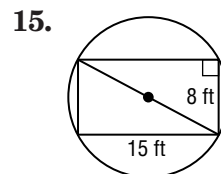
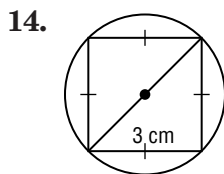


- BF
- AB

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- | | |
|--|--|
| 10. $r = 8$ cm
$d =$ _____, $C \approx$ _____ | 11. $r = 13$ ft
$d =$ _____, $C \approx$ _____ |
| 12. $d = 9$ m
$r =$ _____, $C \approx$ _____ | 13. $C = 35.7$ in.
$d \approx$ _____, $r \approx$ _____ |

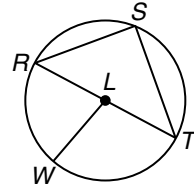
Find the exact circumference of each circle.



10-1 Practice

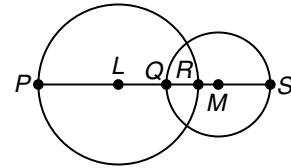
Circles and Circumference

For Exercises 1–5, refer to the circle.



- Name the circle.
- Name a radius.
- Name a chord.
- Name a diameter.
- Name a radius not drawn as part of a diameter.
- Suppose the radius of the circle is 3.5 yards. Find the diameter.
- If $RT = 19$ meters, find LW .

The diameters of $\odot L$ and $\odot M$ are 20 and 13 units, respectively. Find each measure if $QR = 4$.

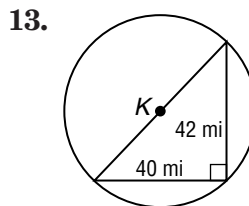
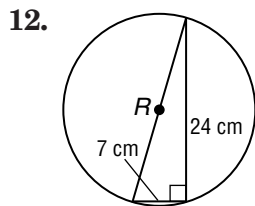


- LQ
- RM

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- $r = 7.5$ mm
 $d = \underline{\hspace{2cm}}, C \approx \underline{\hspace{2cm}}$
- $C = 227.6$ yd
 $d \approx \underline{\hspace{2cm}}, r \approx \underline{\hspace{2cm}}$

Find the exact circumference of each circle.



SUNDIALS For Exercises 14 and 15, use the following information.

Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

- Find the radius of the sundial.
- Find the circumference of the sundial to the nearest hundredth.

10-1

Reading to Learn Mathematics

Circles and Circumference

Pre-Activity How far does a carousel animal travel in one rotation?

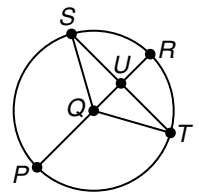
Read the introduction to Lesson 10-1 at the top of page 522 in your textbook.

How could you measure the approximate distance around the circular carousel using everyday measuring devices?

Reading the Lesson

1. Refer to the figure.

- Name the circle.
- Name four radii of the circle.
- Name a diameter of the circle.
- Name two chords of the circle.



2. Match each description from the first column with the best term from the second column. (Some terms in the second column may be used more than once or not at all.)

- | | |
|---|------------------|
| a. a segment whose endpoints are on a circle | i. radius |
| b. the set of all points in a plane that are the same distance from a given point | ii. diameter |
| c. the distance between the center of a circle and any point on the circle | iii. chord |
| d. a chord that passes through the center of a circle | iv. circle |
| e. a segment whose endpoints are the center and any point on a circle | v. circumference |
| f. a chord made up of two collinear radii | |
| g. the distance around a circle | |

3. Which equations correctly express a relationship in a circle?

A. $d = 2r$

B. $C = \pi r$

C. $C = 2d$

D. $d = \frac{C}{\pi}$

E. $r = \frac{d}{\pi}$

F. $C = r^2$

G. $C = 2\pi r$

H. $d = \frac{1}{2}r$

Helping You Remember

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *diameter* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part.

10-1 Enrichment

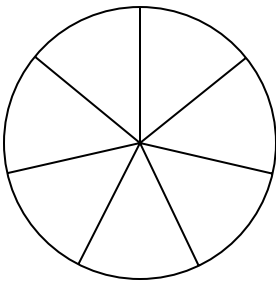
The Four Color Problem

Mapmakers have long believed that only four colors are necessary to distinguish among any number of different countries on a plane map. Countries that meet only at a point may have the same color provided they do not have an actual border. The conjecture that four colors are sufficient for every conceivable plane map eventually attracted the attention of mathematicians and became known as the “four-color problem.” Despite extraordinary efforts over many years to solve the problem, no definite answer was obtained until the 1980s. Four colors are indeed sufficient, and the proof was accomplished by making ingenious use of computers.

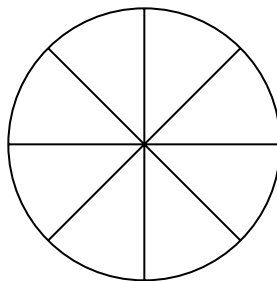
The following problems will help you appreciate some of the complexities of the four-color problem. For these “maps,” assume that each closed region is a different country.

1. What is the minimum number of colors necessary for each map?

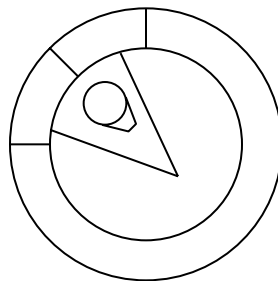
a.



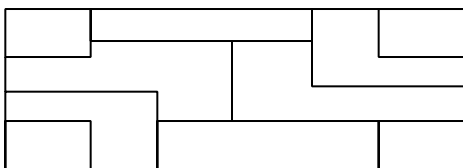
b.



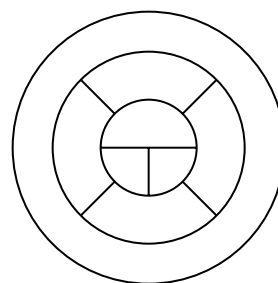
c.



d.



e.

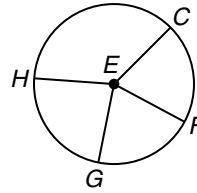


2. Draw some plane maps on separate sheets. Show how each can be colored using four colors. Then determine whether fewer colors would be enough.

10-2 Study Guide and Intervention

Angles and Arcs

Angles and Arcs A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.



\widehat{GF} is a minor arc.
 \widehat{CHG} is a major arc.
 $\angle GEF$ is a central angle.

Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360.
- The measure of a minor arc equals the measure of its central angle.
- The measure of a major arc is 360 minus the measure of the minor arc.
- Two arcs are congruent if and only if their corresponding central angles are congruent.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (**Arc Addition Postulate**)

$$m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$$

$$m\widehat{CF} = m\angle CEF$$

$$m\widehat{CGF} = 360 - m\widehat{CF}$$

$$\widehat{CF} \cong \widehat{FG} \text{ if and only if } \angle CEF \cong \angle FEG.$$

$$m\widehat{CF} + m\widehat{FG} = m\widehat{CG}$$

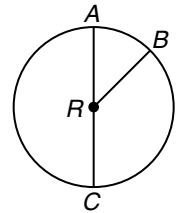
Example

In $\odot R$, $m\angle ARB = 42$ and \overline{AC} is a diameter.

Find $m\widehat{AB}$ and $m\widehat{ACB}$.

$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\widehat{AB} = 42$.

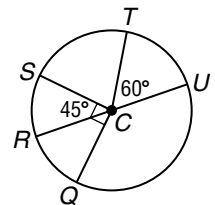
Thus $m\widehat{ACB} = 360 - 42$ or 318.



Exercises

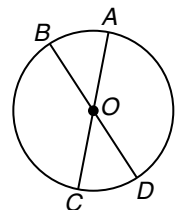
Find each measure.

- | | |
|------------------|------------------|
| 1. $m\angle SCT$ | 2. $m\angle SCU$ |
| 3. $m\angle SCQ$ | 4. $m\angle QCT$ |



If $m\angle BOA = 44$, find each measure.

- | | |
|---------------------|---------------------|
| 5. $m\widehat{BA}$ | 6. $m\widehat{BC}$ |
| 7. $m\widehat{CD}$ | 8. $m\widehat{ACB}$ |
| 9. $m\widehat{BCD}$ | 10. $m\widehat{AD}$ |



10-2 Study Guide and Intervention *(continued)*

Angles and Arcs

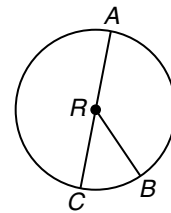
Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

Example In $\odot R$, $m\angle ARB = 135$, $RB = 8$, and \overline{AC} is a diameter. Find the length of \widehat{AB} .

$m\angle ARB = 135$, so $m\widehat{AB} = 135$. Using the formula $C = 2\pi r$, the circumference is $2\pi(8)$ or 16π . To find the length of \widehat{AB} , write a proportion to compare each part to its whole.

$\frac{\text{length of } \widehat{AB}}{\text{circumference}} = \frac{\text{degree measure of arc}}{\text{degree measure of circle}}$	Proportion
$\frac{\ell}{16\pi} = \frac{135}{360}$	Substitution
$\ell = \frac{(16\pi)(135)}{360}$	Multiply each side by 16π .
$\ell = 6\pi$	Simplify.

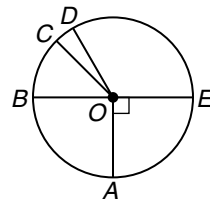
The length of \widehat{AB} is 6π or about 18.85 units.



Exercises

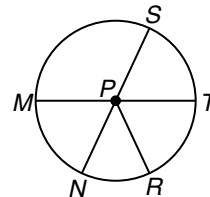
The diameter of $\odot O$ is 24 units long. Find the length of each arc for the given angle measure.

1. \widehat{DE} if $m\angle DOE = 120$
2. \widehat{DEA} if $m\angle DOE = 120$
3. \widehat{BC} if $m\angle COB = 45$
4. \widehat{CBA} if $m\angle COB = 45$



The diameter of $\odot P$ is 15 units long and $\angle SPT \cong \angle RPT$. Find the length of each arc for the given angle measure.

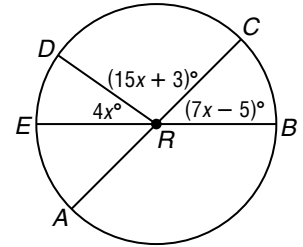
5. \widehat{RT} if $m\angle SPT = 70$
6. \widehat{NR} if $m\angle RPT = 50$
7. \widehat{MST}
8. \widehat{MRS} if $m\angle MPS = 140$



10-2 Skills Practice

Angles and Arcs

ALGEBRA In $\odot R$, \overline{AC} and \overline{EB} are diameters. Find each measure.



1. $m\angle ERD$

2. $m\angle CRD$

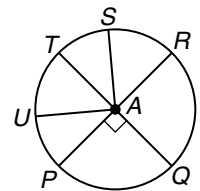
3. $m\angle BRC$

4. $m\angle ARB$

5. $m\angle ARE$

6. $m\angle BRD$

In $\odot A$, $m\angle PAU = 40$, $\angle PAU \cong \angle SAT$, and $\angle RAS \cong \angle TAU$. Find each measure.



7. $m\widehat{PQ}$

8. $m\widehat{PQR}$

9. $m\widehat{ST}$

10. $m\widehat{RS}$

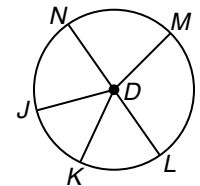
11. $m\widehat{RSU}$

12. $m\widehat{STP}$

13. $m\widehat{PQS}$

14. $m\widehat{PRU}$

The diameter of $\odot D$ is 18 units long. Find the length of each arc for the given angle measure.



15. \widehat{LM} if $m\angle LDM = 100$

16. \widehat{MN} if $m\angle MDN = 80$

17. \widehat{KL} if $m\angle KDL = 60$

18. \widehat{NJK} if $m\angle NDK = 120$

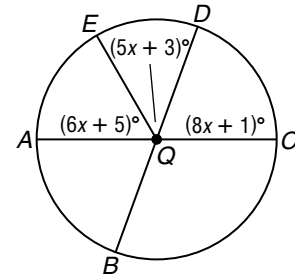
19. \widehat{KLM} if $m\angle KDM = 160$

20. \widehat{JK} if $m\angle JDK = 50$

10-2 Practice

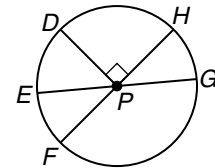
Angles and Arcs

ALGEBRA In $\odot Q$, \overline{AC} and \overline{BD} are diameters. Find each measure.



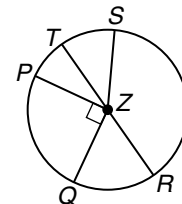
1. $m\angle AQE$
2. $m\angle DQE$
3. $m\angle CQD$
4. $m\angle BQC$
5. $m\angle CQE$
6. $m\angle AQD$

In $\odot P$, $m\angle GPH = 38$. Find each measure.



7. $m\widehat{EF}$
8. $m\widehat{DE}$
9. $m\widehat{FG}$
10. $m\widehat{DHG}$
11. $m\widehat{DFG}$
12. $m\widehat{DGE}$

The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.



13. \widehat{PQT} if $m\angle QZT = 120$
14. \widehat{QR} if $m\angle QZR = 60$
15. \widehat{PQR} if $m\angle PZR = 150$
16. \widehat{QPS} if $m\angle QZS = 160$

HOMEWORK For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

Homework	
Less than 1 hour	8%
1–2 hours	29%
2–3 hours	58%
3–4 hours	3%
Over 4 hours	2%

17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?

18. Describe the arcs associated with each category.

10-2 Reading to Learn Mathematics

Angles and Arcs

Pre-Activity What kinds of angles do the hands on a clock form?

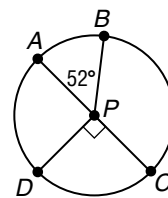
Read the introduction to Lesson 10-2 at the top of page 529 in your textbook.

- What is the measure of the angle formed by the hour hand and the minute hand of the clock at 5:00?
- What is the measure of the angle formed by the hour hand and the minute hand at 10:30? (Hint: How has each hand moved since 10:00?)

Reading the Lesson

1. Refer to $\odot P$. Indicate whether each statement is *true* or *false*.

- \widehat{DAB} is a major arc.
- \widehat{ADC} is a semicircle.
- $\widehat{AD} \cong \widehat{CD}$
- \widehat{DA} and \widehat{AB} are adjacent arcs.
- $\angle BPC$ is an acute central angle.
- $\angle DPA$ and $\angle BPA$ are supplementary central angles.



2. Refer to the figure in Exercise 1. Give each of the following arc measures.

- | | |
|---------------------|---------------------|
| a. $m\widehat{AB}$ | b. $m\widehat{CD}$ |
| c. $m\widehat{BC}$ | d. $m\widehat{ADC}$ |
| e. $m\widehat{DAB}$ | f. $m\widehat{DCB}$ |
| g. $m\widehat{DAC}$ | h. $m\widehat{BDA}$ |

3. Underline the correct word or number to form a true statement.

- The arc measure of a semicircle is (90/180/360).
- Arcs of a circle that have exactly one point in common are (congruent/opposite/adjacent) arcs.
- The measure of a major arc is greater than (0/90/180) and less than (90/180/360).
- Suppose a set of central angles of a circle have interiors that do not overlap. If the angles and their interiors contain all points of the circle, then the sum of the measures of the central angles is (90/270/360).
- The measure of an arc formed by two adjacent arcs is the (sum/difference/product) of the measures of the two arcs.
- The measure of a minor arc is greater than (0/90/180) and less than (90/180/360).

Helping You Remember

- A good way to remember something is to explain it to someone else. Suppose your classmate Luis does not like to work with proportions. What is a way that he can find the length of a minor arc of a circle without solving a proportion?

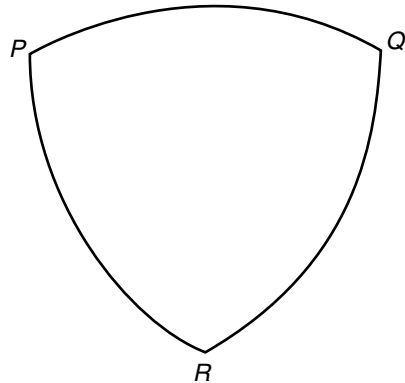
10-2 Enrichment

Curves of Constant Width

A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.

1. Use a metric ruler to find the distance from P to any point on the opposite side.
2. Find the distance from Q to the opposite side.
3. What is the distance from R to the opposite side?



The Reuleaux triangle is made of three arcs. In the example shown, \overline{PQ} has center R , \overline{QR} has center P , and \overline{PR} has center Q .

4. Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the square while keeping its sides in contact with the sides of the square.
5. Make a different curve of constant width by starting with the five points below and following the steps given.

Step 1: Place the point of your compass on D with opening DA . Make an arc with endpoints A and B .



Step 2: Make another arc from B to C that has center E .



Step 3: Continue this process until you have five arcs drawn.



Some countries use shapes like this for coins. They are useful because they can be distinguished by touch, yet they will work in vending machines because of their constant width.

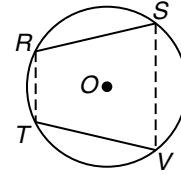
6. Measure the width of the figure you made in Exercise 5. Draw two parallel lines with the distance between them equal to the width you found. On a piece of paper, trace the five-sided figure and cut it out. Show that it will roll between the lines drawn.

10-3 Study Guide and Intervention

Arcs and Chords

Arcs and Chords Points on a circle determine both chords and arcs. Several properties are related to points on a circle.

- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.



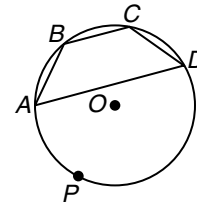
$\overline{RS} \cong \overline{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.
 $RSVT$ is inscribed in $\odot O$.
 $\odot O$ is circumscribed about $RSVT$.

Example

Trapezoid $ABCD$ is inscribed in $\odot O$.

If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $m\widehat{BC} = 50$, what is $m\widehat{APD}$?

Chords \overline{AB} , \overline{BC} , and \overline{CD} are congruent, so \widehat{AB} , \widehat{BC} , and \widehat{CD} are congruent. $m\widehat{BC} = 50$, so $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} = 50 + 50 + 50 = 150$. Then $m\widehat{APD} = 360 - 150$ or 210 .

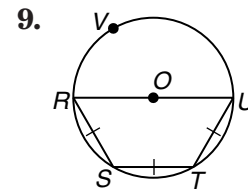
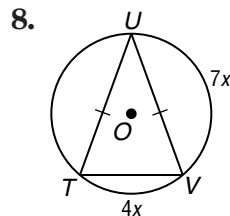
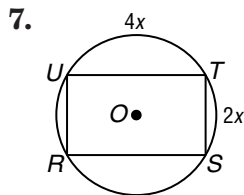


Exercises

Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.

- | | | |
|------------|-------------|-------------|
| 1. hexagon | 2. pentagon | 3. triangle |
| 4. square | 5. octagon | 6. 36-gon |

Determine the measure of each arc of the circle circumscribed about the polygon.

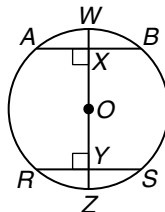


10-3 Study Guide and Intervention *(continued)*

Arcs and Chords

Diameters and Chords

- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
- In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

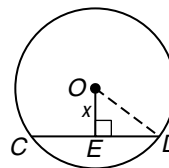


If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\widehat{AW} \cong \widehat{WB}$.
 If $OX = OY$, then $\overline{AB} \cong \overline{RS}$.
 If $\overline{AB} \cong \overline{RS}$, then \overline{AB} and \overline{RS} are equidistant from point O .

Example In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x .

A diameter or radius perpendicular to a chord bisects the chord, so ED is half of CD .

$$\begin{aligned} ED &= \frac{1}{2}(24) \\ &= 12 \end{aligned}$$



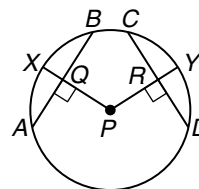
Use the Pythagorean Theorem to find x in $\triangle OED$.

$(OE)^2 + (ED)^2 = (OD)^2$	Pythagorean Theorem
$x^2 + 12^2 = 15^2$	Substitution
$x^2 + 144 = 225$	Multiply.
$x^2 = 81$	Subtract 144 from each side.
$x = 9$	Take the square root of each side.

Exercises

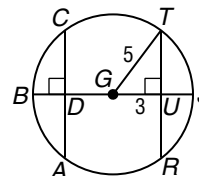
In $\odot P$, $CD = 24$ and $m\widehat{CY} = 45$. Find each measure.

- | | | |
|--------------------|--------------------|--------------------|
| 1. AQ | 2. RC | 3. QB |
| 4. AB | 5. $m\widehat{DY}$ | 6. $m\widehat{AB}$ |
| 7. $m\widehat{AX}$ | 8. $m\widehat{XB}$ | 9. $m\widehat{CD}$ |



In $\odot G$, $DG = GU$ and $AC = RT$. Find each measure.

- | | | |
|----------|----------|---------------------|
| 10. TU | 11. TR | 12. $m\widehat{TS}$ |
| 13. CD | 14. GD | 15. $m\widehat{AB}$ |

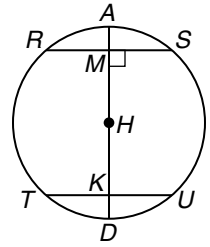


16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius.

10-3 Skills Practice

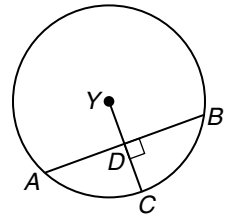
Arcs and Chords

In $\odot H$, $m\widehat{RS} = 82$, $m\widehat{TU} = 82$, $RS = 46$, and $\overline{TU} \cong \overline{RS}$. Find each measure.



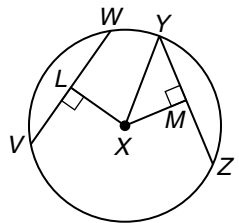
- | | |
|--------------------|--------------------|
| 1. TU | 2. TK |
| 3. MS | 4. $m\angle HKU$ |
| 5. $m\widehat{AS}$ | 6. $m\widehat{AR}$ |
| 7. $m\widehat{TD}$ | 8. $m\widehat{DU}$ |

The radius of $\odot Y$ is 34, $AB = 60$, and $m\widehat{AC} = 71$. Find each measure.



- | | |
|--------------------|---------------------|
| 9. $m\widehat{BC}$ | 10. $m\widehat{AB}$ |
| 11. AD | 12. BD |
| 13. YD | 14. DC |

In $\odot X$, $LX = MX$, $XY = 58$, and $VW = 84$. Find each measure.



- | | |
|----------|----------|
| 15. YZ | 16. YM |
| 17. MX | 18. MZ |
| 19. LV | 20. LX |

10-3 Practice

Arcs and Chords

In $\odot E$, $m\widehat{HQ} = 48$, $HI = JK$, and $JR = 7.5$. Find each measure.

1. $m\widehat{HI}$

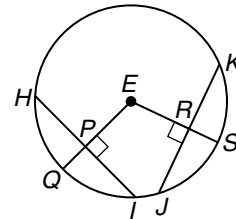
2. $m\widehat{QI}$

3. $m\widehat{JK}$

4. HI

5. PI

6. JK



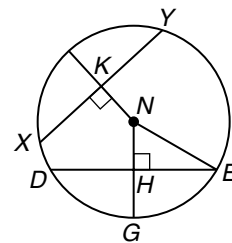
The radius of $\odot N$ is 18, $NK = 9$, and $m\widehat{DE} = 120$. Find each measure.

7. $m\widehat{GE}$

8. $m\angle HNE$

9. $m\angle HEN$

10. HN



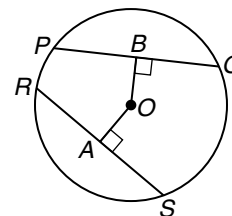
The radius of $\odot O = 32$, $\widehat{PQ} \cong \widehat{RS}$, and $PQ = 56$. Find each measure.

11. PB

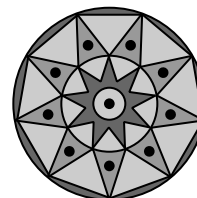
14. BQ

12. OB

16. RS



13. MANDALAS The base figure in a mandala design is a nine-pointed star. Find the measure of each arc of the circle circumscribed about the star.



10-3 Reading to Learn Mathematics

Arcs and Chords

Pre-Activity How do the grooves in a Belgian waffle iron model segments in a circle?

Read the introduction to Lesson 10-3 at the top of page 536 in your textbook.

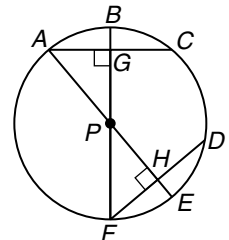
What do you observe about any two of the grooves in the waffle iron shown in the picture in your textbook?

Reading the Lesson

- Supply the missing words or phrases to form true statements.
 - In a circle, if a radius is _____ to a chord, then it bisects the chord and its _____.
 - In a circle or in _____ circles, two _____ are congruent if and only if their corresponding chords are congruent.
 - In a circle or in _____ circles, two chords are congruent if they are _____ from the center.
 - A polygon is inscribed in a circle if all of its _____ lie on the circle.
 - All of the sides of an inscribed polygon are _____ of the circle.

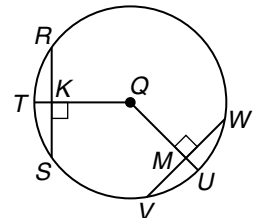
2. If $\odot P$ has a diameter 40 centimeters long, and $AC = FD = 24$ centimeters, find each measure.

- | | |
|---------|---------|
| a. PA | b. AG |
| c. PE | d. PH |
| e. HE | f. FG |



3. In $\odot Q$, $RS = VW$ and $m\widehat{RS} = 70$. Find each measure.

- | | |
|--------------------|--------------------|
| a. $m\widehat{RT}$ | b. $m\widehat{ST}$ |
| c. $m\widehat{VW}$ | d. $m\widehat{VU}$ |



4. Find the measure of each arc of a circle that is circumscribed about the polygon.

- | | |
|----------------------------|-----------------------|
| a. an equilateral triangle | b. a regular pentagon |
| c. a regular hexagon | d. a regular decagon |
| e. a regular dodecagon | f. a regular n -gon |

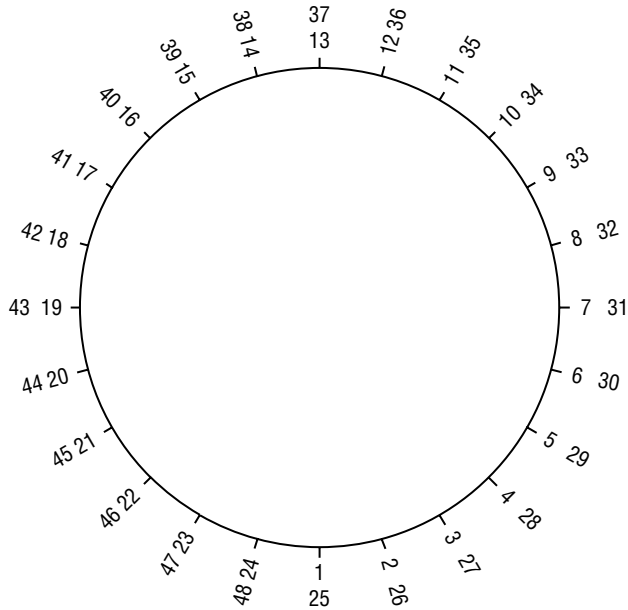
Helping You Remember

5. Some students have trouble distinguishing between *inscribed* and *circumscribed* figures. What is an easy way to remember which is which?

