

---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Sprint Round**  
**Problems 1 – 30**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books, or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

---

Total Correct	Scorer's Initials

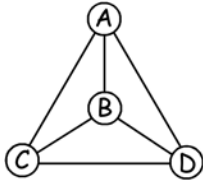
1. You have 40 minutes to complete all 30 questions on this Sprint round. What is the average number of seconds you can use to solve each question? 1. \_\_\_\_\_ seconds

2. How many dots are there on a standard pair of six-faced dice?



2. \_\_\_\_\_ dots

3. Beginning at point A, how many distinct paths can be traced along the lines which visit each of the other letters exactly once before returning to A?



3. \_\_\_\_\_ paths

4. What is the probability of flipping exactly one head on three flips of a fair coin? Express your answer as a common fraction.

4. \_\_\_\_\_

5. Which number could be removed from the following set to make the mean of the remaining set of integers equal to its median? {1, 5, 6, 9, 10, 11, 12, 13, 21, 23}

5. \_\_\_\_\_

6. What is the remainder when  $2^{10}$  is divided by  $10^2$