10-3 Enrichment

Patterns from Chords

Some beautiful and interesting patterns result if you draw chords to connect evenly spaced points on a circle. On the circle shown below, 24 points have been marked to divide the circle into 24 equal parts. Numbers from 1 to 48 have been placed beside the points. Study the diagram to see exactly how this was done.

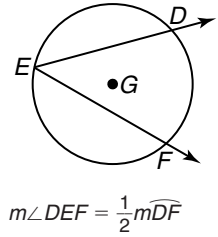


- Use your ruler and pencil to draw chords to connect numbered points as follows: 1 to 2, 2 to 4, 3 to 6, 4 to 8, and so on. Keep doubling until you have gone all the way around the circle. What kind of pattern do you get?
- Copy the original circle, points, and numbers. Try other patterns for connecting points. For example, you might try tripling the first number to get the number for the second endpoint of each chord. Keep special patterns for a possible class display.

10-4 Study Guide and Intervention

Inscribed Angles

Inscribed Angles An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In $\odot G$, inscribed $\angle DEF$ intercepts \widehat{DF} .



Inscribed Angle Theorem	If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.
--------------------------------	---

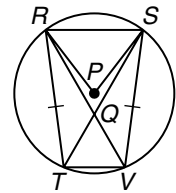
Example In $\odot G$ above, $m\widehat{DF} = 90$. Find $m\angle DEF$.
 $\angle DEF$ is an inscribed angle so its measure is half of the intercepted arc.

$$m\angle DEF = \frac{1}{2}m\widehat{DF}$$

$$= \frac{1}{2}(90) \text{ or } 45$$

Exercises

Use $\odot P$ for Exercises 1–10. In $\odot P$, $\overline{RS} \parallel \overline{TV}$ and $\overline{RT} \cong \overline{SV}$.



1. Name the intercepted arc for $\angle RTS$.
2. Name an inscribed angle that intercepts \widehat{SV} .

In $\odot P$, $m\widehat{SV} = 120$ and $m\angle RPS = 76$. Find each measure.

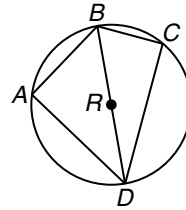
- | | |
|--------------------|---------------------|
| 3. $m\angle PRS$ | 4. $m\widehat{RSV}$ |
| 5. $m\widehat{RT}$ | 6. $m\angle RVT$ |
| 7. $m\angle QRS$ | 8. $m\angle STV$ |
| 9. $m\widehat{TV}$ | 10. $m\angle SVT$ |

10-4 Study Guide and Intervention *(continued)*

Inscribed Angles

Angles of Inscribed Polygons An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.

- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.



If \widehat{BCD} is a semicircle, then $m\angle BCD = 90$.

For inscribed quadrilateral $ABCD$,
 $m\angle A + m\angle C = 180$ and
 $m\angle ABC + m\angle ADC = 180$.

Example

In $\odot R$ above, $BC = 3$ and $BD = 5$. Find each measure.

a. $m\angle C$

$\angle C$ intercepts a semicircle. Therefore $\angle C$ is a right angle and $m\angle C = 90$.

b. CD

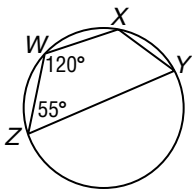
$\triangle BCD$ is a right triangle, so use the Pythagorean Theorem to find CD .

$$\begin{aligned} (CD)^2 + (BC)^2 &= (BD)^2 \\ (CD)^2 + 3^2 &= 5^2 \\ (CD)^2 &= 25 - 9 \\ (CD)^2 &= 16 \\ CD &= 4 \end{aligned}$$

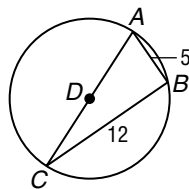
Exercises

Find the measure of each angle or segment for each figure.

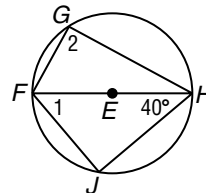
1. $m\angle X$, $m\angle Y$



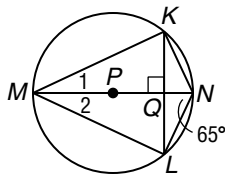
2. AD



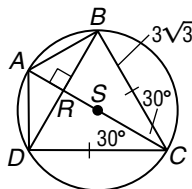
3. $m\angle 1$, $m\angle 2$



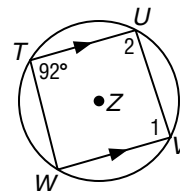
4. $m\angle 1$, $m\angle 2$



5. AB , AC



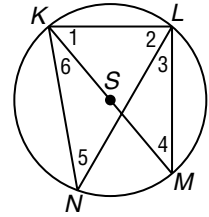
6. $m\angle 1$, $m\angle 2$



10-4 Skills Practice

Inscribed Angles

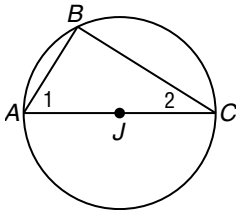
In $\odot S$, $m\widehat{KL} = 80$, $m\widehat{LM} = 100$, and $m\widehat{MN} = 60$. Find the measure of each angle.



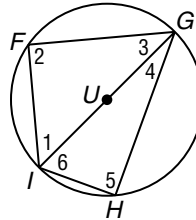
1. $m\angle 1$
2. $m\angle 2$
3. $m\angle 3$
4. $m\angle 4$
5. $m\angle 5$
6. $m\angle 6$

ALGEBRA Find the measure of each numbered angle.

7. $m\angle 1 = 5x - 2$, $m\angle 2 = 2x + 8$

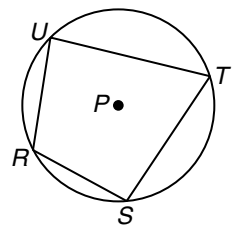


8. $m\angle 1 = 5x$, $m\angle 3 = 3x + 10$,
 $m\angle 4 = y + 7$, $m\angle 6 = 3y + 11$



Quadrilateral $RSTU$ is inscribed in $\odot P$ such that $m\widehat{STU} = 220$ and $m\angle S = 95$. Find each measure.

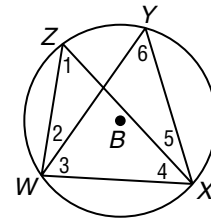
9. $m\angle R$
10. $m\angle T$
11. $m\angle U$
12. $m\widehat{SRU}$
13. $m\widehat{RUT}$
14. $m\widehat{RST}$



10-4 Practice

Inscribed Angles

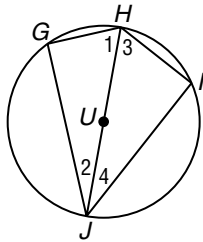
In $\odot B$, $m\widehat{WX} = 104$, $m\widehat{WZ} = 88$, and $m\angle ZWY = 26$. Find the measure of each angle.



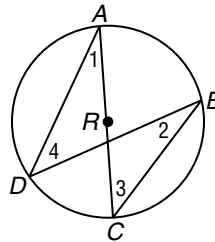
1. $m\angle 1$
2. $m\angle 2$
3. $m\angle 3$
4. $m\angle 4$
5. $m\angle 5$
6. $m\angle 6$

ALGEBRA Find the measure of each numbered angle.

7. $m\angle 1 = 5x + 2$, $m\angle 2 = 2x - 3$
 $m\angle 3 = 7y - 1$, $m\angle 4 = 2y + 10$

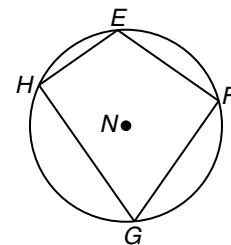


8. $m\angle 1 = 4x - 7$, $m\angle 2 = 2x + 11$,
 $m\angle 3 = 5y - 14$, $m\angle 4 = 3y + 8$

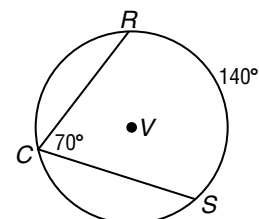


Quadrilateral $EFGH$ is inscribed in $\odot N$ such that $m\widehat{FG} = 97$, $m\widehat{GH} = 117$, and $m\widehat{EHG} = 164$. Find each measure.

9. $m\angle E$
10. $m\angle F$
11. $m\angle G$
12. $m\angle H$



13. PROBABILITY In $\odot V$, point C is randomly located so that it does not coincide with points R or S . If $m\widehat{RS} = 140$, what is the probability that $m\angle RCS = 70$?



10-4 Reading to Learn Mathematics

Inscribed Angles

Pre-Activity How is a socket like an inscribed polygon?

Read the introduction to Lesson 10-4 at the top of page 544 in your textbook.

- Why do you think regular hexagons are used rather than squares for the “hole” in a socket?
- Why do you think regular hexagons are used rather than regular polygons with more sides?

Reading the Lesson

- Underline the correct word or phrase to form a true statement.
 - An angle whose vertex is on a circle and whose sides contain chords of the circle is called a(n) (central/inscribed/circumscribed) angle.
 - Every inscribed angle that intercepts a semicircle is a(n) (acute/right/obtuse) angle.
 - The opposite angles of an inscribed quadrilateral are (congruent/complementary/supplementary).
 - An inscribed angle that intercepts a major arc is a(n) (acute/right/obtuse) angle.
 - Two inscribed angles of a circle that intercept the same arc are (congruent/complementary/supplementary).
 - If a triangle is inscribed in a circle and one of the sides of the triangle is a diameter of the circle, the diameter is (the longest side of an acute triangle/a leg of an isosceles triangle/the hypotenuse of a right triangle).

- Refer to the figure. Find each measure.

a. $m\angle ABC$

b. $m\widehat{CD}$

c. $m\widehat{AD}$

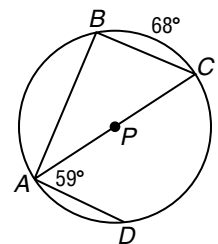
d. $m\angle BAC$

e. $m\angle BCA$

f. $m\widehat{AB}$

g. $m\widehat{BCD}$

h. $m\widehat{BDA}$



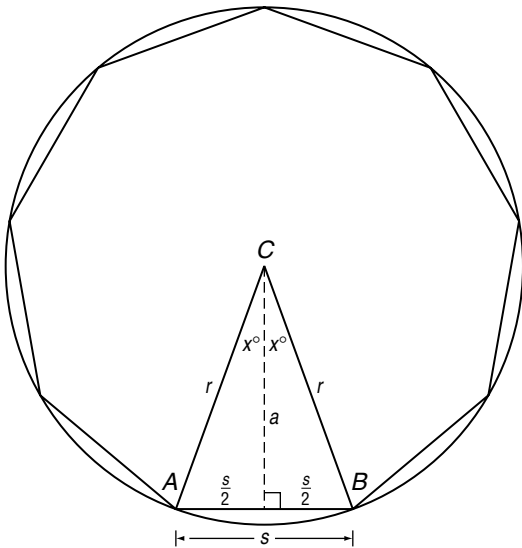
Helping You Remember

- A good way to remember a geometric relationship is to visualize it. Describe how you could make a sketch that would help you remember the relationship between the measure of an inscribed angle and the measure of its intercepted arc.

10-4 Enrichment

Formulas for Regular Polygons

Suppose a regular polygon of n sides is inscribed in a circle of radius r . The figure shows one of the isosceles triangles formed by joining the endpoints of one side of the polygon to the center C of the circle. In the figure, s is the length of each side of the regular polygon, and a is the length of the segment from C perpendicular to \overline{AB} .



Use your knowledge of triangles and trigonometry to solve the following problems.

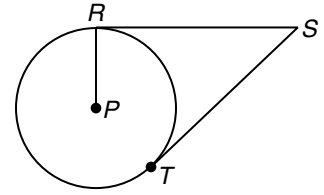
1. Find a formula for x in terms of the number of sides n of the polygon.
2. Find a formula for s in terms of the number of n and r . Use trigonometry.
3. Find a formula for a in terms of n and r . Use trigonometry.
4. Find a formula for the *perimeter* of the regular polygon in terms of n and r .

10-5 Study Guide and Intervention

Tangents

Tangents A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.

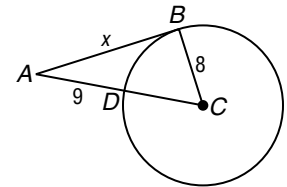


$\overline{RP} \perp \overline{SR}$ if and only if \overline{SR} is tangent to $\odot P$.

If \overline{SR} and \overline{ST} are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

Example \overline{AB} is tangent to $\odot C$. Find x .

\overline{AB} is tangent to $\odot C$, so \overline{AB} is perpendicular to radius \overline{BC} . \overline{CD} is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17 . Use the Pythagorean Theorem with right $\triangle ABC$.

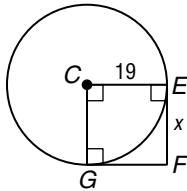


$(AB)^2 + (BC)^2 = (AC)^2$	Pythagorean Theorem
$x^2 + 8^2 = 17^2$	Substitution
$x^2 + 64 = 289$	Multiply.
$x^2 = 225$	Subtract 64 from each side.
$x = 15$	Take the square root of each side.

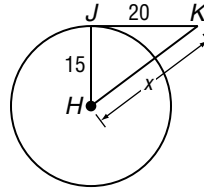
Exercises

Find x . Assume that segments that appear to be tangent are tangent.

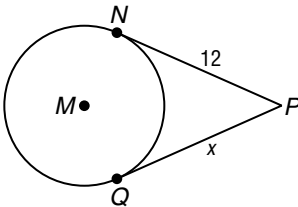
1.



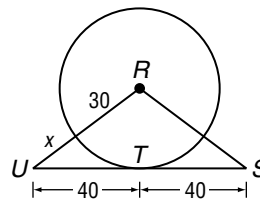
2.



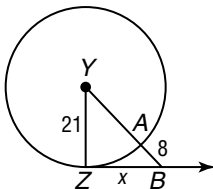
3.



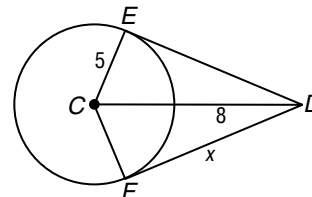
4.



5.



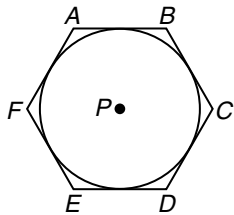
6.



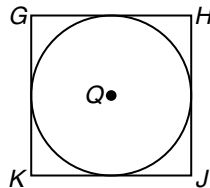
10-5 Study Guide and Intervention *(continued)*

Tangents

Circumscribed Polygons When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.



Hexagon $ABCDEF$ is circumscribed about $\odot P$.
 \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} , \overline{EF} , and \overline{FA} are tangent to $\odot P$.



Square $GHJK$ is circumscribed about $\odot Q$.
 \overline{GH} , \overline{JH} , \overline{JK} , and \overline{KG} are tangent to $\odot Q$.

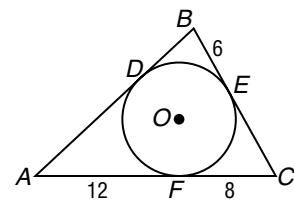
Example $\triangle ABC$ is circumscribed about $\odot O$.

Find the perimeter of $\triangle ABC$.

$\triangle ABC$ is circumscribed about $\odot O$, so points D , E , and F are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.

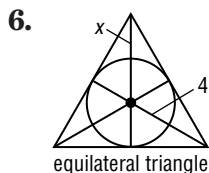
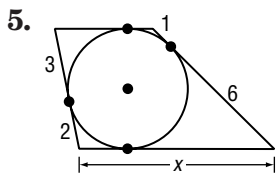
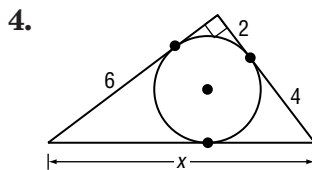
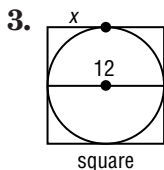
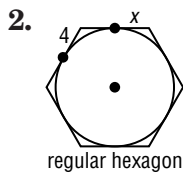
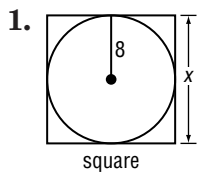
$$\begin{aligned} P &= AD + AF + BE + BD + CF + CE \\ &= 12 + 12 + 6 + 6 + 8 + 8 \\ &= 52 \end{aligned}$$

The perimeter is 52 units.



Exercises

Find x . Assume that segments that appear to be tangent are tangent.

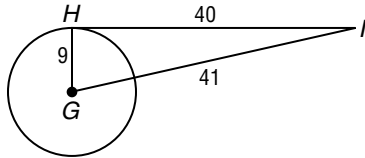


10-5 Skills Practice

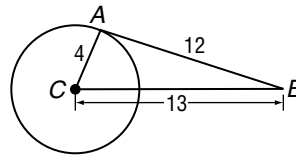
Tangents

Determine whether each segment is tangent to the given circle.

1. \overline{HI}

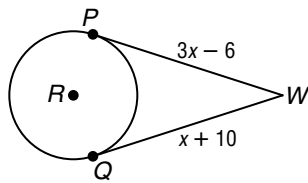


2. \overline{AB}

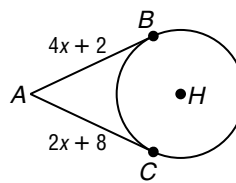


Find x . Assume that segments that appear to be tangent are tangent.

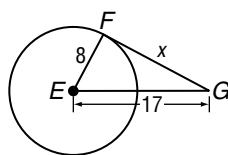
3.



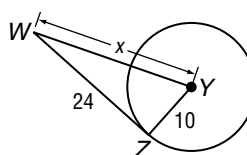
4.



5.

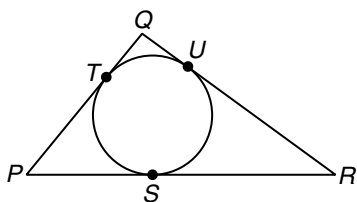


6.

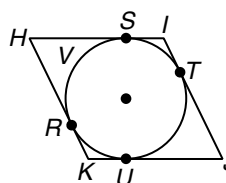


Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

7. $QT = 4$, $PT = 9$, $SR = 13$



8. $H I J K$ is a rhombus, $SI = 5$, $HR = 13$

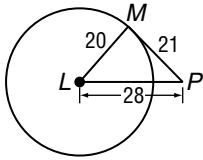


10-5 Practice

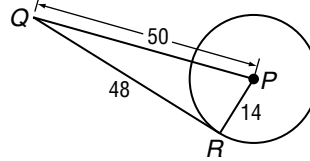
Tangents

Determine whether each segment is tangent to the given circle.

1. \overline{MP}

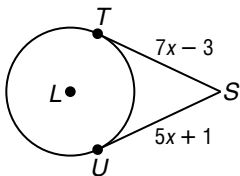


2. \overline{QR}

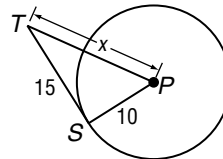


Find x . Assume that segments that appear to be tangent are tangent.

3.

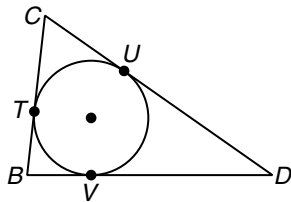


4.

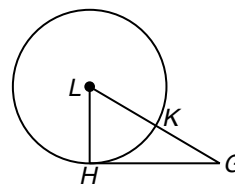


Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

5. $CD = 52$, $CU = 18$, $TB = 12$

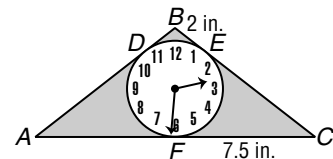


6. $KG = 32$, $HG = 56$



CLOCKS For Exercises 7 and 8, use the following information.

The design shown in the figure is that of a circular clock face inscribed in a triangular base. AF and FC are equal.



7. Find AB .

8. Find the perimeter of the clock.

10-5 Reading to Learn Mathematics

Tangents

Pre-Activity How are tangents related to track and field events?

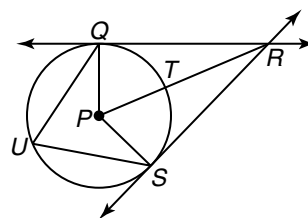
Read the introduction to Lesson 10-5 at the top of page 552 in your textbook.

How is the hammer throw event related to the mathematical concept of a tangent line?

Reading the Lesson

1. Refer to the figure. Name each of the following in the figure.

- two lines that are tangent to $\odot P$
- two points of tangency
- two chords of the circle
- three radii of the circle
- two right angles
- two congruent right triangles
- the hypotenuse or hypotenuses in the two congruent right triangles
- two congruent central angles
- two congruent minor arcs
- an inscribed angle



2. Explain the difference between an *inscribed polygon* and a *circumscribed polygon*. Use the words *vertex* and *tangent* in your explanation.

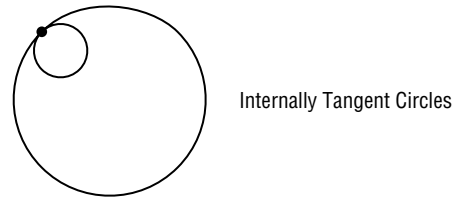
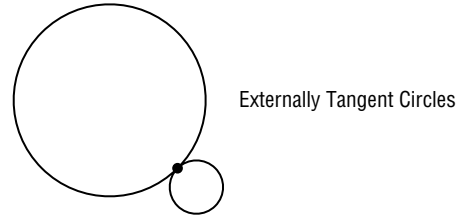
Helping You Remember

3. A good way to remember a mathematical term is to relate it to a word or expression that is used in a nonmathematical way. Sometimes a word or expression used in English is derived from a mathematical term. What does it mean to “go off on a tangent,” and how is this meaning related to the geometric idea of a *tangent* line?

10-5 Enrichment

Tangent Circles

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them are tangent.



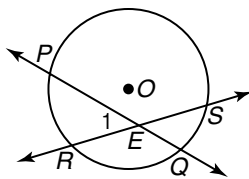
1. Make sketches to show all possible positions of three mutually tangent circles.
2. Make sketches to show all possible positions of four mutually tangent circles.
3. Make sketches to show all possible positions of five mutually tangent circles.
4. Write a conjecture about the number of possible positions for n mutually tangent circles if n is a whole number greater than four.

10-6 Study Guide and Intervention

Secants, Tangents, and Angle Measures

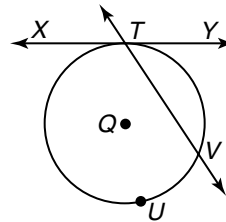
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



$$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$$

- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

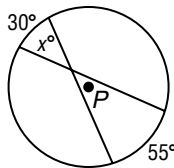


$$m\angle XTV = \frac{1}{2}m\widehat{TUV}$$

$$m\angle YTV = \frac{1}{2}m\widehat{TV}$$

Example 1 Find x .

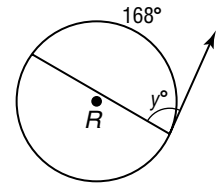
The two secants intersect inside the circle, so x is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.



$$\begin{aligned} x &= \frac{1}{2}(30 + 55) \\ &= \frac{1}{2}(85) \\ &= 42.5 \end{aligned}$$

Example 2 Find y .

The secant and the tangent intersect at the point of tangency, so the measure the angle is one-half the measure of its intercepted arc.

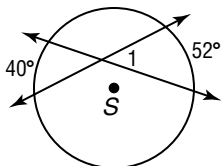


$$\begin{aligned} y &= \frac{1}{2}(168) \\ &= 84 \end{aligned}$$

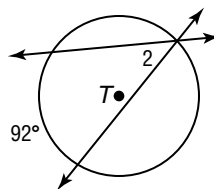
Exercises

Find each measure.

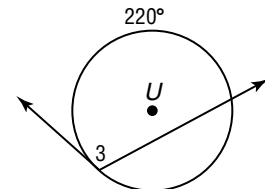
1. $m\angle 1$



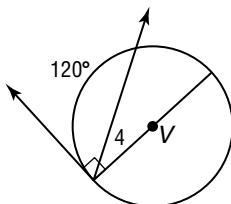
2. $m\angle 2$



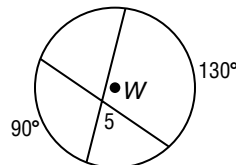
3. $m\angle 3$



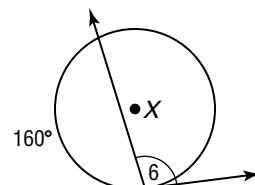
4. $m\angle 4$



5. $m\angle 5$



6. $m\angle 6$

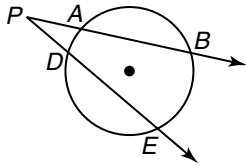


10-6 Study Guide and Intervention *(continued)*

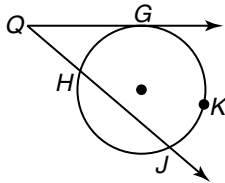
Secants, Tangents, and Angle Measures

Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

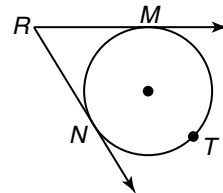
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.



\overline{PB} and \overline{PE} are secants.
 $m\angle P = \frac{1}{2}(m\widehat{BE} - m\widehat{AD})$



\overline{QG} is a tangent. \overline{QJ} is a secant.
 $m\angle Q = \frac{1}{2}(m\widehat{GKJ} - m\widehat{GH})$



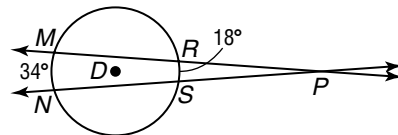
\overline{RM} and \overline{RN} are tangents.
 $m\angle R = \frac{1}{2}(m\widehat{MTN} - m\widehat{MN})$

Example Find $m\angle MPN$.

$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.

$$\begin{aligned} m\angle MPN &= \frac{1}{2}(m\widehat{MN} - m\widehat{RS}) \\ &= \frac{1}{2}(34 - 18) \\ &= \frac{1}{2}(16) \text{ or } 8 \end{aligned}$$

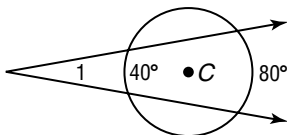
The measure of the angle is 8.



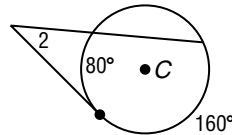
Exercises

Find each measure.

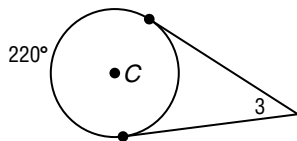
1. $m\angle 1$



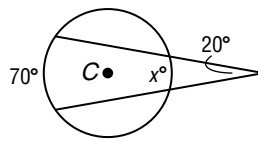
2. $m\angle 2$



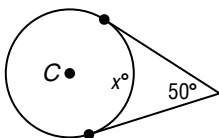
3. $m\angle 3$



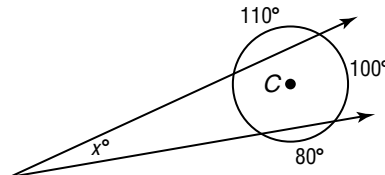
4. x



5. x



6. x

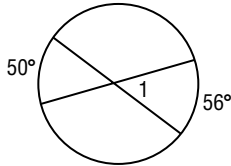


10-6 Skills Practice

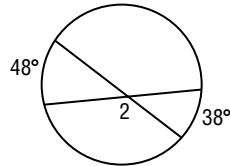
Secants, Tangents, and Angle Measures

Find each measure.

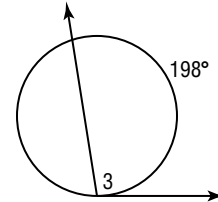
1. $m\angle 1$



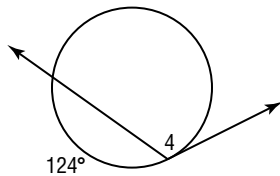
2. $m\angle 2$



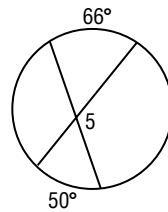
3. $m\angle 3$



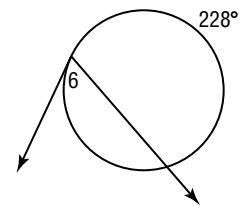
4. $m\angle 4$



5. $m\angle 5$

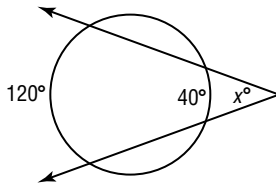


6. $m\angle 6$

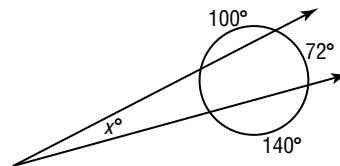


Find x . Assume that any segment that appears to be tangent is tangent.

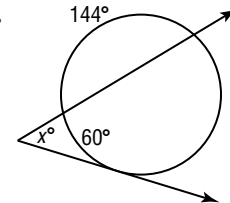
7.



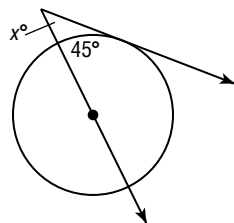
8.



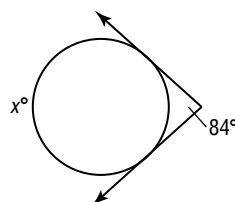
9.



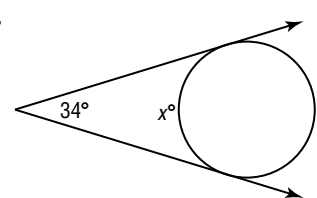
10.



11.



12.

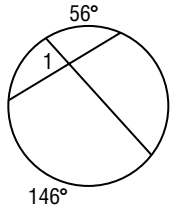


10-6 Practice

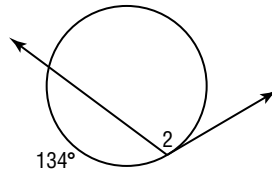
Secants, Tangents, and Angle Measures

Find each measure.

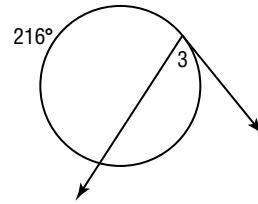
1. $m\angle 1$



2. $m\angle 2$

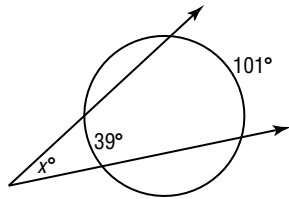


3. $m\angle 3$

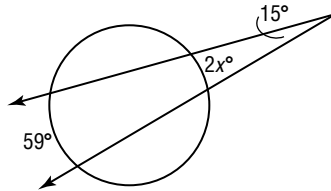


Find x . Assume that any segment that appears to be tangent is tangent.

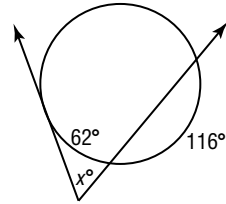
7.



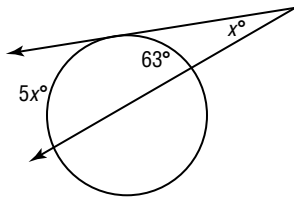
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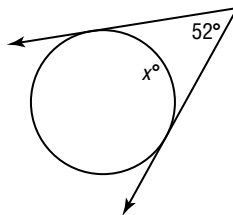
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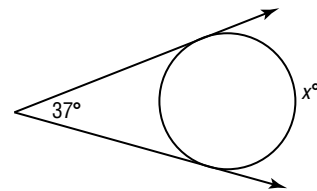
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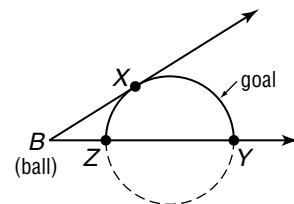
11.



12.



9. RECREATION In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.



10-6

Reading to Learn Mathematics***Secants, Tangents, and Angle Measures*****Pre-Activity** How is a rainbow formed by segments of a circle?

Read the introduction to Lesson 10-6 at the top of page 561 in your textbook.

- How would you describe $\angle C$ in the figure in your textbook?

- When you see a rainbow, where is the sun in relation to the circle of which the rainbow is an arc?

Reading the Lesson

1. Underline the correct word to form a true statement.
 - a. A line can intersect a circle in at most (one/two/three) points.
 - b. A line that intersects a circle in exactly two points is called a (tangent/secant/radius).
 - c. A line that intersects a circle in exactly one point is called a (tangent/secant/radius).
 - d. Every secant of a circle contains a (radius/tangent/chord).

2. Determine whether each statement is *always*, *sometimes*, or *never* true.
 - a. A secant of a circle passes through the center of the circle.
 - b. A tangent to a circle passes through the center of the circle.
 - c. A secant-secant angle is a central angle of the circle.
 - d. A vertex of a secant-tangent angle is a point on the circle.
 - e. A secant-tangent angle passes through the center of the circle.
 - f. The vertex of a tangent-tangent angle is a point on the circle.
 - g. If one side of a secant-tangent angle passes through the center of the circle, the angle is a right angle.
 - h. The measure of a secant-secant angle is one-half the positive difference of the measures of its intercepted arcs.
 - i. The sum of the measures of the arcs intercepted by a tangent-tangent angle is 360.

 - j. The two arcs intercepted by a tangent-tangent angle are congruent.

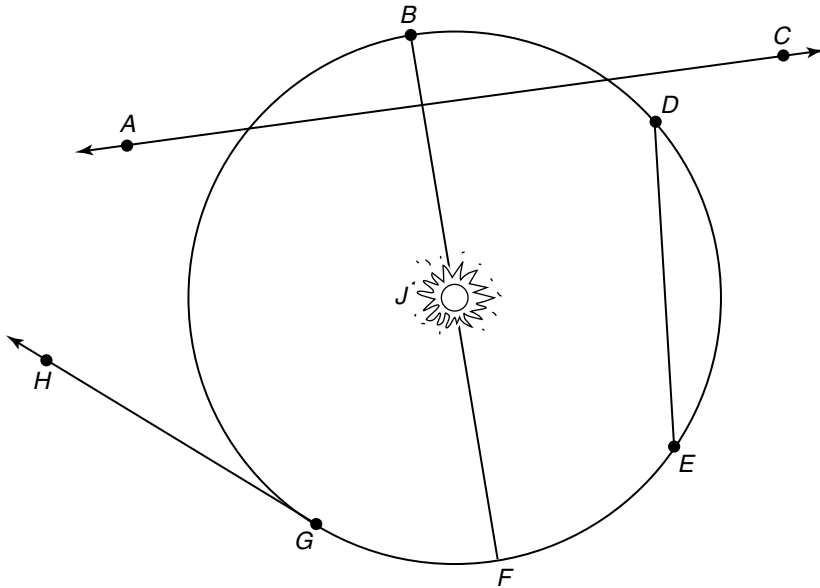
Helping You Remember

4. Some students have trouble remembering the difference between a *secant* and a *tangent*. What is an easy way to remember which is which?

10-6 Enrichment

Orbiting Bodies

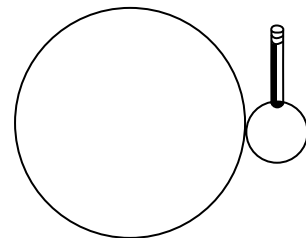
The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



Use the drawing above of the Earth orbiting the sun to name the line or segment described. Then identify it as a *radius*, *diameter*, *chord*, *tangent*, or *secant* of the orbit.

1. the path of an asteroid
2. the distance between the Earth's position in July and the Earth's position in October
3. the distance between the Earth's position in December and the Earth's position in June
4. the path of a rocket shot toward Saturn
5. the path of a sunbeam
6. If a planet has a moon, the moon circles the planet as the planet circles the sun. To visualize the path of the moon, cut two circles from a piece of cardboard, one with a diameter of 4 inches and one with a diameter of 1 inch.

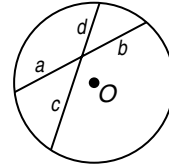
Tape the larger circle firmly to a piece of paper. Poke a pencil point through the smaller circle, close to the edge. Roll the small circle around the outside of the large one. The pencil will trace out the path of a moon circling its planet. This kind of curve is called an epicycloid. To see the path of the planet around the sun, poke the pencil through the center of the small circle (the planet), and roll the small circle around the large one (the sun).



10-7 Study Guide and Intervention

Special Segments in a Circle

Segments Intersecting Inside a Circle If two chords intersect in a circle, then the products of the measures of the chords are equal.



$$a \cdot b = c \cdot d$$

Example

Find x .

The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.

$$AB \cdot BC = EB \cdot BD$$

$$6 \cdot x = 8 \cdot 3$$

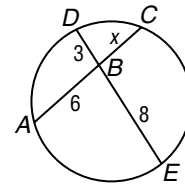
Substitution

$$6x = 24$$

Simplify.

$$x = 4$$

Divide each side by 6.

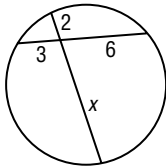


$$AB \cdot BC = EB \cdot BD$$

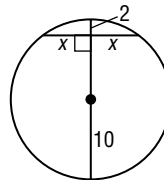
Exercises

Find x to the nearest tenth.

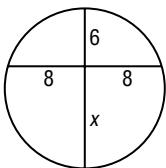
1.



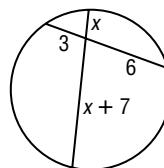
2.



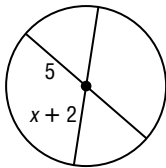
3.



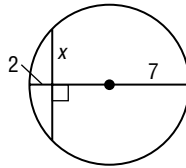
4.



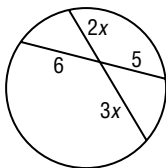
5.



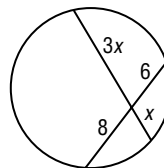
6.



7.



8.

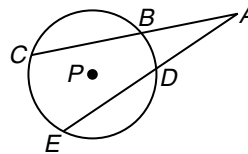


10-7 Study Guide and Intervention *(continued)*

Special Segments in a Circle

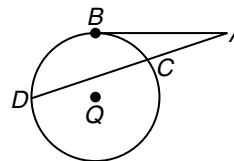
Segments Intersecting Outside a Circle If secants and tangents intersect outside a circle, then two products are equal.

- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



\overline{AC} and \overline{AE} are secant segments.
 \overline{AB} and \overline{AD} are external secant segments.
 $AC \cdot AB = AE \cdot AD$

- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

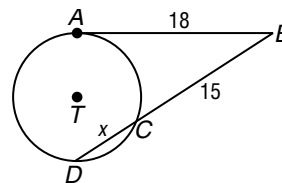


\overline{AB} is a tangent segment.
 \overline{AD} is a secant segment.
 \overline{AC} is an external secant segment.
 $(AB)^2 = AD \cdot AC$

Example Find x to the nearest tenth.

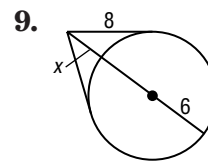
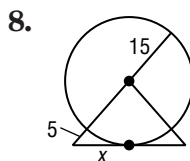
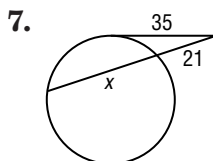
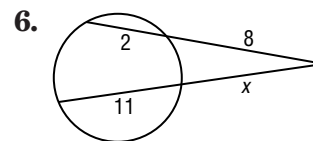
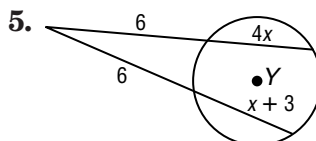
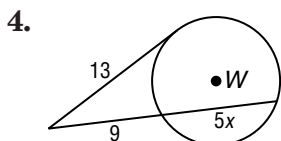
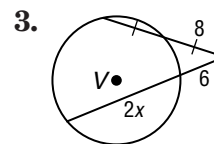
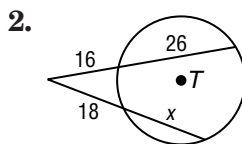
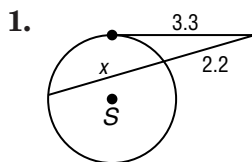
The tangent segment is \overline{AB} , the secant segment is \overline{BD} , and the external secant segment is \overline{BC} .

$$\begin{aligned} (AB)^2 &= BC \cdot BD \\ (18)^2 &= 15(15 + x) \\ 324 &= 225 + 15x \\ 99 &= 15x \\ 6.6 &= x \end{aligned}$$



Exercises

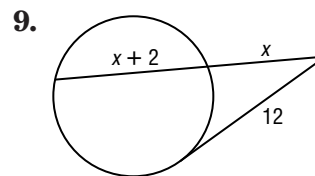
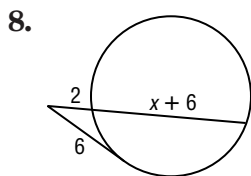
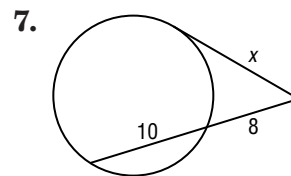
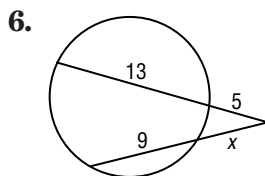
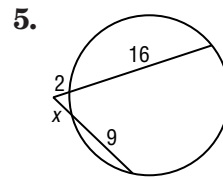
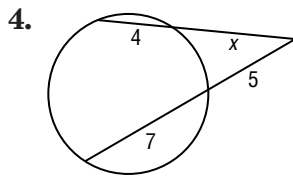
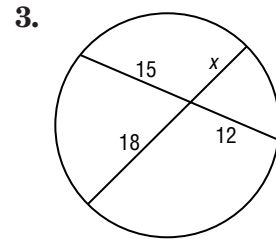
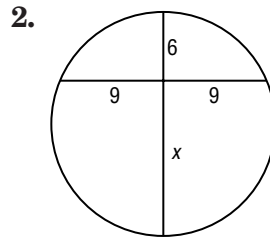
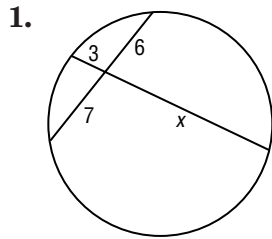
Find x to the nearest tenth. Assume segments that appear to be tangent are tangent.



10-7 Skills Practice

Special Segments in a Circle

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

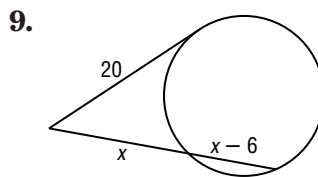
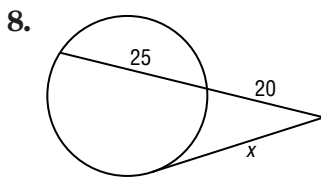
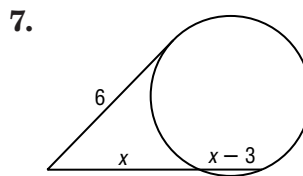
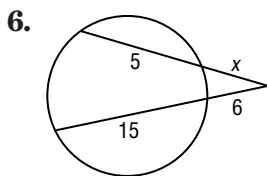
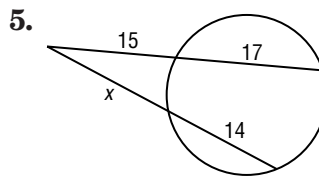
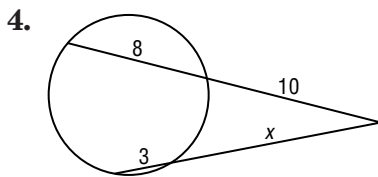
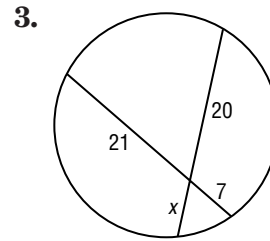
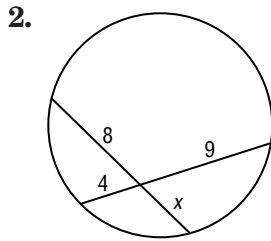
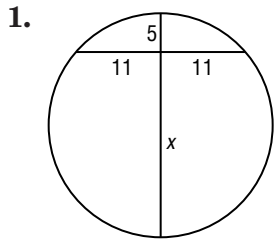


Lesson 10-7

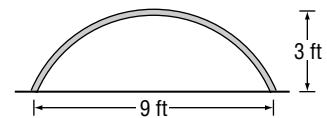
10-7 Practice

Special Segments in a Circle

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.



10. **CONSTRUCTION** An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch.



10-7

Reading to Learn Mathematics

Special Segments in a Circle

Pre-Activity How are lengths of intersecting chords related?

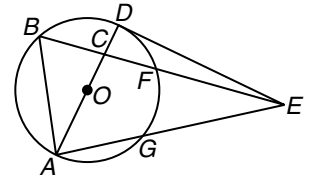
Read the introduction to Lesson 10-7 at the top of page 569 in your textbook.

- What kinds of angles of the circle are formed at the points of the star?
- What is the sum of the measures of the five angles of the star?

Reading the Lesson

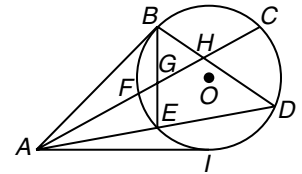
1. Refer to $\odot O$. Name each of the following.

- a diameter
- a chord that is not a diameter
- two chords that intersect in the interior of the circle
- an exterior point
- two secant segments that intersect in the exterior of the circle
- a tangent segment
- a right angle
- an external secant segment
- a secant-tangent angle with vertex on the circle
- an inscribed angle



2. Supply the missing length to complete each equation.

- | | |
|-----------------------------------|-------------------------------------|
| a. $BH \cdot HD = FH \cdot$ _____ | b. $AC \cdot AF = AD \cdot$ _____ |
| c. $AD \cdot AE = AB \cdot$ _____ | d. $AB =$ _____ |
| e. $AF \cdot AC = ($ _____ $)^2$ | f. $EG \cdot$ _____ $= FG \cdot GC$ |



Helping You Remember

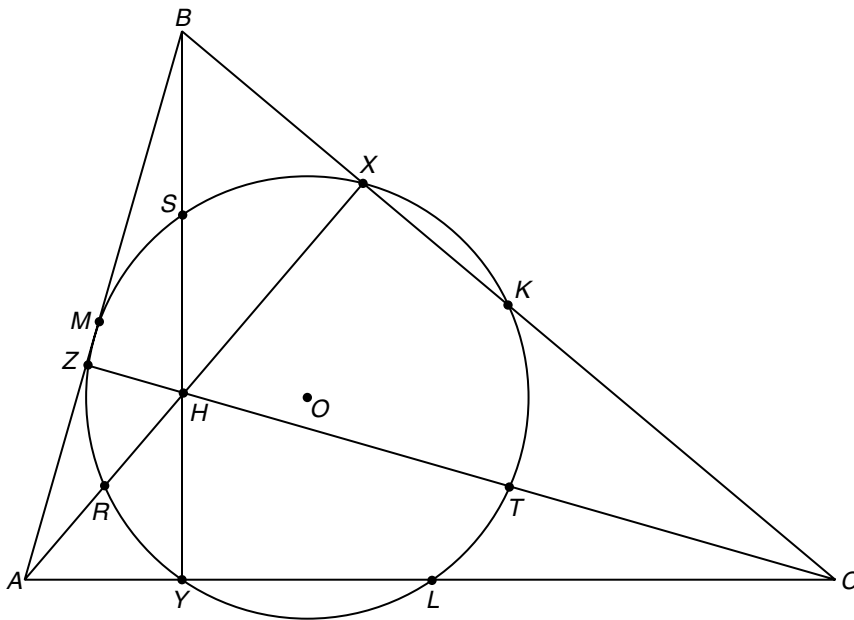
- Some students find it easier to remember geometric theorems if they restate them in their own words. Restate Theorem 10.16 in a way that you find easier to remember.

10-7 Enrichment

The Nine-Point Circle

The figure below illustrates a surprising fact about triangles and circles. Given any $\triangle ABC$, there is a circle that contains all of the following nine points:

- (1) the midpoints K , L , and M of the sides of $\triangle ABC$
- (2) the points X , Y , and Z , where \overline{AX} , \overline{BY} , and \overline{CZ} are the altitudes of $\triangle ABC$
- (3) the points R , S , and T which are the midpoints of the segments \overline{AH} , \overline{BH} , and \overline{CH} that join the vertices of $\triangle ABC$ to the point H where the lines containing the altitudes intersect.



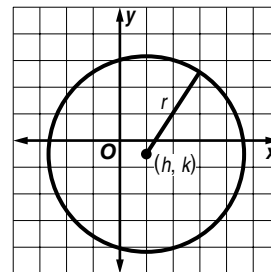
1. On a separate sheet of paper, draw an obtuse triangle ABC . Use your straightedge and compass to construct the circle passing through the midpoints of the sides. Be careful to make your construction as accurate as possible. Does your circle contain the other six points described above?
2. In the figure you constructed for Exercise 1, draw \overline{RK} , \overline{SL} , and \overline{TM} . What do you observe?

10-8 Study Guide and Intervention

Equations of Circles

Equation of a Circle A circle is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.

Standard Equation of a Circle An equation for a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.



Example

Write an equation for a circle with center $(-1, 3)$ and radius 6.

Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 && \text{Equation of a circle} \\ (x - (-1))^2 + (y - 3)^2 &= 6^2 && \text{Substitution} \\ (x + 1)^2 + (y - 3)^2 &= 36 && \text{Simplify.} \end{aligned}$$

Exercises

Write an equation for each circle.

1. center at $(0, 0)$, $r = 8$

2. center at $(-2, 3)$, $r = 5$

3. center at $(2, -4)$, $r = 1$

4. center at $(-1, -4)$, $r = 2$

5. center at $(-2, -6)$, diameter = 8

6. center at $(-\frac{1}{2}, \frac{1}{4})$, $r = \sqrt{3}$

7. center at the origin, diameter = 4

8. center at $(1, -\frac{5}{8})$, $r = \sqrt{5}$

9. Find the center and radius of a circle with equation $x^2 + y^2 = 20$.

10. Find the center and radius of a circle with equation $(x + 4)^2 + (y + 3)^2 = 16$.

10-8 Study Guide and Intervention *(continued)*

Equations of Circles

Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example Graph $(x + 3)^2 + (y - 1)^2 = 9$.

Use the parts of the equation to find (h, k) and r .

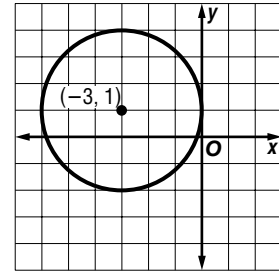
$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - h)^2 = (x + 3)^2 \quad (y - k)^2 = (y - 1)^2 \quad r^2 = 9$$

$$x - h = x + 3 \quad y - k = y - 1 \quad r = 3$$

$$-h = 3 \quad -k = -1$$

$$h = -3 \quad k = 1$$

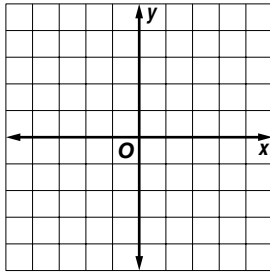


The center is at $(-3, 1)$ and the radius is 3. Graph the center.
Use a compass set at a radius of 3 grid squares to draw the circle.

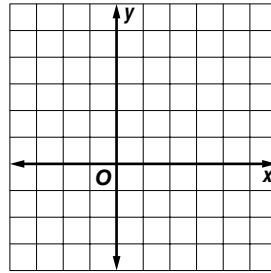
Exercises

Graph each equation.

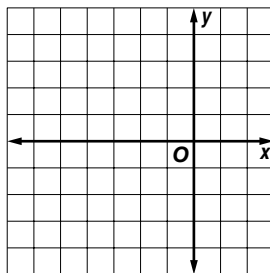
1. $x^2 + y^2 = 16$



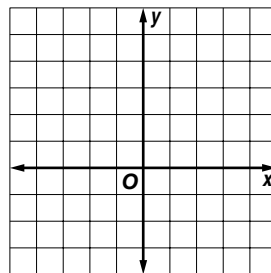
2. $(x - 2)^2 + (y - 1)^2 = 9$



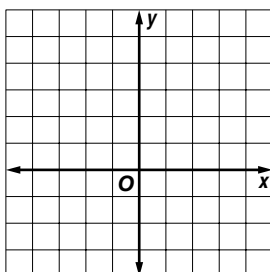
3. $(x + 2)^2 + y^2 = 16$



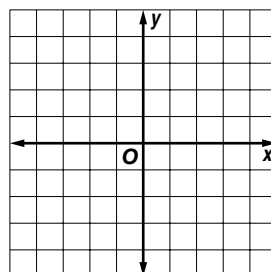
4. $(x + 1)^2 + (y - 2)^2 = 6.25$



5. $(x + \frac{1}{2})^2 + (y - \frac{1}{4})^2 = 4$



6. $x^2 + (y - 1)^2 = 9$



10-8 Skills Practice

Equations of Circles

Write an equation for each circle.

1. center at origin, $r = 6$

2. center at $(0, 0)$, $r = 2$

3. center at $(4, 3)$, $r = 9$

4. center at $(7, 1)$, $d = 24$

5. center at $(-5, 2)$, $r = 4$

6. center at $(6, -8)$, $d = 10$

7. a circle with center at $(8, 4)$ and a radius with endpoint $(0, 4)$

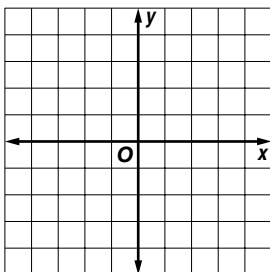
8. a circle with center at $(-2, -7)$ and a radius with endpoint $(0, 7)$

9. a circle with center at $(-3, 9)$ and a radius with endpoint $(1, 9)$

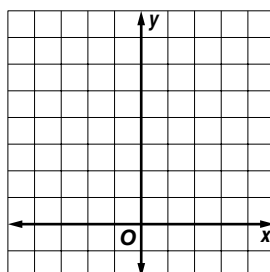
10. a circle whose diameter has endpoints $(-3, 0)$ and $(3, 0)$

Graph each equation.

11. $x^2 + y^2 = 16$



12. $(x - 1)^2 + (y - 4)^2 = 9$



10-8 Practice

Equations of Circles

Write an equation for each circle.

1. center at origin, $r = 7$

2. center at $(0, 0)$, $d = 18$

3. center at $(-7, 11)$, $r = 8$

4. center at $(12, -9)$, $d = 22$

5. center at $(-6, -4)$, $r = \sqrt{5}$

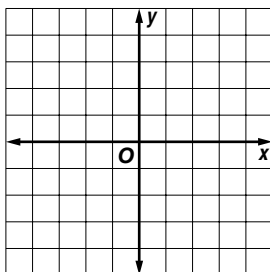
6. center at $(3, 0)$, $d = 28$

7. a circle with center at $(-5, 3)$ and a radius with endpoint $(2, 3)$

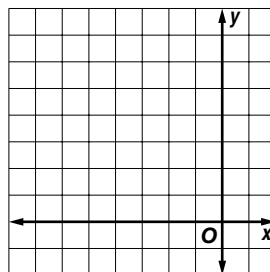
8. a circle whose diameter has endpoints $(4, 6)$ and $(-2, 6)$

Graph each equation.

9. $x^2 + y^2 = 4$



10. $(x + 3)^2 + (y - 3)^2 = 9$



- 11. EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake.

10-8 Reading to Learn Mathematics

Equations of Circles

Pre-Activity What kind of equations describe the ripples of a splash?

Read the introduction to Lesson 10-8 at the top of page 575 in your textbook.

In a series of concentric circles, what is the same about all the circles, and what is different?

Reading the Lesson

1. Identify the center and radius of each circle.

a. $(x - 2)^2 + (y - 3)^2 = 16$

b. $(x + 1)^2 + (y + 5)^2 = 9$

c. $x^2 + y^2 = 49$

d. $(x - 8)^2 + (y + 1)^2 = 36$

e. $x^2 + (y - 10)^2 = 144$

f. $(x + 3)^2 + y^2 = 5$

2. Write an equation for each circle.

a. center at origin, $r = 8$

b. center at $(3, 9)$, $r = 1$

c. center at $(-5, -6)$, $r = 10$

d. center at $(0, -7)$, $r = 7$

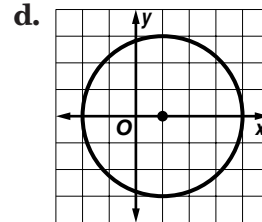
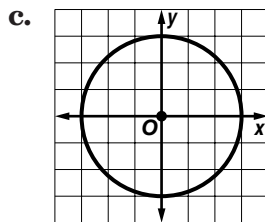
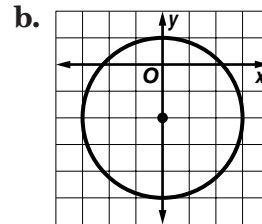
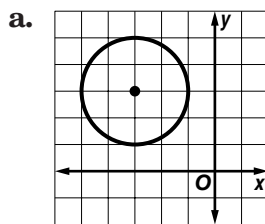
e. center at $(12, 0)$, $d = 12$

f. center at $(-4, 8)$, $d = 22$

g. center at $(4.5, -3.5)$, $r = 1.5$

h. center at $(0, 0)$, $r = \sqrt{13}$

3. Write an equation for each circle.



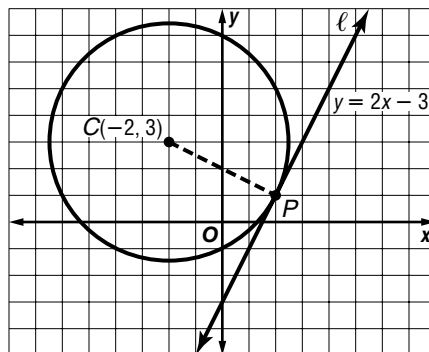
Helping You Remember

4. A good way to remember a new mathematical formula or equation is to relate it to one you already know. How can you use the Distance Formula to help you remember the standard equation of a circle?

10-8 Enrichment

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.

1. State the slope of the line ℓ that has equation $y = 2x - 3$.
2. Suppose $\odot C$ with center $C(-2, 3)$ is tangent to line ℓ at point P . What is the slope of radius \overline{CP} ?
3. Find an equation for the line that contains \overline{CP} .
4. Use your equation from Exercise 3 and the equation $y = 2x - 3$. At what point do the lines for these equations intersect? What are its coordinates?
5. Find the measure of radius \overline{CP} .
6. Use the coordinate pair $C(-2, 3)$ and your answer for Exercise 5 to write an equation for $\odot C$.

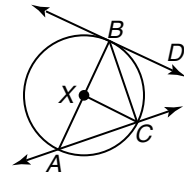
10 Chapter 10 Test, Form 1

Write the letter for the correct answer in the blank at the right of each question.

For Questions 1–3, use $\odot X$.

1. Name a radius.

- A. \overline{XB} B. \overline{AB} C. \overline{BC} D. \overline{AC}



1. _____

2. Name a chord.

- A. \overline{XB} B. \overline{XC} C. \overline{BC} D. \overline{AC}

2. _____

3. Name a tangent.

- A. \overline{AB} B. \overline{BC} C. \overline{AC} D. \overline{BD}

3. _____

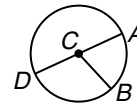
4. If the radius of a circle is 6 feet, find the circumference to the nearest hundredth.

- A. 9.42 ft B. 18.85 ft C. 37.70 ft D. 113.10 ft

4. _____

5. If $m\widehat{AB} = 72$ in $\odot C$, find $m\angle BCD$.

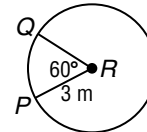
- A. 72 B. 108
C. 144 D. 180



5. _____

6. Find the length of \widehat{PQ} in $\odot R$ to the nearest hundredth.

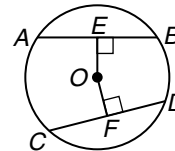
- A. 9.42 m B. 4.71 m
C. 3.14 m D. 1.57 m



6. _____

7. If $AB = 12$ centimeters, $OE = 4$ centimeters, and $OF = 4$ centimeters in $\odot O$, find CF .

- A. 6 cm B. 8 cm
C. 12 cm D. 24 cm



7. _____

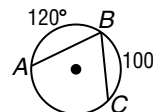
8. Find the radius of a circle if a 48-meter chord is 7 meters from the center.

- A. 14 m B. 24 m C. 25 m D. 41 m

8. _____

9. Find $m\angle ABC$.

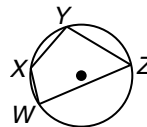
- A. 50 B. 70
C. 90 D. 140



9. _____

10. If $m\angle X = 126$, find $m\angle Z$.

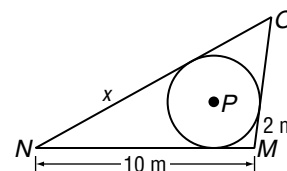
- A. 54 B. 63
C. 90 D. 126



10. _____

11. If \overline{MN} , \overline{NO} , and \overline{MO} are tangent to $\odot P$, find x .

- A. 2 m B. 5 m
C. 6 m D. 8 m



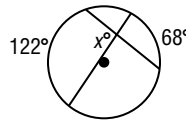
11. _____

10 Chapter 10 Test, Form 1 *(continued)*

12. Find x .

- A. 122
- C. 68

- B. 95
- D. 61

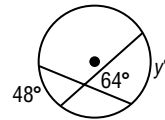


12. _____

13. Find y .

- A. 16
- C. 80

- B. 56
- D. 112

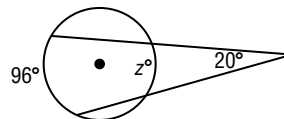


13. _____

14. Find z .

- A. 38
- C. 58

- B. 56
- D. 76

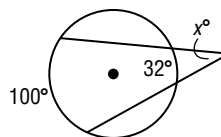


14. _____

15. Find x .

- A. 132
- C. 66

- B. 68
- D. 34

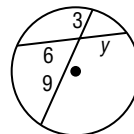


15. _____

16. Find y .

- A. 18
- C. 6

- B. 12
- D. 4.5

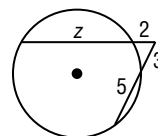


16. _____

17. Find z .

- A. 11.25
- C. 7.5

- B. 10
- D. 4



17. _____

18. Find the radius of the circle whose equation is $(x + 3)^2 + (y - 7)^2 = 289$.

- A. 7
- B. 17
- C. 34
- D. 289

18. _____

19. Find the equation of a circle whose center is at the origin and radius is 4.

- A. $x^2 + y^2 = 4$
- B. $x^2 + y^2 = 16$
- C. $(x - 4)^2 + (y - 4)^2 = 16$
- D. $4x + 4y = 16$

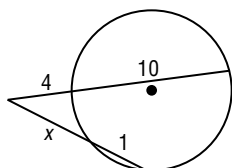
19. _____

20. Identify the graph of $(x - 3)^2 + (y + 2)^2 = 4$.

- A.
- B.
- C.
- D.

20. _____

Bonus Find x .



B: _____

10 Chapter 10 Test, Form 2A

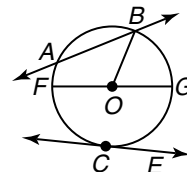
Write the letter for the correct answer in the blank at the right of each question.

For Questions 1–3, use $\odot O$.

1. Name a diameter.

- A. \overline{FG}
- C. \overline{AB}

- B. \overline{AB}
- D. \overline{CE}



1. _____

2. Name a chord.

- A. \overline{FO}

- B. \overline{AB}

- C. \overline{AB}

- D. \overline{CE}

2. _____

3. Name a secant.

- A. \overline{FO}

- B. \overline{AB}

- C. \overline{AB}

- D. \overline{CE}

3. _____

4. If the diameter of a circle is 10 inches, find the circumference to the nearest hundredth.

- A. 15.71 in.

- B. 31.42 in.

- C. 62.83 in.

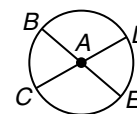
- D. 314.16 in.

4. _____

5. If $m\angle BAD = 110$ in $\odot A$, find $m\widehat{DE}$.

- A. 35
- C. 70

- B. 55
- D. 110



5. _____

6. Points X and Y lie on $\odot P$ so that $PX = 5$ meters and $m\angle XPY = 90$. Find the length of \widehat{XY} to the nearest hundredth.

- A. 3.93 m

- B. 7.85 m

- C. 15.71 m

- D. 19.63 m

6. _____

7. Chords \overline{XY} and \overline{WV} are equidistant from the center of $\odot O$. If $XY = 2x + 30$ and $WV = 5x - 12$, find x .

- A. 58

- B. 28

- C. 14

- D. 6

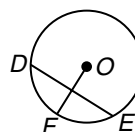
7. _____

8. Find the radius of $\odot O$ if $DE = 12$ inches and \overline{DE} bisects \overline{OF} .

- A. $2\sqrt{3}$ in.
- C. 8 in.

- B. 6 in.

- D. $4\sqrt{3}$ in.



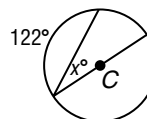
8. _____

9. Find x .

- A. 122
- C. 58

- B. 61

- D. 29



9. _____

10. $EFGH$ is a quadrilateral inscribed in $\odot P$ with $m\angle E = 72$ and $m\angle F = 49$. Find $m\angle H$.

- A. 131

- B. 108

- C. 90

- D. 57

10. _____

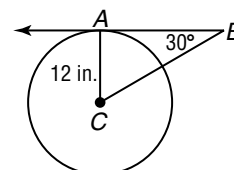
11. If \overline{AB} is tangent to $\odot C$ at A , find BC .

- A. 6 in.

- C. $12\sqrt{3}$ in.

- B. $4\sqrt{3}$ in.

- D. 24 in.



11. _____

10 Chapter 10 Test, Form 2B

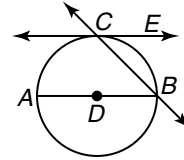
Write the letter for the correct answer in the blank at the right of each question.

For Questions 1–3, use $\odot D$.

1. Name a radius.

- A. \overline{AB}
C. \overline{CB}

- B. \overline{DB}
D. \overline{CE}



1. _____

2. Name a chord that is not a diameter.

- A. \overline{AB}

- B. \overline{DB}

- C. \overline{CB}

- D. \overline{CE}

2. _____

3. Name a secant.

- A. \overline{AB}

- B. \overline{DB}

- C. \overline{CB}

- D. \overline{CE}

3. _____

4. If the circumference of a circle is 20π inches, find the radius.

- A. 10 in.

- B. 20 in.

- C. 40 in.

- D. 100 in.

4. _____

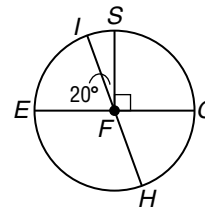
5. Find $m\widehat{GH}$.

- A. 20

- B. 50

- C. 70

- D. 90



5. _____

6. Points G and H lie on $\odot T$ so that $TH = 8$ meters and $m\angle GTH = 45$. Find the length of \widehat{GH} to the nearest hundredth.

- A. 6.28 m

- B. 12.57 m

- C. 25.13 m

- D. 37.70 m

6. _____

7. Chords \overline{AB} and \overline{CD} in $\odot X$ are congruent and \overline{AB} is 9 units from X . Find the distance from \overline{CD} to X .

- A. 4.5 units

- B. 9 units

- C. 18 units

- D. cannot tell

7. _____

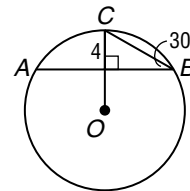
8. Find the radius of $\odot O$.

- A. $4\sqrt{2}$ units

- B. 8 units

- C. $4\sqrt{3}$ units

- D. $4\sqrt{2} + 4$ units



8. _____

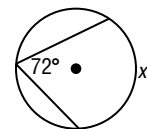
9. Find x .

- A. 36

- B. 72

- C. 144

- D. 180



9. _____

10. $\triangle JKL$ is inscribed in $\odot P$ with diameter \overline{JK} and $m\widehat{JL} = 130$. Find $m\angle KJL$.

- A. 25

- B. 50

- C. 65

- D. 130

10. _____

11. The measure of an angle formed by two tangents to a circle is 90. The radius of the circle is 8 centimeters, how far is the vertex of the angle from the center of the circle?

- A. 8 cm

- B. $8\sqrt{2}$ cm

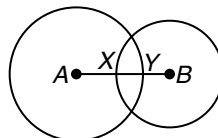
- C. $8\sqrt{3}$ cm

- D. 16 cm

11. _____

10 Chapter 10 Test, Form 2C

1. If the diameter of $\odot A$ is 10 inches, the diameter of $\odot B$ is 8 inches, and $AX = 3$ inches, find YB .

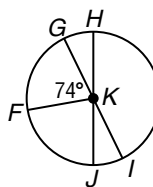


1. _____

2. Find the radius and diameter of a circle whose circumference is 60π meters.

2. _____

3. In $\odot K$, $m\angle HKG = x + 10$ and $m\angle IKJ = 3x - 22$. Find $m\widehat{FJ}$.

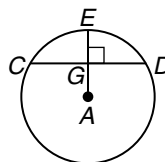


3. _____

4. The diameter of $\odot C$ is 18 units long. Find the length of an arc that has a measure of 100 to the nearest hundredth.

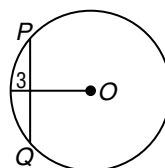
4. _____

5. If $CG = 5x + 2$ and $GD = 7x - 12$, find x .



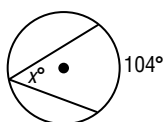
5. _____

6. Find the distance from O to \widehat{PQ} in $\odot O$, if $PQ = 18$ meters.



6. _____

7. Find x .



7. _____

8. A regular decagon is inscribed in a circle. Find the measure of each minor arc.

8. _____

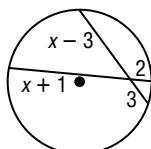
9. \widehat{CD} is tangent to $\odot Z$ at $(1, 7)$. If Z has coordinates $(5, 2)$, find the slope of \widehat{CD} .

9. _____

10. $\triangle DEF$ is circumscribed about $\odot O$ with $DE = 15$ units, $DF = 12$ units, and $EF = 13$ units. Find the length of each segment whose endpoints are D and the points of tangency on \widehat{DE} and \widehat{EF} .

10. _____

11. Find x .

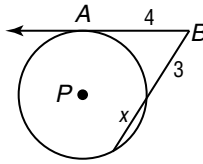


11. _____

10

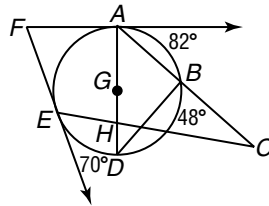
Chapter 10 Test, Form 2C *(continued)*

12. Find x if \overline{AB} is tangent to $\odot P$ at A .



12. _____

For Questions 13–16, use $\odot G$ with \overline{FA} and \overline{FE} tangent at A and E .



13. Find $m\angle ACE$.

13. _____

14. Find $m\angle ADB$.

14. _____

15. Find $m\angle AFE$.

15. _____

16. Find $m\angle EHD$.

16. _____

17. Find the radius of a circle whose equation is $(x - 3)^2 + (y - 2)^2 = r^2$ and contains $(1, 4)$.

17. _____

18. Write the equation of a circle with a diameter having endpoints at $(-2, 6)$ and $(8, 4)$.

18. _____

19. Write the equation of a circle whose center is at $(-4, -9)$ and radius is 10.

19. _____

20. Graph $(x + 1)^2 + (y - 2)^2 = 16$.

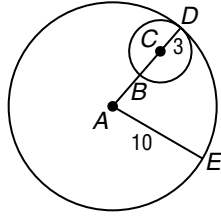
20.

Bonus \overline{AB} is tangent to $\odot P$ at $(5, 1)$. The equation for $\odot P$ is $x^2 + y^2 - 2x + 4y = 20$. Write the equation of \overline{AB} in slope-intercept form.

B: _____

10 Chapter 10 Test, Form 2D

1. Find AB .

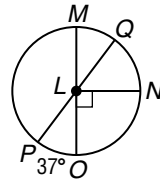


1. _____

2. Find the diameter and the circumference of a circle whose radius is 11 inches, to the nearest hundredth.

2. _____

3. In $\odot L$, $m\angle QLN = 2x - 5$. Find x .

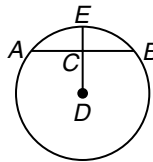


3. _____

4. The radius of $\odot C$ is 16 units long. Find the length of an arc that has a measure of 270° to the nearest hundredth.

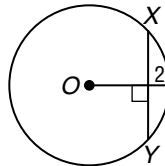
4. _____

5. If \overline{DE} bisects \overline{AB} , what is the measure of $\angle BCE$?



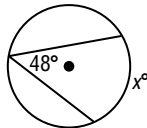
5. _____

6. Find the radius of $\odot O$ if $XY = 10$.



6. _____

7. Find x .



7. _____

8. Regular nonagon $ABCDEFGHI$ is inscribed in a circle. Find $m\widehat{AC}$.

8. _____

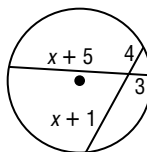
9. \overline{EF} is tangent to circle P at $G(3, 6)$. If the slope of \overline{EF} is $\frac{5}{3}$, what is the slope of \overline{GP} ?

9. _____

10. $\triangle GHI$ is circumscribed about $\odot K$ with $GH = 20$ units, $HI = 14$ units, and $IG = 12$ units. Find the length of each segment whose endpoints are G and the points of tangency on \overline{GH} and \overline{GI} .

10. _____

11. Find x .

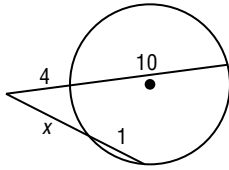


11. _____

10

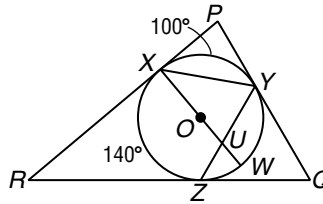
Chapter 10 Test, Form 2D *(continued)*

12. Find x .



12. _____

For Questions 13–16, use $\odot O$ with $\triangle PQR$ circumscribed.



13. Find $m\angle PQR$.

13. _____

14. Find $m\angle XYZ$.

14. _____

15. Find $m\angle PYX$.

15. _____

16. Find $m\angle XUZ$.

16. _____

17. Write the equation of the circle whose center is at $(-7, 8)$ and radius is 9.

17. _____

18. Write the equation of the circle containing the point at $(8, 1)$ whose center is at $(4, -9)$.

18. _____

19. Find the radius of a circle whose equation is $(x + 3)^2 + (y - 2)^2 = r^2$ and contains $(0, 8)$.

19. _____

20. Graph $(x - 3)^2 + (y + 1)^2 = 25$.

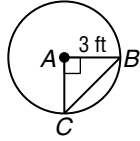
20.

Bonus Find the coordinates of the point(s) of intersection of the circles whose equations are $(x - 2)^2 + y^2 = 13$ and $(x + 3)^2 + y^2 = 8$.

B: _____

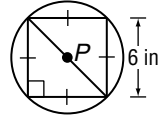
10 Chapter 10 Test, Form 3

1. Find BC .



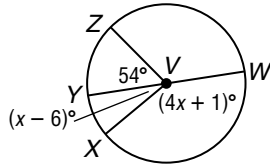
1. _____

2. Find the circumference of $\odot P$ to the nearest hundredth.



2. _____

3. Find $m\widehat{XW}$.

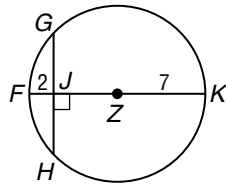


3. _____

4. If the length of an arc of measure 80 is 12π inches long, find the radius of the circle.

4. _____

5. Find GH .

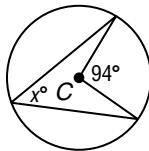


5. _____

6. Two parallel chords 16 centimeters and 30 centimeters long are 23 centimeters apart. Find the radius of the circle.

6. _____

7. Find x .



7. _____

8. Find the radius of a circle if each side of an inscribed square has length 8 centimeters.

8. _____

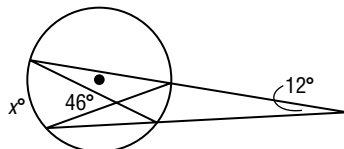
9. In $\odot O$, \overline{OA} and \overline{OB} are radii and $m\angle BOA = 120$. Tangents \overline{PA} and \overline{PB} have length 10. Find OA .

9. _____

10. Quadrilateral $ABCD$ is circumscribed about $\odot O$. If $AB = 7$, $BC = 11$, and $DC = 8$, find AD .

10. _____

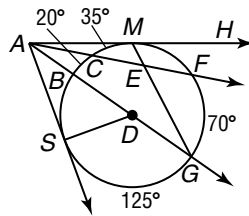
11. Find x .



11. _____

10 Chapter 10 Test, Form 3 *(continued)*

For Questions 12–14, use $\odot D$ with tangents \overline{AS} and \overline{AM} .



12. Find $m\angle GAF$.

12. _____

13. Find $m\angle GMH$.

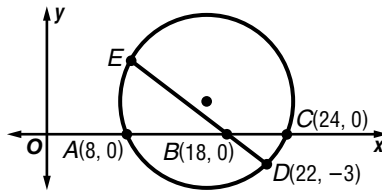
13. _____

14. Find $m\angle AEM$.

14. _____

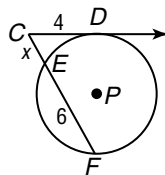
15. Find BE .

15. _____



16. If \overline{CD} is tangent to $\odot P$, find x .

16. _____



17. Find the coordinates of the points of intersection of the line $5x + 6y = 30$ and the circle $x^2 + y^2 = 25$.

17. _____

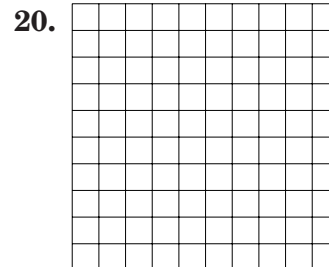
18. Write the equation of the circle whose center is at $(-3, -2)$ and is tangent to the y -axis.

18. _____

19. Find the center and radius of the circle whose equation is $x^2 - 12x + y^2 + 14y + 4 = 0$.

19. _____

20. Graph $x^2 + (y + 6)^2 = 1$.



Bonus Find the coordinates of the center of the circle containing the points at $(0, 0)$, $(-2, 4)$, and $(4, -2)$.

B: _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Make up a set of data, perhaps modeling a survey, that you can represent by a circle graph. Calculate the number of degrees for each sector. Draw and label the circle graph. You must have at least four noncongruent sectors on your graph.
2.
 - a. Explain the difference between the length of an arc and the measure of an arc.
 - b. Is it possible for two arcs to have the same measure but not the same length? Explain your answer.
3. Use a compass to construct a circle. Label the center P . Then draw two chords that are not diameters of $\odot P$. Locate the center of your circle by constructing the perpendicular bisectors of these two chords.
4. An inscribed regular polygon intercepts congruent arcs on the circle. What happens to the measures of these arcs as you increase the number of sides of the polygon?
5.
 - a. Write an equation of a circle in $(x - h)^2 + (y - k)^2 = r^2$ form whose center is not at $(0, 0)$.
 - b. Find the coordinates of any point B that lies on the circle.
 - c. Write an equation of the line through point B that is tangent to the circle. Write your equation in $y = mx + b$ form.

arc	circumference	major arc	radius
center	circumscribed	minor arc	secant
central angle	diameter	pi (π)	semicircle
chord	inscribed	point of tangency	tangent
circle	intercepted		

Write whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

- The vertex of a(n) central angle lies on the circle. 1. _____
- A(n) circle is the locus of all points in a plane equidistant from a given point. 2. _____
- $C = 2\pi r$ is the formula for the circumference of a circle. 3. _____
- The diameter of a circle is a segment with one endpoint at the center and the other endpoint on the circle. 4. _____
- A major arc has measure greater than 0 but less than 180. 5. _____
- The point of tangency is the point where a tangent line intersects a circle. 6. _____
- A(n) chord is a line that intersects a circle in two points. 7. _____
- A(n) tangent is a line that intersects a circle in one point. 8. _____
- A(n) semicircle is an arc with measure 180. 9. _____
- Pi is an irrational number equal to the ratio of the circumference to the diameter of a circle. 10. _____

Define each term.

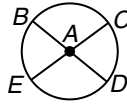
- congruent arcs 11. _____
- circumscribed polygon 12. _____
- inscribed polygon 13. _____

10 Chapter 10 Quiz

(Lessons 10-1 and 10-2)

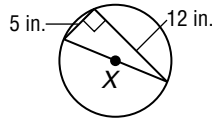
SCORE _____

1. In $\odot A$, if $BA = 4$, find CE .



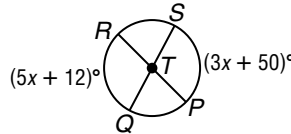
1. _____

2. Find the circumference of $\odot X$ to the nearest hundredth.



2. _____

3. If \overline{QS} and \overline{PR} are diameters of $\odot T$, find $m\widehat{RS}$.



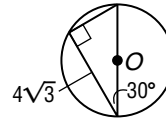
3. _____

4. The diameter of a clock's face is 6 inches. Find the length of the minor arc formed by the hands of the clock at 4:00 to the nearest hundredth.

4. _____

5. **STANDARDIZED TEST PRACTICE** Find the circumference of $\odot O$ to the nearest hundredth.

- A. 4.00 in. B. 8.00 in.
C. 12.57 in. D. 25.13 in.



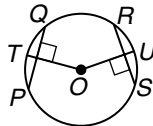
5. _____

10 Chapter 10 Quiz

(Lessons 10-3 and 10-4)

SCORE _____

1. In $\odot O$, $PQ = 20$, $RS = 20$, and $m\widehat{PT} = 35$. Find $m\widehat{RS}$.

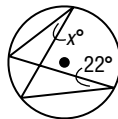


1. _____

2. Find the radius of a circle if a 24-inch chord is 9 inches from the center.

2. _____

3. Find x .



3. _____

4. Find the length of each side of a regular hexagon inscribed in a circle with radius 12 centimeters.

4. _____

5. Each side of an inscribed equilateral triangle has length 18 meters. Find the length of one of the minor arcs to the nearest hundredth.

5. _____

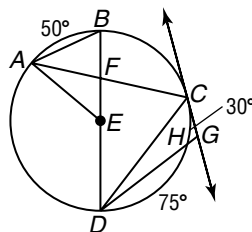
10 Chapter 10 Quiz

(Lessons 10-5 and 10-6)

SCORE _____

- Two segments from P are tangent to $\odot O$. If $m\angle P = 60$ and the radius of $\odot O$ is 12 feet, find the length of each tangent segment. 1. _____
- Each side of a circumscribed equilateral triangle is 16 meters. Find the radius of the circle. 2. _____

For Questions 3-5, use $\odot E$ with \overline{CG} tangent at C .



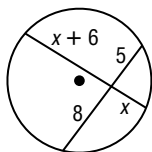
- Find $m\angle ABD$. 3. _____
- Find $m\angle AFB$. 4. _____
- Find $m\angle CGD$. 5. _____

10 Chapter 10 Quiz

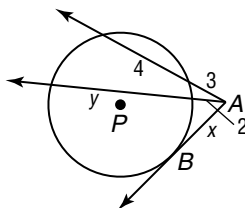
(Lessons 10-7 and 10-8)

SCORE _____

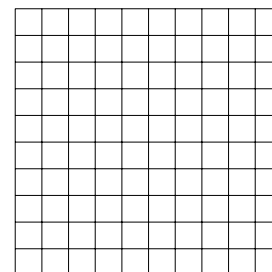
- Find x . 1. _____



- If \overline{AB} is tangent to $\odot P$ at B , find x and y . 2. _____



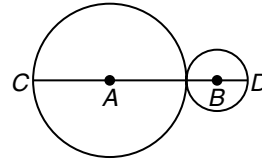
- Find the coordinates of the center of a circle whose equation is $(x + 11)^2 + (y - 13)^2 = 4$. 3. _____
- Find the radius of a circle whose equation is $(x + 12)^2 + (y + 3)^2 = 225$. 4. _____
- Graph $x^2 + (y - 1)^2 = 9$. 5. _____



Part I Write the letter for the correct answer in the blank at the right of each question.

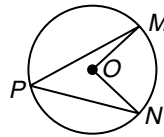
1. What is the name of the longest chord in a circle? 1. _____
 A. diameter B. radius C. secant D. tangent

2. The radius of $\odot B$ is 4 centimeters and the circumference of $\odot A$ is 20π centimeters. Find CD . 2. _____
 A. 10 cm B. 14 cm
 C. 24 cm D. 28 cm

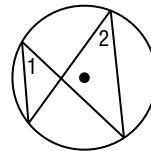


3. A chord of $\odot P$ has length 8 inches and the distance from the center to the chord is 3 inches. Find the radius of $\odot P$. 3. _____
 A. 3 in. B. 5 in. C. $\sqrt{73}$ in. D. 10 in.

4. If $m\angle MON = 86$, find $m\angle MPN$. 4. _____
 A. 86 B. 45
 C. 43 D. 30

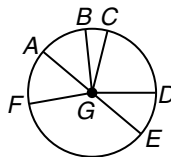


5. Find x if $m\angle 1 = 2x + 10$ and $m\angle 2 = 3x - 6$. 5. _____
 A. 4 B. 16
 C. 24 D. 42



Part II

6. \overline{AE} is a diameter of $\odot G$ and $m\angle BGE = 136$. Find $m\widehat{AB}$. 6. _____

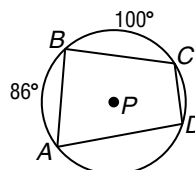


7. A circle with radius 12 inches has an arc that measures 8π inches. Find the measure of the central angle determined by this arc. 7. _____

8. Chord \overline{AB} measures $4x - 6$ centimeters and chord \overline{CD} measures $6x - 12$ centimeters in $\odot P$. If \overline{AB} and \overline{CD} are each 4 centimeters from P , find AP . 8. _____

9. Rectangle $WXYZ$ with length 15 meters and width 8 meters is inscribed in $\odot P$. Find the radius of $\odot P$. 9. _____

10. Quadrilateral $ABCD$ is inscribed in $\odot P$. Find $m\angle ABC$. 10. _____



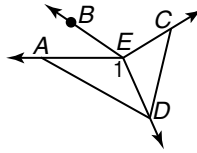
10

Chapter 10 Cumulative Review

SCORE _____

(Chapters 1–10)

1. Name the sides of $\angle 1$. (Lesson 1-4)



1. _____

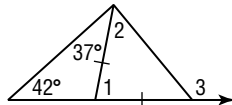
2. If p is true and q is false, find the truth value of $p \wedge \sim q$.
(Lesson 2-2)

2. _____

3. Toby Toy Company sells an average of 560 toys over the internet each week. There are presently 8500 toys in stock. Write an equation in slope-intercept form that describes how many toys they will have in stock after x weeks if no new toys are added. (Lesson 3-4)

3. _____

4. Find the measures of the numbered angles. (Lesson 4-6)

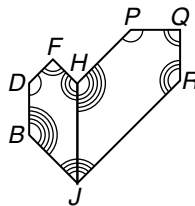


4. _____

5. State the assumption you would make to start an indirect proof of the statement *If $3a - 4 < 11$, then $a < 5$* . (Lesson 5-3)

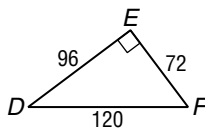
5. _____

6. Write a similarity statement.
(Lesson 6-2)



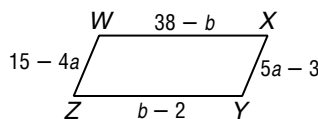
6. _____

7. Find $\sin D$, $\cos D$, and $\tan D$.
(Lesson 7-4)



7. _____

8. Find a and b so that $WXYZ$ is a parallelogram. (Lesson 8-3)



8. _____

9. Find the image of \overline{AB} with $A(-4, 2)$ and $B(-2, 4)$ under a rotation of 90° clockwise about the origin. (Lesson 9-3)

9. _____

10. Write the equation of a circle with center $(4, -1)$ and diameter 24. (Lesson 10-8)

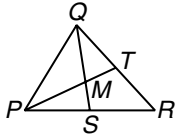
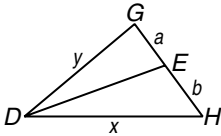
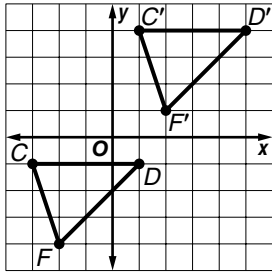
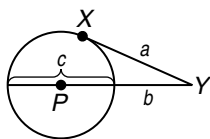
10. _____

Standardized Test Practice

(Chapters 1–10)

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Find the slope of a segment with endpoints at $(2a, -b)$ and $(-a, -3b)$. (Lesson 3-6)
- A. $\frac{-a}{4b}$ B. $\frac{3a}{2b}$ C. $\frac{2b}{3a}$ D. $\frac{-4b}{a}$
1. (A) (B) (C) (D)
2. If \overline{PT} and \overline{QS} are medians of $\triangle PQR$, which term describes M ? (Lesson 5-1)
- E. incenter F. centroid
G. orthocenter H. segment bisector
- 
2. (E) (F) (G) (H)
3. If \overline{DE} is an angle bisector of $\angle GDH$, which is a true statement? (Lesson 6-5)
- A. $\frac{a}{b} = \frac{y}{x}$
B. $\frac{a}{b} = \frac{x}{y}$
C. $(a + b)^2 = x^2 + y^2$
D. $DE = DH$
- 
3. (A) (B) (C) (D)
4. A plane flies at an altitude of 350 meters and then starts to descend when it is 6 kilometers from the runway. What is the angle of depression for the descent of the plane? (Lesson 7-5)
- E. about 3.3° F. about 33.4° G. about 8.9° H. about 89°
4. (E) (F) (G) (H)
5. Which statement is *not* true for all rectangles? (Lesson 8-4)
- A. The diagonals are congruent and bisect each other.
B. Opposite sides are congruent and parallel.
C. The diagonals are perpendicular.
D. Opposite angles are congruent.
5. (A) (B) (C) (D)
6. What transformation relates $\triangle CDF$ and $\triangle C'D'F'$? (Lesson 9-2)
- E. reflection F. translation
G. rotation H. dilation
- 
6. (E) (F) (G) (H)
7. Which is a true statement if \overline{XY} is tangent to $\odot P$? (Lesson 10-7)
- A. $ab = bc$ B. $a = bc$
C. $a^2 = bc$ D. $a^2 = b(b + c)$
- 
7. (A) (B) (C) (D)

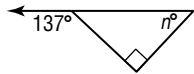
10

Standardized Test Practice (continued)

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

8. Find n . (Lesson 4-2)



8.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9. Find the length of \overline{XY} if $\overline{XY} \parallel \overline{BC}$, $BC = 15$, and \overline{XY} is a midsegment of $\triangle ABC$. (Lesson 6-4)

10. If $ABCD$ is an isosceles trapezoid with bases \overline{BC} and \overline{AD} , median \overline{EF} , $EF = 43$, and $BC = 12$, find AD . (Lesson 8-6)

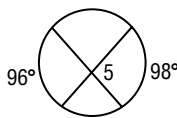
10.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11. Find $m\angle 5$. (Lesson 10-6)



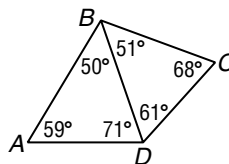
Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

12. Two parallel lines are cut by a transversal so that $\angle 1$ and $\angle 2$ are alternate interior angles. Find $m\angle 1$ if $m\angle 1 = 3y - 5$ and $m\angle 2 = y + 7$. (Lesson 3-2)

12. _____

13. Determine the relationship between \overline{AB} and \overline{BC} . (Lesson 5-2)

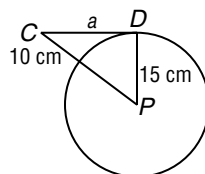


13. _____

14. Determine whether $\triangle GHJ$ is a right triangle given $G(3, 7)$, $H(-2, 5)$, and $J(-4, 10)$. (Lesson 7-2)

14. _____

15. Find a so that \overline{CD} is tangent to $\odot P$. (Lesson 10-5)



15. _____

10 Unit 3 Review

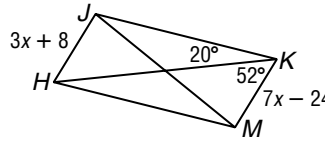
(Chapters 8–10)

SCORE _____

1. The measure of an interior angle of a regular polygon is 140. Find the number of sides in the polygon.

1. _____

2. If $JKLMH$ is a parallelogram, find $m\angle JHK$, $m\angle HMK$, and x .



2. _____

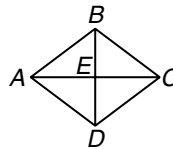
3. Determine whether the vertices of quadrilateral $DEFG$ form a parallelogram given $D(-3, 5)$, $E(3, 6)$, $F(-1, 0)$, and $G(6, 1)$.

3. _____

4. If $WXYZ$ is a rectangle with diagonals \overline{WY} and \overline{XZ} , $WY = 3d + 4$, and $XZ = 4d - 1$, find d .

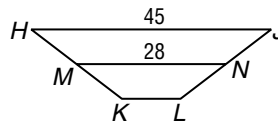
4. _____

5. If $m\angle BEC = 9z + 45$ in rhombus $ABCD$, find z .



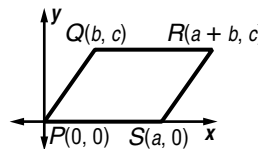
5. _____

6. In trapezoid $HJLK$, M and N are midpoints of the legs. Find KL .



6. _____

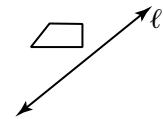
7. Prove that quadrilateral $PQRS$ is a parallelogram.



7. _____

8. Construct the reflected image of the quadrilateral in line ℓ .

8. _____



9. Triangle QST with vertices $Q(9, 5)$, $S(12, -8)$, and $T(6, -3)$ is translated so that S' is at $(17, -9)$. Find the coordinates of Q' and T' .

9. _____

10. Determine the order and magnitude of the rotational symmetry of a regular decagon.

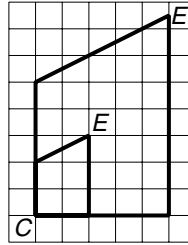
10. _____

11. Can an isosceles trapezoid tessellate the plane?

11. _____

10 Unit 3 Review *(continued)*

12. Determine the scale factor used for the dilation of the figure with center C . Then state whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.



12. _____

13. Find the coordinates of the image of $B(3, -5)$ under the translation $\vec{v} = \langle -6, 2 \rangle$.

13. _____

14. Use a matrix to find the coordinates of the vertices of the image of $\triangle PQR$ with $P(-1, 8)$, $Q(5, 5)$, and $R(3, -6)$ after a reflection in the line $y = x$.

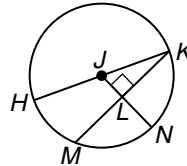
14. _____

15. Find the diameter and circumference of a circle with radius 47 centimeters.

15. _____

For Questions 16–18, refer to the figure.

16. In $\odot J$, if $HK = 28$ centimeters and $m\widehat{NK} = 72$, find $m\angle NJK$ and the length of \widehat{NK} .



16. _____

17. If radius \overline{HJ} measures 20 units, $JL = 12$, and $m\angle HJN = 126.9$, find LK , MK , and $m\widehat{MNK}$.

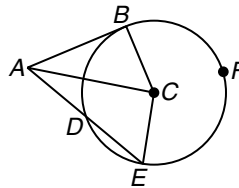
17. _____

18. Find $m\angle HKM$ if $m\widehat{HM} = 42$.

18. _____

For Questions 19–21, refer to the figure.

19. If \overline{AB} is tangent to $\odot C$, $BC = 8$, and $AB = 10$, find AC to the nearest tenth.



19. _____

20. What is $m\widehat{BFE}$ if $m\angle BAE = 64$ and $m\widehat{BD} = 68$?

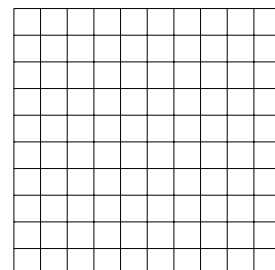
20. _____

21. In $\odot C$, if $AB = 12$, $AD = x$, and $DE = x + 12$, find x .

21. _____

22. Graph $(x - 1)^2 + (y + 2)^2 = 4$.

22. _____



Standardized Test Practice

Student Record Sheet (Use with pages 588–589 of the Student Edition.)

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

- | | | | | | | | | | | | | | | |
|---|-----|-----|-----|-----|---|-----|-----|-----|-----|---|-----|-----|-----|-----|
| 1 | (A) | (B) | (C) | (D) | 4 | (A) | (B) | (C) | (D) | 7 | (A) | (B) | (C) | (D) |
| 2 | (A) | (B) | (C) | (D) | 5 | (A) | (B) | (C) | (D) | 8 | (A) | (B) | (C) | (D) |
| 3 | (A) | (B) | (C) | (D) | 6 | (A) | (B) | (C) | (D) | 9 | (A) | (B) | (C) | (D) |

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 11, 12, 13, 14, and 15, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 _____

11 _____ (grid in)

12 _____ (grid in)

13 _____ (grid in)

14 _____ (grid in)

15 _____ (grid in)

11

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12

.	.	.	.
0	0	0	0
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2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

13

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

14

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

15

.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3 Open-Ended

Record your answers for Questions 16–17 on the back of this paper.

NAME _____

DATE _____

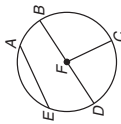
PERIOD _____

10-1 Study Guide and Intervention

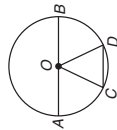
Circles and Circumference

Parts of Circles A circle consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.

- A segment or line can intersect a circle in several ways.
- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.



chord: \overline{AE} , \overline{BD}
radius: \overline{FB} , \overline{FC} , \overline{FD}
diameter: \overline{AD}



Example

- Name the circle.**
The name of the circle is $\odot O$.
- Name radii of the circle.**
 \overline{AO} , \overline{BO} , \overline{CO} , and \overline{DO} are radii.
- Name chords of the circle.**
 \overline{AB} and \overline{CD} are chords.
- Name a diameter of the circle.**
 \overline{AB} is a diameter.

Exercises

- Name the circle. $\odot R$
- Name radii of the circle. \overline{RA} , \overline{RB} , \overline{RY} , and \overline{RX}
- Name chords of the circle. \overline{BY} , \overline{AX} , \overline{AB} , and \overline{XY}
- Name diameters of the circle. \overline{AB} and \overline{XY}
- Find \overline{AR} if \overline{AB} is 18 millimeters. **9 mm**
- Find \overline{AR} and \overline{AB} if \overline{RY} is 10 inches. **$\overline{AR} = 10$ in.; $\overline{AB} = 20$ in.**
- Is $\overline{AB} \cong \overline{XY}$? Explain. **Yes; all diameters of the same circle are congruent.**

NAME _____

DATE _____

PERIOD _____

10-1 Study Guide and Intervention

Circles and Circumference

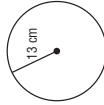
Circumference The circumference of a circle is the distance around the circle.

For a circumference of C units and a diameter of d units or a radius of r units,
 $C = \pi d$ or $C = 2\pi r$.

Example Find the circumference of the circle to the nearest hundredth.

$C = 2\pi r$ Circumference formula
 $= 2\pi(13)$ $r = 13$
 ≈ 81.68 Use a calculator.

The circumference is about 81.68 centimeters.



Exercises

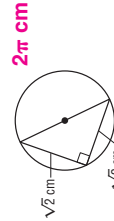
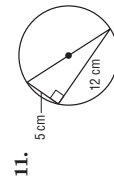
Find the circumference of a circle with the given radius or diameter. Round to the nearest hundredth.

- $r = 8$ cm **50.27 cm**
- $r = 3\sqrt{2}$ ft **26.66 ft**
- $r = 4.1$ cm **25.76 cm**
- $d = 10$ in. **31.42 in.**
- $d = \frac{1}{3}$ m **1.05 m**
- $d = 18$ yd **56.55 yd**

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- $r = 4$ cm **8 cm**, $C = 25.13$ cm
- $d = 6$ ft **3 ft**, $C = 18.85$ ft
- $r = 12$ cm **24 cm**, $C = 75.40$ cm
- $d = 15$ in. **7.5 in.**, $C = 47.12$ in.

Find the exact circumference of each circle.



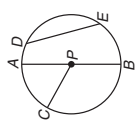
- 13π cm**
- 2π cm**

NAME _____ DATE _____ PERIOD _____

10-1 Skills Practice

Circles and Circumference

For Exercises 1–5, refer to the circle.

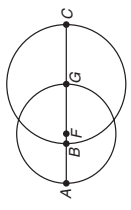


- Name the circle. **⊙P**
- Name a radius. **\overline{PA} , \overline{PB} , or \overline{PC}**
- Name a chord. **\overline{AB} or \overline{DE}**
- Name a diameter not drawn as part of a diameter. **\overline{PC}**

6. Suppose the diameter of the circle is 16 centimeters. Find the radius. **8 cm**

7. If $PC = 11$ inches, find AB . **22 in.**

The diameters of $\odot F$ and $\odot G$ are 5 and 6 units, respectively. Find each measure.

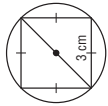
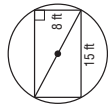


- BF **0.5**
- AB **2**

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- $r = 8$ cm
 $d =$ **16 cm**, $C \approx$ **50.27 cm**
- $d = 9$ m
 $r =$ **4.5 m**, $C \approx$ **28.27 m**
- $r = 13$ ft
 $d =$ **26 ft**, $C \approx$ **81.68 ft**
- $C = 35.7$ in.
 $d \approx$ **11.36 in.**, $r \approx$ **5.68 in.**

Find the exact circumference of each circle.

-  **$3\pi\sqrt{2}$ cm**
-  **17π ft**

© Glencoe/McGraw-Hill **543** *Glencoe Geometry*

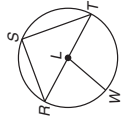
Lesson 10-1

NAME _____ DATE _____ PERIOD _____

10-1 Practice (Average)

Circles and Circumference

For Exercises 1–5, refer to the circle.

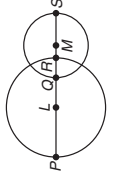


- Name the circle. **⊙L**
- Name a radius. **\overline{LR} , \overline{LT} , or \overline{LW}**
- Name a chord. **\overline{RT} , \overline{RS} , or \overline{ST}**
- Name a diameter not drawn as part of a diameter. **\overline{LW}**

6. Suppose the radius of the circle is 3.5 yards. Find the diameter. **7 yd**

7. If $RT = 19$ meters, find LW . **9.5 m**

The diameters of $\odot L$ and $\odot M$ are 20 and 13 units, respectively. Find each measure if $QR = 4$.

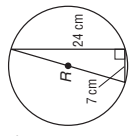
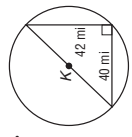


- LQ **6**
- RM **2.5**

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

- $r = 7.5$ mm
 $d =$ **15 mm**, $C \approx$ **47.12 mm**
- $C = 227.6$ yd
 $d \approx$ **72.45 yd**, $r \approx$ **36.22 yd**

Find the exact circumference of each circle.

-  **25π cm**
-  **58π mi**

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10-1

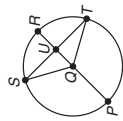
Reading to Learn Mathematics
Circles and Circumference

Pre-Activity How far does a carousel animal travel in one rotation?

Read the introduction to Lesson 10-1 at the top of page 522 in your textbook. How could you measure the approximate distance around the circular carousel using everyday measuring devices? **Sample answer: Place a piece of string along the rim of the carousel. Cut off a length of string that covers the perimeter of the circle. Straighten the string and measure it with a yardstick.**

Reading the Lesson

- Refer to the figure.
 - Name the circle. $\odot Q$
 - Name four radii of the circle. \overline{QP} , \overline{QR} , \overline{QS} , and \overline{QT}
 - Name a diameter of the circle. \overline{PR}
 - Name two chords of the circle. \overline{PT} and \overline{ST}
- Match each description from the first column with the best term from the second column. (Some terms in the second column may be used more than once or not at all.)
 - a segment whose endpoints are on a circle **iii**
 - the set of all points in a plane that are the same distance from a given point **iv**
 - the distance between the center of a circle and any point on the circle **i**
 - a chord that passes through the center of a circle **ii**
 - a segment whose endpoints are the center and any point on a circle **i**
 - a chord made up of two collinear radii **ii**
 - the distance around a circle **v**
- Which equations correctly express a relationship in a circle? **A, D, G**
 - $d = 2r$
 - $C = \pi r$
 - $C = 2\pi r$
 - $d = \frac{C}{\pi}$
 - $r = \frac{d}{\pi}$
 - $C = r^2$
 - $C = 2\pi r$
 - $d = \frac{C}{2}$
 - $d = \frac{1}{2}r$



Helping You Remember

- A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *diameter* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part. **Sample answer: The first part comes from dia, which means across or through, as in diagonal. The second part comes from metron, which means measure, as in geometry.**

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10-1

Enrichment

The Four Color Problem

Mapmakers have long believed that only four colors are necessary to distinguish among any number of different countries on a plane map. Countries that meet only at a point may have the same color provided they do not have an actual border. The conjecture that four colors are sufficient for every conceivable plane map eventually attracted the attention of mathematicians and became known as the “four-color problem.” Despite extraordinary efforts over many years to solve the problem, no definite answer was obtained until the 1980s. Four colors are indeed sufficient, and the proof was accomplished by making ingenious use of computers.

The following problems will help you appreciate some of the complexities of the four-color problem. For these “maps,” assume that each closed region is a different country.

- What is the minimum number of colors necessary for each map?
 - 3**
 - 2**
 - 4**
 - 3**
 - 4**
- Draw some plane maps on separate sheets. Show how each can be colored using four colors. Then determine whether fewer colors would be enough. **See students' work.**

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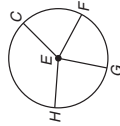
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Glencoe Geometry

10-2 Study Guide and Intervention

Angles and Arcs

Angles and Arcs A central angle is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.



\widehat{EF} is a minor arc.
 \widehat{EFG} is a major arc.
 $\angle FEG$ is a central angle.

- Here are some properties of central angles and arcs.
- The sum of the measures of the central angles of a circle with no interior points in common is 360.
 - The measure of a minor arc equals the measure of its central angle.
 - The measure of a major arc is 360 minus the measure of the minor arc.
 - Two arcs are congruent if and only if their corresponding central angles are congruent.
 - The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (Arc Addition Postulate)

$$m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$$

$$m\widehat{CF} = m\angle CEF$$

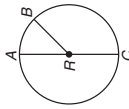
$$m\widehat{CGF} = 360 - m\widehat{CF}$$

$$\widehat{CF} \cong \widehat{FG} \text{ if and only if } \angle CEF \cong \angle FEG.$$

$$m\widehat{CF} + m\widehat{FG} = m\widehat{CG}$$

Example In $\odot R$, $m\angle ARB = 42$ and \widehat{AC} is a diameter. Find $m\widehat{AB}$ and $m\widehat{ACB}$.

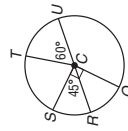
$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\widehat{AB} = 42$. Thus $m\widehat{ACB} = 360 - 42$ or 318.



Exercises

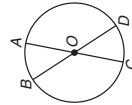
Find each measure.

- $m\angle SCT$ **75**
- $m\angle SCU$ **135**
- $m\angle SCQ$ **90**
- $m\angle QCT$ **165**



If $m\angle BOA = 44$, find each measure.

- $m\widehat{BA}$ **44**
- $m\widehat{BC}$ **136**
- $m\widehat{CD}$ **44**
- $m\widehat{BCD}$ **180**
- $m\widehat{AD}$ **136**



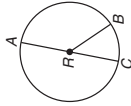
10-2 Study Guide and Intervention

Angles and Arcs

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

Example In $\odot R$, $m\angle ARB = 135$, $RB = 8$, and \widehat{AC} is a diameter. Find the length of \widehat{AB} .

$m\angle ARB = 135$, so $m\widehat{AB} = 135$. Using the formula $C = 2\pi r$, the circumference is $2\pi(8)$ or 16π . To find the length of \widehat{AB} , write a proportion to compare each part to its whole.



$$\frac{\text{length of } \widehat{AB}}{\text{circumference}} = \frac{\text{degree measure of arc}}{\text{degree measure of circle}}$$

$$\frac{\ell}{16\pi} = \frac{135}{360}$$

$$\ell = \frac{(16\pi)(135)}{360}$$

$$= 6\pi$$

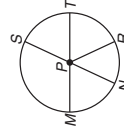
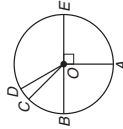
Proportion
Substitution
Multiply each side by 16π .
Simplify.

The length of \widehat{AB} is 6π or about 18.85 units.

Exercises

The diameter of $\odot O$ is 24 units long. Find the length of each arc for the given angle measure.

- \widehat{DE} if $m\angle DOE = 120$ **8π or 25.1**
 - \widehat{DEA} if $m\angle DOE = 120$ **14π or 44.0**
 - \widehat{BC} if $m\angle COB = 45$ **3π or 9.4**
 - \widehat{CBA} if $m\angle COB = 45$ **9π or 28.3**
- The diameter of $\odot P$ is 15 units long and $\angle SPT \cong \angle RPT$. Find the length of each arc for the given angle measure.
- \widehat{RT} if $m\angle SPT = 70$ **$\frac{35}{12}\pi$ or 9.2**
 - \widehat{NR} if $m\angle RPT = 50$ **$\frac{10}{3}\pi$ or 10.5**
 - \widehat{MST} **7.5π or 23.6**
 - \widehat{MRS} if $m\angle MPS = 140$ **$\frac{55}{6}\pi$ or 28.8**



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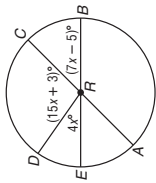
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10-2 Skills Practice

Angles and Arcs

ALGEBRA In $\odot R$, \overline{AC} and \overline{EB} are diameters. Find each measure.

- $m\angle ERD$ **28**
- $m\angle CRD$ **108**
- $m\angle BRC$ **44**
- $m\angle ARB$ **136**
- $m\angle ARE$ **44**
- $m\angle BRD$ **152**



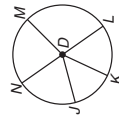
In $\odot A$, $m\angle PAU = 40$, $\angle PAU \cong \angle SAT$, and $\angle RAS \cong \angle TAU$. Find each measure.

- $m\overline{PQ}$ **90**
- $m\overline{QR}$ **180**
- $m\overline{ST}$ **40**
- $m\overline{RS}$ **50**
- $m\overline{RSU}$ **140**
- $m\overline{STP}$ **130**
- $m\overline{PRU}$ **320**



The diameter of $\odot D$ is 18 units long. Find the length of each arc for the given angle measure.

- \overline{LM} if $m\angle LDM = 100$
 $5\pi \approx 15.71$ units
- \overline{KL} if $m\angle KDL = 60$
 $3\pi \approx 9.42$ units
- \overline{MN} if $m\angle MDN = 80$
 $4\pi \approx 12.57$ units
- \overline{JK} if $m\angle NDK = 120$
 $6\pi \approx 18.85$ units
- \overline{JK} if $m\angle JDK = 50$
 $2.5\pi \approx 7.85$ units
- \overline{KL} if $m\angle KDM = 160$
 $8\pi \approx 25.13$ units



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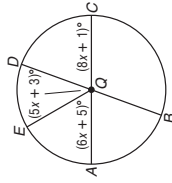
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10-2 Practice (Average)

Angles and Arcs

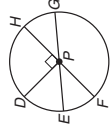
ALGEBRA In $\odot Q$, \overline{AC} and \overline{BD} are diameters. Find each measure.

- $m\angle AQE$ **59**
- $m\angle DQE$ **48**
- $m\angle CQD$ **73**
- $m\angle BQC$ **107**
- $m\angle CQE$ **121**
- $m\angle AQD$ **107**



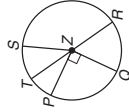
In $\odot P$, $m\angle GPH = 38$. Find each measure.

- $m\overline{EF}$ **38**
- $m\overline{DE}$ **52**
- $m\overline{FG}$ **142**
- $m\overline{DHG}$ **128**
- $m\overline{DFG}$ **232**
- $m\overline{DGE}$ **308**



The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.

- \overline{QT} if $m\angle QZT = 120$
 $9\pi \approx 28.27$ units
- \overline{QR} if $m\angle QZR = 60$
 $4.5\pi \approx 14.14$ units
- \overline{PS} if $m\angle QZS = 160$
 $11.25\pi \approx 35.34$ units
- \overline{PS} if $m\angle QZS = 160$
 $12\pi \approx 37.70$ units



HOMEWORK For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?
 28.8° , 104.4° , 208.8° , 10.8° , 7.2°

18. Describe the arcs associated with each category.

The arc associated with 2–3 hours is a major arc; minor arcs are associated with the remaining categories.

Homework	
Less than 1 hour	8%
1–2 hours	29%
2–3 hours	58%
3–4 hours	3%
Over 4 hours	2%

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10-2 Reading to Learn Mathematics

Angles and Arcs

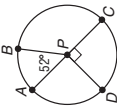
Pre-Activity What kinds of angles do the hands on a clock form?

Read the introduction to Lesson 10-2 at the top of page 529 in your textbook.

- What is the measure of the angle formed by the hour hand and the minute hand of the clock at 5:00? **150**
- What is the measure of the angle formed by the hour hand and the minute hand at 10:30? (Hint: How has each hand moved since 10:00?) **135**

Reading the Lesson

- Refer to $\odot P$. Indicate whether each statement is *true* or *false*.
 - \widehat{DAB} is a major arc. **false**
 - \widehat{ADC} is a semicircle. **true**
 - $\widehat{AD} \cong \widehat{CD}$ **true**
 - \widehat{DA} and \widehat{AB} are adjacent arcs. **true**
 - $\angle BPC$ is an acute central angle. **false**
 - $\angle DPA$ and $\angle BPA$ are supplementary central angles. **false**



2. Refer to the figure in Exercise 1. Give each of the following arc measures.

- $m\widehat{AB}$ **52**
- $m\widehat{BC}$ **128**
- $m\widehat{DAB}$ **142**
- $m\widehat{DAC}$ **270**
- $m\widehat{CD}$ **90**
- $m\widehat{ADC}$ **180**
- $m\widehat{DCB}$ **218**
- $m\widehat{BDA}$ **308**

3. Underline the correct word or number to form a true statement.

- The arc measure of a semicircle is (90/180/360).
- Arcs of a circle that have exactly one point in common are (congruent/opposite/adjacent) arcs.
- The measure of a major arc is greater than (0/90/180) and less than (90/180/360).
- Suppose a set of central angles of a circle have interiors that do not overlap. If the angles and their interiors contain all points of the circle, then the sum of the measures of the central angles is (90/270/360).
- The measure of an arc formed by two adjacent arcs is the (sum/difference/product) of the measures of the two arcs.
- The measure of a minor arc is greater than (0/90/180) and less than (90/180/360).

Helping You Remember

- A good way to remember something is to explain it to someone else. Suppose your classmate Luis does not like to work with proportions. What is a way that he can find the length of a minor arc of a circle without solving a proportion? **Sample answer: Divide the measure of the central angle of the arc by 360 to form a fraction. Multiply this fraction by the circumference of the circle to find the length of the arc.**

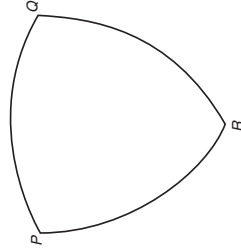
10-2 Enrichment

Curves of Constant Width

A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.

- Use a metric ruler to find the distance from P to any point on the opposite side. **4.6 cm**
- Find the distance from Q to the opposite side. **4.6 cm**
- What is the distance from R to the opposite side? **4.6 cm**

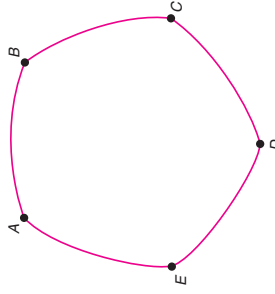


The Reuleaux triangle is made of three arcs. In the example shown, \widehat{PQ} has center R , \widehat{QR} has center P , and \widehat{PR} has center Q .

4. Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the square while keeping its sides in contact with the sides of the square. **See students' work.**

5. Make a different curve of constant width by starting with the five points below and following the steps given.

- Step 1:** Place the point of your compass on D with opening DA . Make an arc with endpoints A and B .
- Step 2:** Make another arc from B to C that has center E .
- Step 3:** Continue this process until you have five arcs drawn.



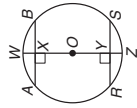
Some countries use shapes like this for coins. They are useful because they can be distinguished by touch, yet they will work in vending machines because of their constant width.

6. Measure the width of the figure you made in Exercise 5. Draw two parallel lines with the distance between them equal to the width you found. On a piece of paper, trace the five-sided figure and cut it out. Show that it will roll between the lines drawn. **5.3 cm**

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10-3 Study Guide and Intervention (continued)

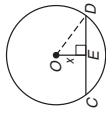
Arcs and Chords



- Diameters and Chords**
- In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.
 - In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

If $\overline{WZ} \perp \overline{AB}$, then $\overline{AX} \cong \overline{XB}$ and $\overline{AW} \cong \overline{WB}$.
 If $\overline{OX} = \overline{OY}$, then $\overline{AB} \cong \overline{RS}$.
 If $\overline{AB} \cong \overline{RS}$, then \overline{AB} and \overline{RS} are equidistant from point O.

Example In $\odot O$, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x .
 A diameter or radius perpendicular to a chord bisects the chord, so ED is half of CD .



$$ED = \frac{1}{2}(24) = 12$$

Use the Pythagorean Theorem to find x in $\triangle OED$.

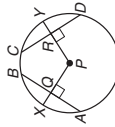
$$(OE)^2 + (ED)^2 = (OD)^2$$

Pythagorean Theorem
 $x^2 + 12^2 = 15^2$ Substitution
 $x^2 + 144 = 225$ Multiply
 $x^2 = 81$ Subtract 144 from each side.
 $x = 9$ Take the square root of each side.

Exercises

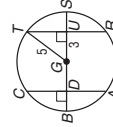
In $\odot P$, $CD = 24$ and $m\widehat{CY} = 45$. Find each measure.

- AQ 12
- RC 12
- QB 12
- AB 24
- $m\widehat{DY}$ 45
- $m\widehat{AB}$ 90
- $m\widehat{AX}$ 45
- $m\widehat{XB}$ 45
- $m\widehat{CD}$ 90



In $\odot G$, $DG = GU$ and $AC = RT$. Find each measure.

- TU 4
- TR 8
- TS 53.1
- CD 4
- GD 3
- $m\widehat{AB}$ 53.1



16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius. **26 in.**

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Glencoe Geometry

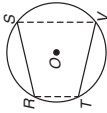
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10-3 Study Guide and Intervention

Arcs and Chords

Arcs and Chords Points on a circle determine both chords and arcs. Several properties are related to points on a circle.

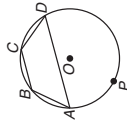


- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.

$\overline{RS} \cong \overline{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.
 $RSVT$ is inscribed in $\odot O$.
 $\odot O$ is circumscribed about $RSVT$.

Example Trapezoid $ABCD$ is inscribed in $\odot O$. If $\overline{AB} \cong \overline{BC}$ and $m\widehat{BC} = 50$, what is $m\widehat{APD}$?

Chords \overline{AB} , \overline{BC} , and \overline{CD} are congruent, so \overline{AB} , \overline{BC} , and \overline{CD} are congruent. $m\widehat{BC} = 50$, so $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} = 50 + 50 + 50 = 150$. Then $m\widehat{APD} = 360 - 150$ or 210 .

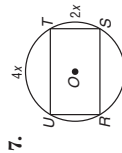


Exercises

Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.

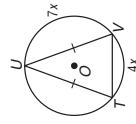
- hexagon 60
- pentagon 72
- triangle 120
- square 90
- octagon 45
- 36-gon 10

Determine the measure of each arc of the circle circumscribed about the polygon.



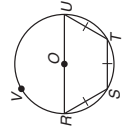
$$m\widehat{UT} = m\widehat{RS} = 120$$

$$m\widehat{ST} = m\widehat{RU} = 60$$



$$m\widehat{UT} = m\widehat{UV} = 140$$

$$m\widehat{TV} = 80$$



$$m\widehat{RS} = m\widehat{ST} = 60$$

$$m\widehat{RVU} = 180$$

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10-3

Skills Practice
Arcs and Chords

In $\odot H$, $m\widehat{RS} = 82$, $m\widehat{TU} = 82$, $RS = 46$, and $\widehat{TU} \cong \widehat{RS}$. Find each measure.

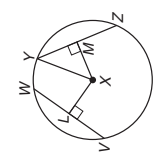
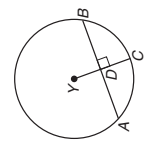
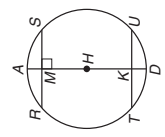
1. TU **46**
3. MS **23**
5. $m\widehat{AS}$ **41**
7. $m\widehat{TD}$ **41**

The radius of $\odot Y$ is 34, $AB = 60$, and $m\widehat{AC} = 71$. Find each measure.

9. $m\widehat{BC}$ **71**
11. AD **30**
13. YD **11**

In $\odot X$, $LX = MX$, $XY = 58$, and $VW = 84$. Find each measure.

15. YZ **84**
17. MX **40**
19. LV **42**



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10-3

Practice (Average)
Arcs and Chords

In $\odot E$, $m\widehat{HQ} = 48$, $HI = JK$, and $JR = 7.5$. Find each measure.

1. $m\widehat{HI}$ **96**
2. $m\widehat{QI}$ **48**
3. $m\widehat{JK}$ **96**
4. HI **15**
5. PI **7.5**
6. JK **15**

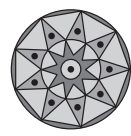
The radius of $\odot N$ is 18, $NK = 9$, and $m\widehat{DE} = 120$. Find each measure.

7. $m\widehat{GE}$ **60**
8. $m\angle HNE$ **60**
9. $m\angle HEN$ **30**
10. HN **9**

The radius of $\odot O = 32$, $PQ \cong RS$, and $PQ = 56$. Find each measure.

11. PB **28**
14. BQ **28**
12. OB $4\sqrt{15} \approx 15.49$
16. RS **56**

13. **MANDALAS** The base figure in a mandala design is a nine-pointed star. Find the measure of each arc of the circle circumscribed about the star. **Each arc measures 40° .**



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10-3 Reading to Learn Mathematics

Arcs and Chords

Pre-Activity How do the grooves in a Belgian waffle iron model segments in a circle?

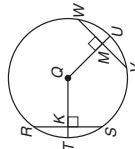
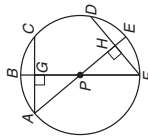
Read the introduction to Lesson 10-3 at the top of page 536 in your textbook. What do you observe about any two of the grooves in the waffle iron shown in the picture in your textbook? **They are either parallel or perpendicular.**

Reading the Lesson

- Supply the missing words or phrases to form true statements.
 - In a circle, if a radius is **perpendicular** to a chord, then it bisects the chord and its **arc**.
 - In a circle or in **congruent** circles, two **minor arcs** are congruent if and only if their corresponding chords are congruent.
 - In a circle or in **congruent** circles, two chords are congruent if they are **equidistant** from the center.
 - A polygon is inscribed in a circle if all of its **vertices** lie on the circle.
 - All of the sides of an inscribed polygon are **chords** of the circle.

2. If $\odot P$ has a diameter 40 centimeters long, and $AC = FD = 24$ centimeters, find each measure.

- PA **20 cm**
- AG **12 cm**
- PE **20 cm**
- PH **16 cm**
- HE **4 cm**
- $m\widehat{ST}$ **35**
- $m\widehat{VW}$ **70**
- $m\widehat{VT}$ **35**



4. Find the measure of each arc of a circle that is circumscribed about the polygon.

- an equilateral triangle **120**
- a regular pentagon **72**
- a regular hexagon **60**
- a regular decagon **36**
- a regular dodecagon **30**
- a regular n -gon **$\frac{360}{n}$**

Helping You Remember

- Some students have trouble distinguishing between *inscribed* and *circumscribed* figures. What is an easy way to remember which is which? **Sample answer: The inscribed figure is inside the circle.**

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Glencoe Geometry

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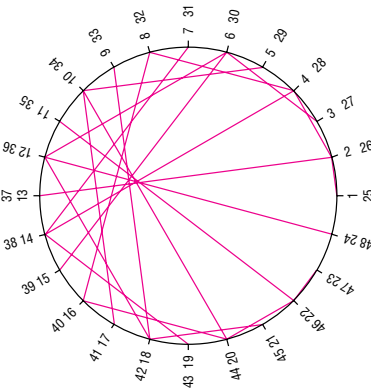
DATE _____

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10-3 Enrichment

Patterns from Chords

Some beautiful and interesting patterns result if you draw chords to connect evenly spaced points on a circle. On the circle shown below, 24 points have been marked to divide the circle into 24 equal parts. Numbers from 1 to 48 have been placed beside the points. Study the diagram to see exactly how this was done.



- Use your ruler and pencil to draw chords to connect numbered points as follows: 1 to 2, 2 to 4, 3 to 6, 4 to 8, and so on. Keep doubling until you have gone all the way around the circle. What kind of pattern do you get?

For figure, see above. The pattern is a heart-shaped figure.

- Copy the original circle, points, and numbers. Try other patterns for connecting points. For example, you might try tripling the first number to get the number for the second endpoint of each chord. Keep special patterns for a possible class display.

See students' work.

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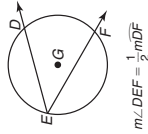
Glencoe Geometry

Lesson 10-3

10-4 Study Guide and Intervention

Inscribed Angles

Inscribed Angles An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle. In $\odot G$, inscribed $\angle DEF$ intercepts \widehat{DF} .

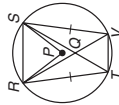


Inscribed Angle Theorem If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc.

Example In $\odot G$ above, $m\widehat{DF} = 90$. Find $m\angle DEF$.
 $\angle DEF$ is an inscribed angle so its measure is half of the intercepted arc.
 $m\angle DEF = \frac{1}{2}m\widehat{DF}$
 $= \frac{1}{2}(90)$ or 45

Exercises

Use $\odot P$ for Exercises 1–10. In $\odot P$, $\overline{RS} \parallel \overline{TV}$ and $\overline{RT} \cong \overline{SV}$.



- Name the intercepted arc for $\angle RTS$. **RS**
- Name an inscribed angle that intercepts \overline{SV} . **$\angle SRV$ or $\angle STV$**

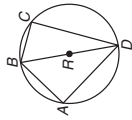
In $\odot P$, $m\widehat{SV} = 120$ and $m\angle RPS = 76$. Find each measure.

- $m\angle PRS$ **52**
- $m\widehat{RS}$ **196**
- $m\angle RVT$ **60**
- $m\angle STV$ **60**
- $m\angle QRS$ **60**
- $m\angle SVT$ **98**

10-4 Study Guide and Intervention

Inscribed Angles

Angles of Inscribed Polygons An inscribed polygon is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.



- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle. If \widehat{BCD} is a semicircle, then $m\angle BCD = 90$.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. For inscribed quadrilateral ABCD, $m\angle A + m\angle C = 180$ and $m\angle ABC + m\angle ADC = 180$.

Example In $\odot R$ above, $BC = 3$ and $BD = 5$. Find each measure.

- $m\angle C$
 $\angle C$ intercepts a semicircle. Therefore $\angle C$ is a right angle and $m\angle C = 90$.
- CD
 $\triangle BCD$ is a right triangle, so use the Pythagorean Theorem to find CD .
 $(CD)^2 + (BC)^2 = (BD)^2$
 $(CD)^2 + 3^2 = 5^2$
 $(CD)^2 = 25 - 9$
 $(CD)^2 = 16$
 $CD = 4$

Exercises

Find the measure of each angle or segment for each figure.

- $m\angle X$, $m\angle Y$

 **$m\angle X = 125$;
 $m\angle Y = 60$**
- AD

6.5
- $m\angle 1$, $m\angle 2$

 **$m\angle 1 = 50$;
 $m\angle 2 = 90$**
- AB , AC

 $AB = 3$; $AC = 6$
- $m\angle 1$, $m\angle 2$

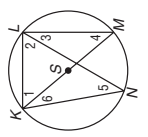
 $m\angle 1 = 88$; $m\angle 2 = 92$

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10-4 Skills Practice

Inscribed Angles

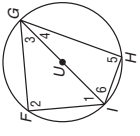
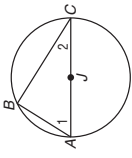
In $\odot S$, $m\widehat{KL} = 80$, $m\widehat{LM} = 100$, and $m\widehat{MN} = 60$. Find the measure of each angle.



- $m\angle 1$ **50**
- $m\angle 2$ **60**
- $m\angle 3$ **30**
- $m\angle 4$ **40**
- $m\angle 5$ **40**
- $m\angle 6$ **30**

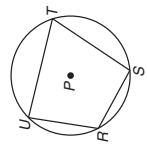
ALGEBRA Find the measure of each numbered angle.

- $m\angle 1 = 5x - 2$, $m\angle 2 = 2x + 8$
- $m\angle 1 = 5x$, $m\angle 3 = 3x + 10$, $m\angle 4 = y + 7$, $m\angle 6 = 3y + 11$



- $m\angle 1 = 58$, $m\angle 2 = 32$
- $m\angle 1 = 50$, $m\angle 2 = 90$, $m\angle 3 = 40$, $m\angle 4 = 25$, $m\angle 5 = 90$, $m\angle 6 = 65$

Quadrilateral $RSTU$ is inscribed in $\odot P$ such that $m\widehat{STU} = 220$ and $m\angle S = 95$. Find each measure.



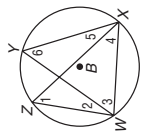
- $m\angle R$ **110**
- $m\angle T$ **70**
- $m\angle U$ **85**
- $m\widehat{RST}$ **170**

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10-4 Practice (Average)

Inscribed Angles

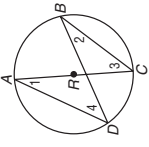
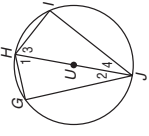
In $\odot B$, $m\widehat{WX} = 104$, $m\widehat{WZ} = 88$, and $m\angle ZWY = 26$. Find the measure of each angle.



- $m\angle 1$ **52**
- $m\angle 2$ **26**
- $m\angle 3$ **58**
- $m\angle 4$ **44**
- $m\angle 5$ **26**
- $m\angle 6$ **52**

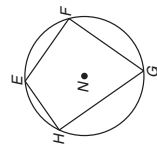
ALGEBRA Find the measure of each numbered angle.

- $m\angle 1 = 5x + 2$, $m\angle 2 = 2x - 3$, $m\angle 1 = 4x - 7$, $m\angle 2 = 2x + 11$, $m\angle 3 = 7y - 1$, $m\angle 4 = 2y + 10$, $m\angle 3 = 5y - 14$, $m\angle 4 = 3y + 8$



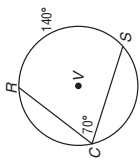
- $m\angle 1 = 67$, $m\angle 2 = 23$, $m\angle 3 = 62$, $m\angle 4 = 28$
- $m\angle 1 = 29$, $m\angle 2 = 29$, $m\angle 3 = 41$, $m\angle 4 = 41$

Quadrilateral $EFGH$ is inscribed in $\odot N$ such that $m\widehat{FG} = 97$, $m\angle H = 117$, and $m\angle EHG = 164$. Find each measure.



- $m\angle E$ **107**
- $m\angle F$ **82**
- $m\angle G$ **73**
- $m\angle H$ **98**

13. PROBABILITY In $\odot V$, point C is randomly located so that it does not coincide with points R or S . If $m\widehat{RS} = 140$, what is the probability that $m\angle RCS = 70^\circ$?



- $\frac{11}{18}$

10-4 Reading to Learn Mathematics

Inscribed Angles

Pre-Activity How is a socket like an inscribed polygon?

Read the introduction to Lesson 10-4 at the top of page 544 in your textbook.

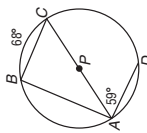
- Why do you think regular hexagons are used rather than squares for the "hole" in a socket? **Sample answer: If a square were used, the points might be too sharp for the tool to work smoothly.**
- Why do you think regular hexagons are used rather than regular polygons with more sides? **Sample answer: If there are too many sides, the polygon would be too close to a circle, so the wrench might slip.**

Reading the Lesson

- Underline the correct word or phrase to form a true statement.
 - An angle whose vertex is on a circle and whose sides contain chords of the circle is called a(n) (central/inscribed/circumscribed) angle.
 - Every inscribed angle that intercepts a semicircle is a(n) (acute/right/obtuse) angle.
 - The opposite angles of an inscribed quadrilateral are (congruent/complementary/supplementary).
 - An inscribed angle that intercepts a major arc is a(n) (acute/right/obtuse) angle.
 - Two inscribed angles of a circle that intercept the same arc are (congruent/complementary/supplementary).
 - If a triangle is inscribed in a circle and one of the sides of the triangle is a diameter of the circle, the diameter is (the longest side of an acute triangle/a leg of an isosceles triangle/the hypotenuse of a right triangle).

2. Refer to the figure. Find each measure.

- $m\angle ABC$ **90**
- $m\widehat{CD}$ **118**
- $m\widehat{AD}$ **62**
- $m\angle BAC$ **34**
- $m\angle BCA$ **56**
- $m\widehat{AB}$ **112**
- $m\widehat{BCD}$ **186**
- $m\widehat{BDA}$ **248**



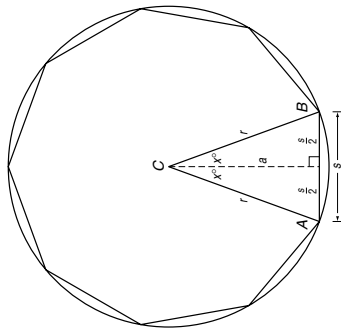
Helping You Remember

- A good way to remember a geometric relationship is to visualize it. Describe how you could make a sketch that would help you remember the relationship between the measure of an inscribed angle and the measure of its intercepted arc. **Sample answer: Draw a diameter of the circle to divide it into two semicircles. Inscribe an angle in one of the semicircles; this angle will intercept the other semicircle. From your sketch, you can see that the inscribed angle is a right angle. The measure of the semicircle arc is 180, so the measure of the inscribed angle is half the measure of its intercepted arc.**

10-4 Enrichment

Formulas for Regular Polygons

Suppose a regular polygon of n sides is inscribed in a circle of radius r . The figure shows one of the isosceles triangles formed by joining the endpoints of one side of the polygon to the center C of the circle. In the figure, s is the length of each side of the regular polygon, and a is the length of the segment from C perpendicular to \overline{AB} .



Use your knowledge of triangles and trigonometry to solve the following problems.

- Find a formula for x in terms of the number of sides n of the polygon.
$$x = \frac{180^\circ}{n}$$
- Find a formula for s in terms of the number of sides n and r . Use trigonometry.
$$s = 2r \sin\left(\frac{180^\circ}{n}\right)$$
- Find a formula for a in terms of n and r . Use trigonometry.
$$a = r \cos\left(\frac{180^\circ}{n}\right)$$
- Find a formula for the perimeter of the regular polygon in terms of n and r .
$$\text{perimeter} = 2nr \sin\left(\frac{180^\circ}{n}\right)$$

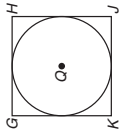
Lesson 10-4

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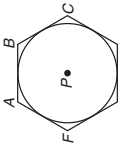
10-5 Study Guide and Intervention *(continued)*

Tangents

Circumscribed Polygons When a polygon is circumscribed about a circle, all of the sides of the polygon are tangent to the circle.



Square $GHJK$ is circumscribed about $\odot O$. \overline{GH} , \overline{JK} , and \overline{KG} are tangent to $\odot O$.



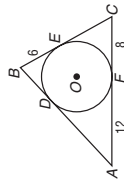
Hexagon $ABCDEF$ is circumscribed about $\odot P$. \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} , \overline{EF} , and \overline{FA} are tangent to $\odot P$.

Example $\triangle ABC$ is circumscribed about $\odot O$.

Find the perimeter of $\triangle ABC$.

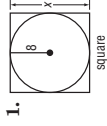
$\triangle ABC$ is circumscribed about $\odot O$, so points D , E , and F are points of tangency. Therefore $AD = AF$, $BE = BD$, and $CF = CE$.
 $P = AD + AF + BE + BD + CF + CE$
 $= 12 + 12 + 6 + 6 + 8 + 8$
 $= 52$

The perimeter is 52 units.

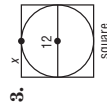


Exercises

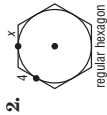
Find x . Assume that segments that appear to be tangent are tangent.



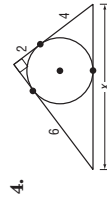
16



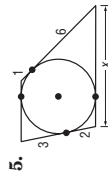
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4



10



8



4

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10-5 Study Guide and Intervention

Tangents

Tangents A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example \overline{AB} is tangent to $\odot C$. Find x .

\overline{AB} is tangent to $\odot C$, so \overline{AB} is perpendicular to radius \overline{BC} . CD is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17 . Use the Pythagorean Theorem with right $\triangle ABC$.

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$x^2 + 8^2 = 17^2$$

$$x^2 + 64 = 289$$

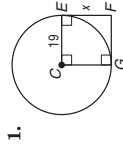
$$x^2 = 225$$

$$x = 15$$

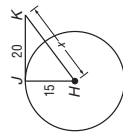
Pythagorean Theorem
 Substitution
 Multiply.
 Subtract 64 from each side.
 Take the square root of each side.

Exercises

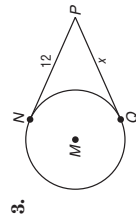
Find x . Assume that segments that appear to be tangent are tangent.



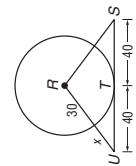
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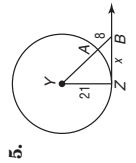
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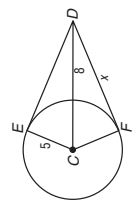
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565

Glencoe Geometry

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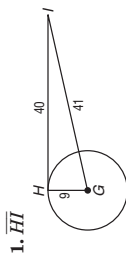
Lesson 10-5

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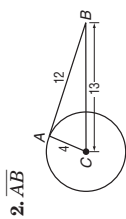
10-5 Skills Practice

Tangents

Determine whether each segment is tangent to the given circle.

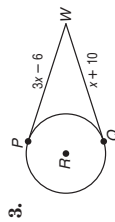


yes

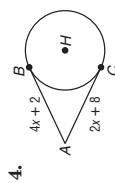


no

Find x . Assume that segments that appear to be tangent are tangent.



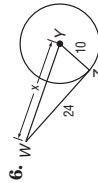
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3

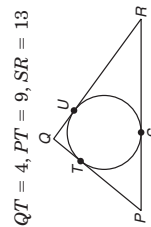


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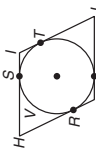


26

Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.



52 units



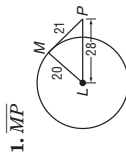
72 units

10-5 Practice (Average)

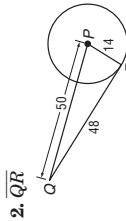
Tangents

NAME _____ DATE _____ PERIOD _____

Determine whether each segment is tangent to the given circle.

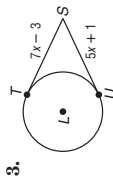


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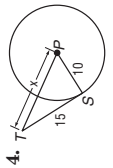


yes

Find x . Assume that segments that appear to be tangent are tangent.



2

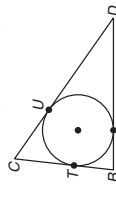


$5\sqrt{13}$

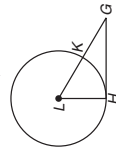
Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.

5. $CD = 52$, $CU = 18$, $TB = 12$

6. $KG = 32$, $HG = 56$



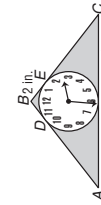
128 units



154 units

CLOCKS For Exercises 7 and 8, use the following information.

The design shown in the figure is that of a circular clock face inscribed in a triangular base. AP and FC are equal.



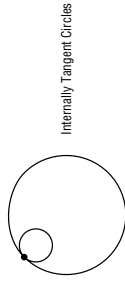
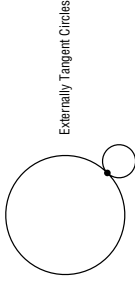
7. Find AB . **9.5 in.**

8. Find the perimeter of the clock. **34 in.**

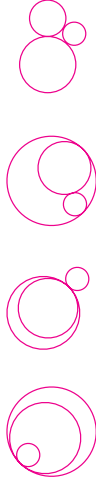
10-5 Enrichment

Tangent Circles

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them are tangent.



1. Make sketches to show all possible positions of three mutually tangent circles.



2. Make sketches to show all possible positions of four mutually tangent circles.



3. Make sketches to show all possible positions of five mutually tangent circles.



4. Write a conjecture about the number of possible positions for n mutually tangent circles if n is a whole number greater than four.

Possible answer: For $n > 4$, there are $\frac{n}{2}$ positions if n is even and $\frac{1}{2}(n + 1)$ positions if n is odd.

10-5 Reading to Learn Mathematics

Tangents

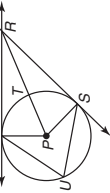
Pre-Activity How are tangents related to track and field events?

Read the introduction to Lesson 10-5 at the top of page 552 in your textbook. How is the hammer throw event related to the mathematical concept of a tangent line?

Sample answer: When the hammer is released, its initial path is a good approximation of a tangent line to the circular path around which it was traveling just before it was released.

Reading the Lesson

1. Refer to the figure. Name each of the following in the figure.



- a. two lines that are tangent to $\odot P$ \overline{RQ} and \overline{RS}
- b. two points of tangency Q, S
- c. two chords of the circle \overline{UQ} and \overline{US}
- d. three radii of the circle \overline{PQ} , \overline{PS} , and \overline{PT}
- e. two right angles $\angle PQR$ and $\angle PSR$
- f. two congruent right triangles $\triangle PQR$ and $\triangle PSR$
- g. the hypotenuse or hypotenuses in the two congruent right triangles \overline{PR}
- h. two congruent central angles $\angle QPT$ and $\angle SPT$
- i. two congruent minor arcs \overline{QT} and \overline{ST}
- j. an inscribed angle $\angle QUS$

2. Explain the difference between an *inscribed polygon* and a *circumscribed polygon*. Use the words *vertex* and *tangent* in your explanation.

Sample answer: If a polygon is *inscribed* in a circle, every vertex of the polygon lies on the circle. If a polygon is *circumscribed* about a circle, every side of the polygon is tangent to the circle.

Helping You Remember

3. A good way to remember a mathematical term is to relate it to a word or expression that is used in a nonmathematical way. Sometimes a word or expression used in English is derived from a mathematical term. What does it mean to “go off on a tangent,” and how is this meaning related to the geometric idea of a *tangent* line?

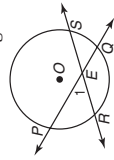
Sample answer: To “go off on a tangent” means to suddenly change the subject when you are talking or writing. You can visualize this as being like a tangent line “going off” from a circle as you go farther from the point of tangency.

10-6 Study Guide and Intervention

Secants, Tangents, and Angle Measures

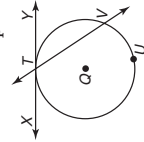
Intersections On or Inside a Circle A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.



$$m\angle 1 = \frac{1}{2}(m\overline{PR} + m\overline{QS})$$

- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.



$$m\angle XTV = \frac{1}{2}m\overline{TUV}$$

$$m\angle YTV = \frac{1}{2}m\overline{TUV}$$

Example 1 Find x.

The two secants intersect inside the circle, so x is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

$$x = \frac{1}{2}(30 + 55)$$

$$= \frac{1}{2}(85)$$

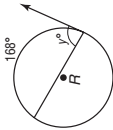
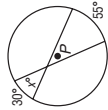
$$= 42.5$$

Example 2 Find y.

The secant and the tangent intersect at the point of tangency, so the measure the angle is one-half the measure of its intercepted arc.

$$y = \frac{1}{2}(168)$$

$$= 84$$

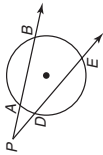


10-6 Study Guide and Intervention

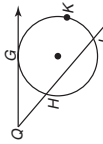
Secants, Tangents, and Angle Measures

Intersections Outside a Circle If secants and tangents intersect outside a circle, they form an angle whose measure is related to the intercepted arcs.

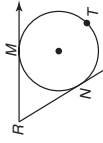
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.



$$m\angle P = \frac{1}{2}(m\overline{BE} - m\overline{AD})$$



$$m\angle Q = \frac{1}{2}(m\overline{GK} - m\overline{GH})$$



$$m\angle R = \frac{1}{2}(m\overline{MTN} - m\overline{MN})$$

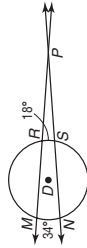
Example Find $m\angle MPN$.

$\angle MPN$ is formed by two secants that intersect in the exterior of a circle.

$$m\angle MPN = \frac{1}{2}(m\overline{MN} - m\overline{RS})$$

$$= \frac{1}{2}(34 - 18)$$

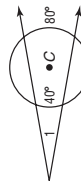
$$= \frac{1}{2}(16) \text{ or } 8$$



Exercises

Find each measure.

1. $m\angle 1$ **20**



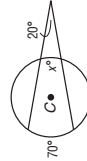
2. $m\angle 2$ **40**



3. $m\angle 3$ **40**



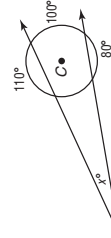
4. x **30**



5. x **130**



6. x **15**

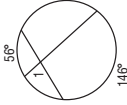
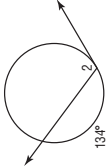
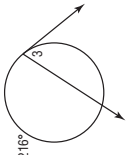


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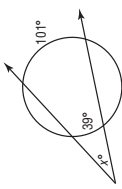
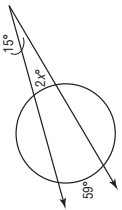
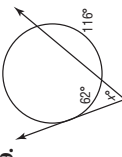
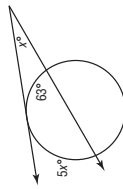
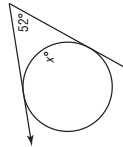
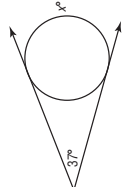
10-6 Practice (Average)

Secants, Tangents, and Angle Measures

Find each measure.

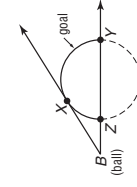
- 1. $m\angle 1$  **79**
- 2. $m\angle 2$  **113**
- 3. $m\angle 3$  **72**

Find x . Assume that any segment that appears to be tangent is tangent.

- 7.  **31**
- 8.  **14.5**
- 9.  **60**
- 10.  **21**
- 11.  **128**
- 12.  **217**

9. RECREATION In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.

Rickie must kick the ball at an angle less than 32° since the measure of the angle from the ground that a tangent would make with the goal post is 32° .

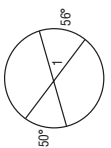
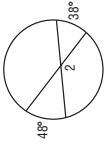
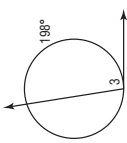


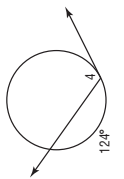
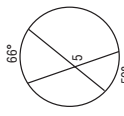
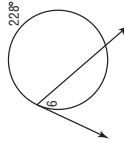
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10-6 Skills Practice

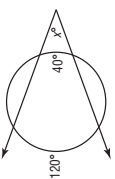
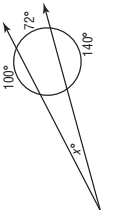
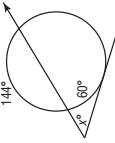
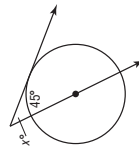
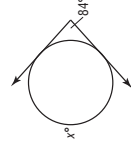
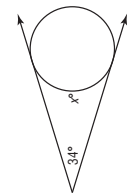
Secants, Tangents, and Angle Measures

Find each measure.

- 1. $m\angle 1$  **53**
- 2. $m\angle 2$  **137**
- 3. $m\angle 3$  **99**

- 4. $m\angle 4$  **118**
- 5. $m\angle 5$  **122**
- 6. $m\angle 6$  **66**

Find x . Assume that any segment that appears to be tangent is tangent.

- 7.  **40**
- 8.  **12**
- 9.  **48**
- 10.  **45**
- 11.  **264**
- 12.  **146**

10-6 Reading to Learn Mathematics
Secants, Tangents, and Angle Measures

Pre-Activity How is a rainbow formed by segments of a circle?

Read the introduction to Lesson 10-6 at the top of page 561 in your textbook.

- How would you describe $\angle C$ in the figure in your textbook?
Sample answer: $\angle C$ is an inscribed angle in the circle that represents the raindrop.
- When you see a rainbow, where is the sun in relation to the circle of which the rainbow is an arc? **Sample answer: behind you and opposite the center of the circle**

Reading the Lesson

- Underline the correct word to form a true statement.
 - A line can intersect a circle in at most (one/two/three) points.
 - A line that intersects a circle in exactly two points is called a (tangent/secant/radius).
 - A line that intersects a circle in exactly one point is called a (tangent/secant/radius).
 - Every secant of a circle contains a (radius/tangent/chord).
- Determine whether each statement is *always*, *sometimes*, or *never* true.
 - A secant of a circle passes through the center of the circle. **sometimes**
 - A tangent to a circle passes through the center of the circle. **never**
 - A secant-secant angle is a central angle of the circle. **sometimes**
 - A vertex of a secant-tangent angle is a point on the circle. **sometimes**
 - A secant-tangent angle passes through the center of the circle. **sometimes**
 - The vertex of a tangent-tangent angle is a point on the circle. **never**
 - If one side of a secant-tangent angle passes through the center of the circle, the angle is a right angle. **always**
 - The measure of a secant-secant angle is one-half the positive difference of the measures of its intercepted arcs. **sometimes**
 - The sum of the measures of the arcs intercepted by a tangent-tangent angle is 360. **always**
 - The two arcs intercepted by a tangent-tangent angle are congruent. **never**

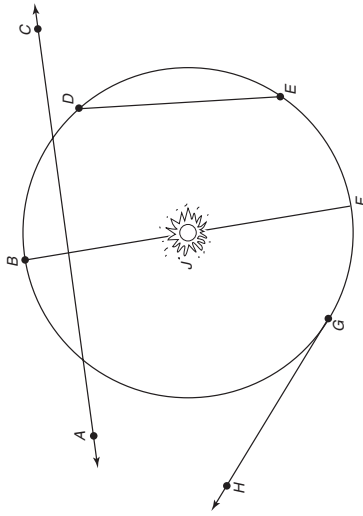
Helping You Remember

- Some students have trouble remembering the difference between a *secant* and a *tangent*. What is an easy way to remember which is which?
Sample answer: A secant cuts a circle, while a tangent just touches it at one point. You can associate tangent with touches because they both start with t. Then associate secant with cuts.

10-6 Enrichment

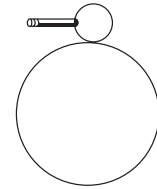
Orbiting Bodies

The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



Use the drawing above of the Earth orbiting the sun to name the line or segment described. Then identify it as a *radius*, *diameter*, *chord*, *tangent*, or *secant* of the orbit.

- the path of an asteroid **\overline{AC} , secant**
- the distance between the Earth's position in July and the Earth's position in October **\overline{DE} , chord**
- the distance between the Earth's position in December and the Earth's position in June **\overline{BF} , diameter**
- the path of a rocket shot toward Saturn **\overline{GH} , tangent**
- the path of a sunbeam **\overline{JB} or \overline{JF} , radius**
- If a planet has a moon, the moon circles the planet as the planet circles the sun. To visualize the path of the moon, cut two circles from a piece of cardboard, one with a diameter of 4 inches and one with a diameter of 1 inch.



Tape the larger circle firmly to a piece of paper. Poke a pencil point through the smaller circle, close to the edge. Roll the small circle around the outside of the large one. The pencil will trace out the path of a moon circling its planet. This kind of curve is called an epicycloid. To see the path of the planet around the sun, poke the pencil through the center of the small circle (the planet), and roll the small circle around the large one (the sun). **See students' work.**

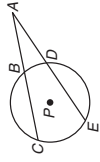
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10-7 Study Guide and Intervention (continued)

Special Segments in a Circle

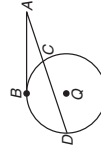
Segments Intersecting Outside a Circle If secants and tangents intersect outside a circle, then two products are equal.

- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



AC and AE are secant segments.
 AB and AD are external secant segments.
 $AC \cdot AB = AE \cdot AD$

- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



AB is a tangent segment.
 AD is a secant segment.
 AC is an external secant segment.
 $(AB)^2 = AD \cdot AC$

Example Find x to the nearest tenth.

The tangent segment is AB , the secant segment is BD , and the external secant segment is BC .

$$(AB)^2 = BC \cdot BD$$

$$(18)^2 = 15(15 + x)$$

$$324 = 225 + 15x$$

$$99 = 15x$$

$$6.6 = x$$

Exercises

Find x to the nearest tenth. Assume segments that appear to be tangent are tangent.

1. 2.8
2. 19.3
3. 7.7
4. 2.0
5. 1
6. 5
7. 37.3
8. 13.2
9. 4

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578

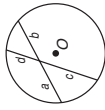
Glencoe Geometry

NAME _____ DATE _____ PERIOD _____

10-7 Study Guide and Intervention

Special Segments in a Circle

Segments Intersecting Inside a Circle If two chords intersect in a circle, then the products of the measures of the chords are equal.



$$a \cdot b = c \cdot d$$

Example Find x .

The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.

$$AB \cdot BC = EB \cdot BD$$

$$6 \cdot x = 8 \cdot 3$$

$$6x = 24$$

$$x = 4$$

Substitution
Simplify
Divide each side by 6.

Lesson 10-7

Exercises

Find x to the nearest tenth.

1. 9
2. 6
3. 10.7
4. 2
5. 3
6. 4.9
7. 2.2
8. 4

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Glencoe Geometry

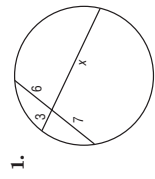
NAME _____ DATE _____ PERIOD _____

10-7

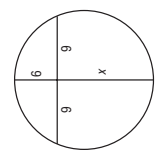
Skills Practice

Special Segments in a Circle

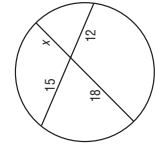
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.



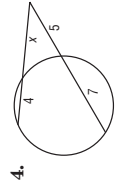
14



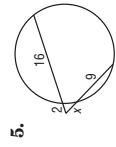
13.5



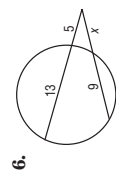
10



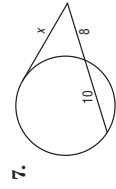
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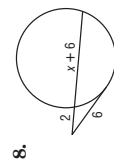
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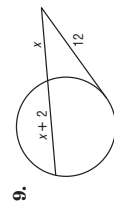
6



12



10



8

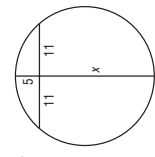
NAME _____ DATE _____ PERIOD _____

10-7

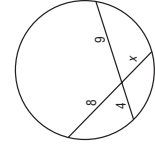
Practice (Average)

Special Segments in a Circle

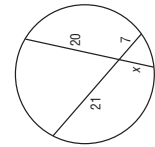
Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.



24.2



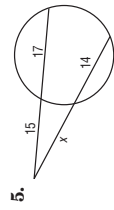
4.5



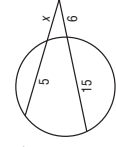
7.4



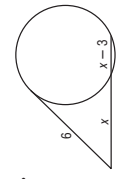
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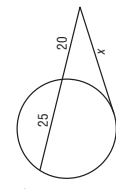
16



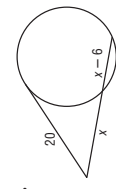
9



5.1



30



15.7



10. CONSTRUCTION An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch. **4.875 ft**

10-7

Reading to Learn Mathematics

Special Segments in a Circle

Pre-Activity How are lengths of intersecting chords related?

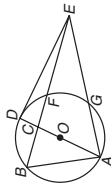
Read the introduction to Lesson 10-7 at the top of page 569 in your textbook.

- What kinds of angles of the circle are formed at the points of the star?
inscribed angles
- What is the sum of the measures of the five angles of the star? **180**

Reading the Lesson

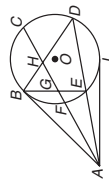
1. Refer to $\odot O$. Name each of the following.

- a diameter **\overline{AD}**
- a chord that is not a diameter **\overline{AB} , \overline{BF} , or \overline{AG}**
- two chords that intersect in the interior of the circle **\overline{AD} and \overline{BF}**
- an exterior point **E**
- two secant segments that intersect in the exterior of the circle **\overline{EA} and \overline{EB}**
- a tangent segment **\overline{ED}**
- a right angle **$\angle ADE$**
- an external secant segment **\overline{EF} or \overline{EG}**
- a secant-tangent angle with vertex on the circle **$\angle ADE$**
- an inscribed angle **$\angle BAD$, $\angle DAG$, $\angle BAG$, or $\angle ABF$**



2. Supply the missing length to complete each equation.

- $BH \cdot HD = FH \cdot \underline{HC}$ b. $AC \cdot AF = AD \cdot \underline{AE}$
- $AD \cdot AE = AB \cdot \underline{AB}$ d. $AB = \underline{AI}$
- $AF \cdot AC = (\underline{AI \text{ or } AB})^2$ f. $EG \cdot \underline{GB} = FG \cdot GC$



Helping You Remember

3. Some students find it easier to remember geometric theorems if they restate them in their own words. Restate Theorem 10.16 in a way that you find easier to remember.

Sample answer: Suppose you draw a secant to a circle through a point A outside the circle. Multiply the distances from point A to the points where the secant intersects the circle. The corresponding product will be the same for any other secant through point A to the same circle.

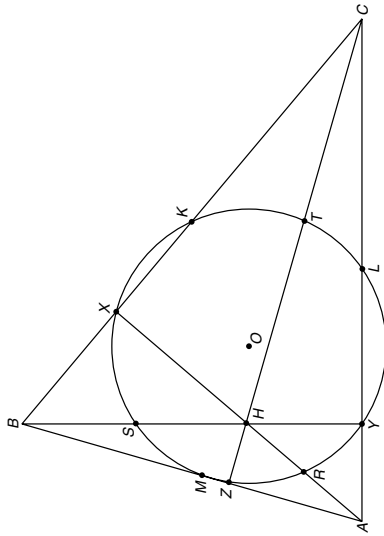
10-7

Enrichment

The Nine-Point Circle

The figure below illustrates a surprising fact about triangles and circles. Given any $\triangle ABC$, there is a circle that contains all of the following nine points:

- (1) the midpoints K , L , and M of the sides of $\triangle ABC$
- (2) the points X , Y , and Z , where \overline{AX} , \overline{BY} , and \overline{CZ} are the altitudes of $\triangle ABC$
- (3) the points R , S , and T which are the midpoints of the segments \overline{AH} , \overline{BH} , and \overline{CH} that join the vertices of $\triangle ABC$ to the point H where the lines containing the altitudes intersect.



1. On a separate sheet of paper, draw an obtuse triangle ABC . Use your straightedge and compass to construct the circle passing through the midpoints of the sides. Be careful to make your construction as accurate as possible. Does your circle contain the other six points described above?

For constructions, see students' work; yes.

2. In the figure you constructed for Exercise 1, draw \overline{RK} , \overline{SL} , and \overline{TM} . What do you observe?

The segments intersect at the center of the nine-point circle.

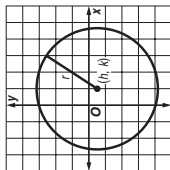
NAME _____ DATE _____ PERIOD _____

10-8 Study Guide and Intervention

Equations of Circles

Equation of a Circle A circle is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.

Standard Equation An equation for a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.



Example Write an equation for a circle with center $(-1, 3)$ and radius 6. Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 && \text{Equation of a circle} \\ (x - (-1))^2 + (y - 3)^2 &= 6^2 && \text{Substitution} \\ (x + 1)^2 + (y - 3)^2 &= 36 && \text{Simplify.} \end{aligned}$$

Exercises

Write an equation for each circle.

- center at $(0, 0)$, $r = 8$
 $x^2 + y^2 = 64$
- center at $(2, -4)$, $r = 1$
 $(x - 2)^2 + (y + 4)^2 = 1$
- center at $(-2, -6)$, diameter = 8
 $(x + 2)^2 + (y + 6)^2 = 16$
- center at $(-\frac{1}{2}, \frac{1}{4})$, $r = \sqrt{3}$
 $(x + \frac{1}{2})^2 + (y - \frac{1}{4})^2 = 3$
- center at $(1, -\frac{5}{8})$, $r = \sqrt{5}$
 $(x - 1)^2 + (y + \frac{5}{8})^2 = 5$

9. Find the center and radius of a circle with equation $x^2 + y^2 = 20$.

center $(0, 0)$; radius $2\sqrt{5}$

10. Find the center and radius of a circle with equation $(x + 4)^2 + (y + 3)^2 = 16$.

center $(-4, -3)$; radius 4

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Glencoe Geometry

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NAME _____ DATE _____ PERIOD _____

10-8 Study Guide and Intervention

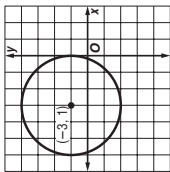
Equations of Circles

Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example Graph $(x + 3)^2 + (y - 1)^2 = 9$.

Use the parts of the equation to find (h, k) and r .

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 && (x + 3)^2 = (x - (-3))^2 && (y - 1)^2 = (y - 1)^2 && r^2 = 9 \\ x - h &= x + 3 && && && r &= 3 \\ -h &= 3 && && && & \\ h &= -3 && && && & \\ & && && && -k &= -1 \\ & && && && k &= 1 \end{aligned}$$



The center is at $(-3, 1)$ and the radius is 3. Graph the center. Use a compass set at a radius of 3 grid squares to draw the circle.

Exercises

Graph each equation.

- $x^2 + y^2 = 16$
- $(x - 2)^2 + (y - 1)^2 = 9$
- $(x + 2)^2 + y^2 = 16$
- $(x + 1)^2 + (y - 2)^2 = 6.25$
- $(x + \frac{1}{2})^2 + (y - \frac{1}{4})^2 = 4$
- $x^2 + (y - 1)^2 = 9$

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Glencoe Geometry

NAME _____ DATE _____ PERIOD _____

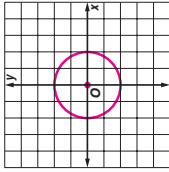
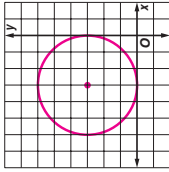
10-8 Practice (Average)

Equations of Circles

Write an equation for each circle.

- center at origin, $r = 7$
 $x^2 + y^2 = 49$
- center at $(0, 0)$, $d = 18$
 $x^2 + y^2 = 81$
- center at $(-7, 11)$, $r = 8$
 $(x + 7)^2 + (y - 11)^2 = 64$
- center at $(12, -9)$, $d = 22$
 $(x - 12)^2 + (y + 9)^2 = 121$
- center at $(-6, -4)$, $r = \sqrt{5}$
 $(x + 6)^2 + (y + 4)^2 = 5$
- center at $(3, 0)$, $d = 28$
 $(x - 3)^2 + y^2 = 196$
- a circle with center at $(-5, 3)$ and a radius with endpoint $(2, 3)$
 $(x + 5)^2 + (y - 3)^2 = 49$
- a circle whose diameter has endpoints $(4, 6)$ and $(-2, 6)$
 $(x - 1)^2 + (y - 6)^2 = 9$

Graph each equation.

- $x^2 + y^2 = 4$

- $(x + 3)^2 + (y - 3)^2 = 9$


Lesson 10-8

NAME _____ DATE _____ PERIOD _____

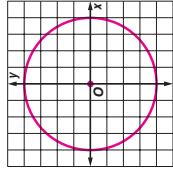
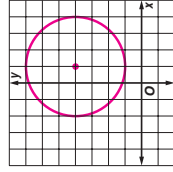
10-8 Skills Practice

Equations of Circles

Write an equation for each circle.

- center at origin, $r = 6$
 $x^2 + y^2 = 36$
- center at $(0, 0)$, $r = 2$
 $x^2 + y^2 = 4$
- center at $(4, 3)$, $r = 9$
 $(x - 4)^2 + (y - 3)^2 = 81$
- center at $(7, 1)$, $d = 24$
 $(x - 7)^2 + (y - 1)^2 = 144$
- center at $(-5, 2)$, $r = 4$
 $(x + 5)^2 + (y - 2)^2 = 16$
- center at $(6, -8)$, $d = 10$
 $(x - 6)^2 + (y + 8)^2 = 25$
- a circle with center at $(8, 4)$ and a radius with endpoint $(0, 4)$
 $(x - 8)^2 + (y - 4)^2 = 64$
- a circle with center at $(-2, -7)$ and a radius with endpoint $(0, 7)$
 $(x + 2)^2 + (y + 7)^2 = 200$
- a circle with center at $(-3, 9)$ and a radius with endpoint $(1, 9)$
 $(x + 3)^2 + (y - 9)^2 = 16$
- a circle whose diameter has endpoints $(-3, 0)$ and $(3, 0)$
 $x^2 + y^2 = 9$

Graph each equation.

- $x^2 + y^2 = 16$

- $(x - 1)^2 + (y - 4)^2 = 9$


Lesson 10-8

NAME _____ DATE _____ PERIOD _____

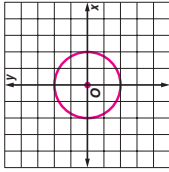
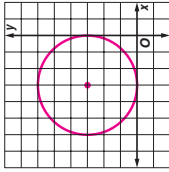
10-8 Practice (Average)

Equations of Circles

Write an equation for each circle.

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 $x^2 + y^2 = 49$
- center at $(0, 0)$, $d = 18$
 $x^2 + y^2 = 81$
- center at $(-7, 11)$, $r = 8$
 $(x + 7)^2 + (y - 11)^2 = 64$
- center at $(12, -9)$, $d = 22$
 $(x - 12)^2 + (y + 9)^2 = 121$
- center at $(-6, -4)$, $r = \sqrt{5}$
 $(x + 6)^2 + (y + 4)^2 = 5$
- center at $(3, 0)$, $d = 28$
 $(x - 3)^2 + y^2 = 196$
- a circle with center at $(-5, 3)$ and a radius with endpoint $(2, 3)$
 $(x + 5)^2 + (y - 3)^2 = 49$
- a circle whose diameter has endpoints $(4, 6)$ and $(-2, 6)$
 $(x - 1)^2 + (y - 6)^2 = 9$

Graph each equation.

- $x^2 + y^2 = 4$

- $(x + 3)^2 + (y - 3)^2 = 9$


Lesson 10-8

NAME _____ DATE _____ PERIOD _____

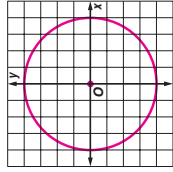
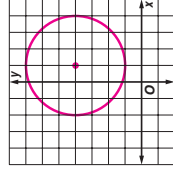
10-8 Skills Practice

Equations of Circles

Write an equation for each circle.

- center at origin, $r = 6$
 $x^2 + y^2 = 36$
- center at $(0, 0)$, $r = 2$
 $x^2 + y^2 = 4$
- center at $(4, 3)$, $r = 9$
 $(x - 4)^2 + (y - 3)^2 = 81$
- center at $(7, 1)$, $d = 24$
 $(x - 7)^2 + (y - 1)^2 = 144$
- center at $(-5, 2)$, $r = 4$
 $(x + 5)^2 + (y - 2)^2 = 16$
- center at $(6, -8)$, $d = 10$
 $(x - 6)^2 + (y + 8)^2 = 25$
- a circle with center at $(8, 4)$ and a radius with endpoint $(0, 4)$
 $(x - 8)^2 + (y - 4)^2 = 64$
- a circle with center at $(-2, -7)$ and a radius with endpoint $(0, 7)$
 $(x + 2)^2 + (y + 7)^2 = 200$
- a circle with center at $(-3, 9)$ and a radius with endpoint $(1, 9)$
 $(x + 3)^2 + (y - 9)^2 = 16$
- a circle whose diameter has endpoints $(-3, 0)$ and $(3, 0)$
 $x^2 + y^2 = 9$

Graph each equation.

- $x^2 + y^2 = 16$

- $(x - 1)^2 + (y - 4)^2 = 9$


Lesson 10-8

11. EARTHQUAKES When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake. $x^2 + y^2 = 2500$

10-8 Reading to Learn Mathematics
Equations of Circles

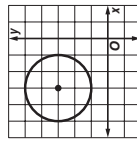
Pre-Activity What kind of equations describe the ripples of a splash?

Read the introduction to Lesson 10-8 at the top of page 575 in your textbook. In a series of concentric circles, what is the same about all the circles, and what is different? **Sample answer: They all have the same center, but different radii.**

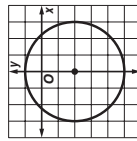
Reading the Lesson

- Identify the center and radius of each circle.
 - $(x - 2)^2 + (y - 3)^2 = 16$ **(2, 3); 4**
 - $x^2 + y^2 = 49$ **(0, 0); 7**
 - $x^2 + (y - 10)^2 = 144$ **(0, 10); 12**
- Write an equation for each circle.
 - center at origin, $r = 8$ **$x^2 + y^2 = 64$**
 - center at (3, 9), $r = 1$ **$(x - 3)^2 + (y - 9)^2 = 1$**
 - center at (-5, -6), $r = 10$ **$(x + 5)^2 + (y + 6)^2 = 100$**
 - center at (0, -7), $r = 7$ **$x^2 + (y + 7)^2 = 49$**
 - center at (12, 0), $d = 12$ **$(x - 12)^2 + y^2 = 36$**
 - center at (-4, 8), $d = 22$ **$(x + 4)^2 + (y - 8)^2 = 121$**
 - center at (4.5, -3.5), $r = 1.5$ **$(x - 4.5)^2 + (y + 3.5)^2 = 2.25$**
 - center at (0, 0), $r = \sqrt{13}$ **$x^2 + y^2 = 13$**

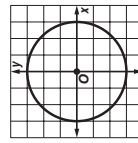
3. Write an equation for each circle.



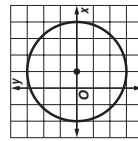
$(x + 3)^2 + (y - 3)^2 = 4$



$x^2 + (y + 2)^2 = 9$



$x^2 + y^2 = 9$

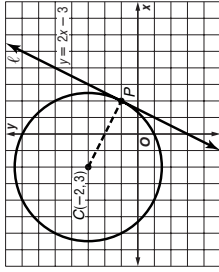


$(x - 1)^2 + y^2 = 9$

10-8 Enrichment

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.

- State the slope of the line ℓ that has equation $y = 2x - 3$.
2
- Suppose $\odot C$ with center $C(-2, 3)$ is tangent to line ℓ at point P . What is the slope of radius \overline{CP} ?
 $-\frac{1}{2}$
- Find an equation for the line that contains \overline{CP} .
 $y = -\frac{1}{2}x + 2$
- Use your equation from Exercise 3 and the equation $y = 2x - 3$. At what point do the lines for these equations intersect? What are its coordinates?
 $P(-2, 1)$
- Find the measure of radius \overline{CP} .
 $\sqrt{20}$
- Use the coordinate pair $C(-2, 3)$ and your answer for Exercise 5 to write an equation for $\odot C$.
 $(x - (-2))^2 + (y - 3)^2 = 20$ or $(x + 2)^2 + (y - 3)^2 = 20$

Lesson 10-8

10-8 Reading to Learn Mathematics
Equations of Circles

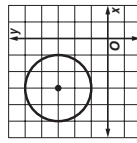
Pre-Activity What kind of equations describe the ripples of a splash?

Read the introduction to Lesson 10-8 at the top of page 575 in your textbook. In a series of concentric circles, what is the same about all the circles, and what is different? **Sample answer: They all have the same center, but different radii.**

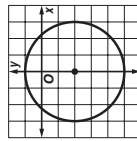
Reading the Lesson

- Identify the center and radius of each circle.
 - $(x - 2)^2 + (y - 3)^2 = 16$ **(2, 3); 4**
 - $x^2 + y^2 = 49$ **(0, 0); 7**
 - $x^2 + (y - 10)^2 = 144$ **(0, 10); 12**
- Write an equation for each circle.
 - center at origin, $r = 8$ **$x^2 + y^2 = 64$**
 - center at (3, 9), $r = 1$ **$(x - 3)^2 + (y - 9)^2 = 1$**
 - center at (-5, -6), $r = 10$ **$(x + 5)^2 + (y + 6)^2 = 100$**
 - center at (0, -7), $r = 7$ **$x^2 + (y + 7)^2 = 49$**
 - center at (12, 0), $d = 12$ **$(x - 12)^2 + y^2 = 36$**
 - center at (-4, 8), $d = 22$ **$(x + 4)^2 + (y - 8)^2 = 121$**
 - center at (4.5, -3.5), $r = 1.5$ **$(x - 4.5)^2 + (y + 3.5)^2 = 2.25$**
 - center at (0, 0), $r = \sqrt{13}$ **$x^2 + y^2 = 13$**

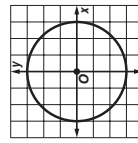
3. Write an equation for each circle.



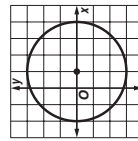
$(x + 3)^2 + (y - 3)^2 = 4$



$x^2 + (y + 2)^2 = 9$



$x^2 + y^2 = 9$



$(x - 1)^2 + y^2 = 9$

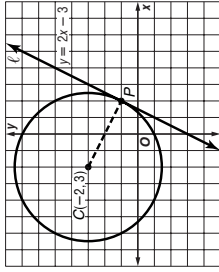
Helping You Remember

4. A good way to remember a new mathematical formula or equation is to relate it to one you already know. How can you use the Distance Formula to help you remember the standard equation of a circle? **Sample answer: Use the Distance Formula to find the distance between the center (h, k) and a general point (x, y) on the circle. Square each side to obtain the standard equation of a circle.**

10-8 Enrichment

Equations of Circles and Tangents

Recall that the circle whose radius is r and whose center has coordinates (h, k) is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.



Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.

- State the slope of the line ℓ that has equation $y = 2x - 3$.
2
- Suppose $\odot C$ with center $C(-2, 3)$ is tangent to line ℓ at point P . What is the slope of radius \overline{CP} ?
 $-\frac{1}{2}$
- Find an equation for the line that contains \overline{CP} .
 $y = -\frac{1}{2}x + 2$
- Use your equation from Exercise 3 and the equation $y = 2x - 3$. At what point do the lines for these equations intersect? What are its coordinates?
 $P(-2, 1)$
- Find the measure of radius \overline{CP} .
 $\sqrt{20}$
- Use the coordinate pair $C(-2, 3)$ and your answer for Exercise 5 to write an equation for $\odot C$.
 $(x - (-2))^2 + (y - 3)^2 = 20$ or $(x + 2)^2 + (y - 3)^2 = 20$

Chapter 10 Assessment Answer Key

Form 1
Page 589

1. A

2. C

3. D

4. C

5. B

6. C

7. A

8. C

9. B

10. A

11. D

Page 590

12. B

13. C

14. B

15. D

16. D

17. B

18. B

19. B

20. D

B: 7

Form 2A
Page 591

1. A

2. B

3. C

4. B

5. C

6. B

7. C

8. D

9. D

10. A

11. D

(continued on the next page)

Chapter 10 Assessment Answer Key

Form 2A (continued)

Page 592

12. C

13. C

14. A

15. B

16. D

17. B

18. A

19. D

20. A

B: 10

Form 2B

Page 593

1. B

2. C

3. C

4. A

5. C

6. A

7. B

8. B

9. C

10. A

11. B

Page 594

12. C

13. A

14. C

15. A

16. D

17. C

18. D

19. D

20. B

B: outside

Chapter 10 Assessment Answer Key

Form 2C

Page 595

Page 596 $\frac{7}{3}$

1. 2 in.

12. _____

2. radius: 30 m,
diameter: 60 m

3. 80

13. 31

4. 15.71 units

14. 41

5. 7

15. 70

16. 100

6. 12 m

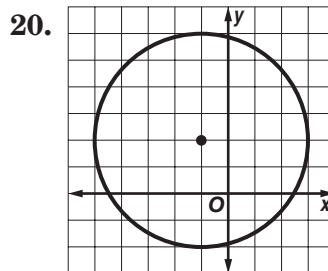
17. $2\sqrt{2}$

18.
$$\frac{(x - 3)^2 + (y - 5)^2}{26} = 26$$

7. 52°

19.
$$\frac{(x + 4)^2 + (y + 9)^2}{100} = 100$$

8. 36



9. $\frac{4}{5}$

10. 7 units

11. 11

B: $y = -\frac{4}{3}x + \frac{23}{3}$

Chapter 10 Assessment Answer Key

Form 2D

Page 597

1. 4

diameter: 22 in.,
circumference:

2. 69.12 in.

3. 29

4. 75.40 units

5. 90

6. $\frac{29}{4}$

7. 96°

8. 80

9. $-\frac{3}{5}$

10. 9 units

11. 11

Page 598

12. 7

13. 60

14. 70

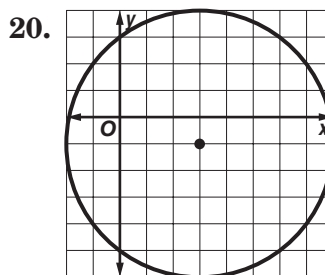
15. 50

16. 110

17. $\frac{(x + 7)^2 + (y - 8)^2}{(y - 8)^2} = 81$

18. $\frac{(x - 4)^2 + (y + 9)^2}{(y + 9)^2} = 116$

19. $3\sqrt{5}$



B: $(-1, 2)$ $(-1, -2)$

Chapter 10 Assessment Answer Key

Form 3

Page 599

1. $3\sqrt{2}$ ft

2. 26.66 in.

3. 149

4. 27 in.

5. $4\sqrt{6}$

6. 17 cm

7. 47

8. $4\sqrt{2}$ cm

9. $\frac{10\sqrt{3}}{3}$

10. 4

11. 58

Page 600

12. 25

13. 62.5

14. 52.5

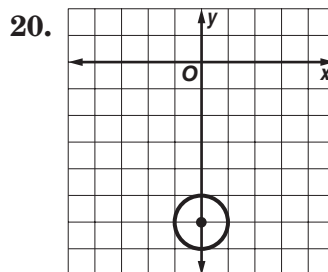
15. 12

16. 2

17. $(0, 5), \left(\frac{300}{61}, \frac{55}{61}\right)$

18. $(x + 3)^2 + (y + 2)^2 = 9$

19. center: $(6, -7)$,
radius: 9



B: $(5, 5)$

Chapter 10 Assessment Answer Key

Page 601, Open-Ended Assessment Scoring Rubric

Score	General Description	Specific Criteria
4	Superior A correct solution that is supported by well-developed, accurate explanations	<ul style="list-style-type: none"> Shows thorough understanding of the concepts of <i>circles, arcs, chords, tangents, secants, inscribed and circumscribed polygons, and equations of circles.</i> Uses appropriate strategies to solve problems. Computations are correct. Written explanations are exemplary. Figures and graphs are accurate and appropriate. Goes beyond requirements of some or all problems.
3	Satisfactory A generally correct solution, but may contain minor flaws in reasoning or computation	<ul style="list-style-type: none"> Shows an understanding of the concepts of <i>circles, arcs, chords, tangents, secants, inscribed and circumscribed polygons, and equations of circles.</i> Uses appropriate strategies to solve problems. Computations are mostly correct. Written explanations are effective. Figures and graphs are mostly accurate and appropriate. Satisfies all requirements of problems.
2	Nearly Satisfactory A partially correct interpretation and/or solution to the problem	<ul style="list-style-type: none"> Shows an understanding of most of the concepts of <i>circles, arcs, chords, tangents, secants, inscribed and circumscribed polygons, and equations of circles.</i> May not use appropriate strategies to solve problems. Computations are mostly correct. Written explanations are satisfactory. Figures and graphs are mostly accurate. Satisfies the requirements of most of the problems.
1	Nearly Unsatisfactory A correct solution with no supporting evidence or explanation	<ul style="list-style-type: none"> Final computation is correct. No written explanations or work shown to substantiate the final computation. Figures and graphs may be accurate but lack detail or explanation. Satisfies minimal requirements of some of the problems.
0	Unsatisfactory An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	<ul style="list-style-type: none"> Shows little or no understanding of most of the concepts of <i>circles, arcs, chords, tangents, secants, inscribed and circumscribed polygons, and equations of circles.</i> Does not use appropriate strategies to solve problems. Computations are incorrect. Written explanations are unsatisfactory. Figures and graphs are inaccurate or inappropriate. Does not satisfy requirements of problems. No answer given.

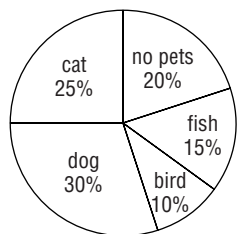
Chapter 10 Assessment Answer Key

Page 601, Open-Ended Assessment Sample Answers

In addition to the scoring rubric found on page A31, the following sample answers may be used as guidance in evaluating open-ended assessment items.

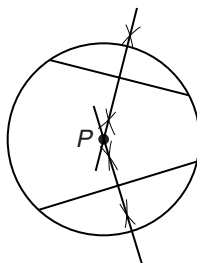
1. 100 families were surveyed about the type of pet they own. The results are:

no pets	20	$\frac{20}{100} = \frac{x}{360}$	72°
dogs	30	$\frac{30}{100} = \frac{x}{360}$	108°
cats	25	$\frac{25}{100} = \frac{x}{360}$	90°
fish	15	$\frac{15}{100} = \frac{x}{360}$	54°
birds	10	$\frac{10}{100} = \frac{x}{360}$	36°



2. a. Arc length is the measure of the distance around part of a circle. It is a fraction of the circumference of the circle. Arc length is measured in centimeters or inches or feet, etc. Arc measure is the number of degrees in an arc. It is measured with a protractor.
- b. Yes. The arcs could have the same measure, for example 60, but could be arcs in circles with different radii. The arc in the circle with the greater radius would have a greater length.

3.



4. The measures decrease.

5. a. $(x - 2)^2 + (y + 3)^2 = 25$

b. $B(-1, 1)$

c. center: $(2, -3)$

The slope of the segment, having endpoints at B and the point of tangency to the center, is $-\frac{4}{3}$.

The slope of tangent line is $\frac{3}{4}$.

equation: $y - 1 = \frac{3}{4}(x + 1)$ or

$$y = \frac{3}{4}x + \frac{7}{4}$$

Chapter 10 Assessment Answer Key

Vocabulary Test/Review Page 602

1. false, inscribed
2. true
3. true
4. false, radius
5. false, minor arc
6. true
7. false, secant
8. true
9. true
10. true
11. arcs in the same \odot or $\cong \odot$ s that have the same measure
12. A polygon is circumscribed about a \odot if all of its sides are tangent to the \odot .
13. A polygon is inscribed in a \odot if all of its vertices lie on the \odot .

Quiz 1 Page 603

1. 8
2. 40.84 in.
3. 73
4. 6.28 in.
5. D

Quiz 2 Page 603

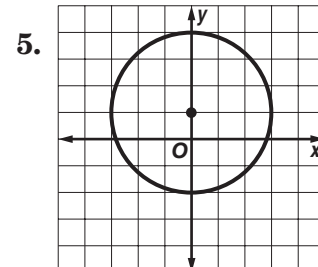
1. 70
2. 15 in.
3. 22
4. 12 cm
5. 21.77 m

Quiz 3 Page 604

1. $12\sqrt{3}$ ft
2. $\frac{8\sqrt{3}}{3}$ m
3. 65
4. 77.5
5. 112.5

Quiz 4 Page 604

1. 4
2. $x = \sqrt{21}, y = \frac{17}{2}$
3. (-11, 13)
4. 15



Chapter 10 Assessment Answer Key

Mid-Chapter Test

Page 605

Part I

1. A

2. D

3. B

4. C

5. B

Part II

6. 44

7. 120°

8. 5 cm

9. 8.5 m

10. 87

Cumulative Review

Page 606

1. \overrightarrow{EA} and \overrightarrow{ED}

2. true

3. $y = -560x + 8500$

4. $m\angle 1 = 79,$
 $m\angle 2 = 50.5,$
 $m\angle 3 = 129.5$

5. $a \geq 5$

6. polygon $FHJBD \sim$
polygon $QRJHP$

7. 0.6, 0.8, 0.75

8. $a = 2; b = 20$

9. $A'(2, 4), B'(4, 2)$

10. $(x - 4)^2 +$
 $(y + 1)^2 = 144$

Chapter 10 Assessment Answer Key

Standardized Test Practice

Page 607

Page 608

1. A B C D

2. E F G H

3. A B C D

4. E F G H

5. A B C D

6. E F G H

7. A B C D

8.

4	7		
.	/	/	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9.

7	.	5	
.	/	/	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

10.

7	4		
.	/	/	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11.

9	7		
.	/	/	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12. $m\angle 1 = 13$

13. $AB > BC$

14. yes

15. 20 cm

Chapter 10 Assessment Answer Key

Unit 3 Test/Review (Ch. 8–10)

Page 609

1. 9

2. $m\angle JHK = 52;$
 $m\angle HMK = 108,$
and $x = 8$

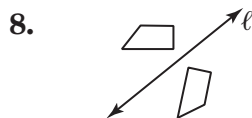
3. No; opp. sides
are not \parallel .

4. 5

5. 5

6. 11

7. Slopes of \overline{QR} and
 \overline{PS} are both 0, and
 $QR = PS = a$, so
 $PQRS$ is a \square .



9. $Q'(14, 4),$
 $T'(11, -4)$

10. order: 10;
magnitude: 36°

11. yes

Page 610
12. $r = \frac{5}{2};$
enlargement

13. $B'(-3, -3)$

14. $P'(8, -1),$
 $Q'(5, 5), R'(-6, 3)$

15. diameter: 94 cm;
circumference:
about 295.3 cm

16. $m\angle NJK = 72;$
length of \widehat{NK} is
about 17.6 cm.

17. $LK = 16,$
 $MK = 32,$ and
 $m\widehat{MNK} = 106.2$

18. 21

19. 12.8 cm

20. 196

21. 6

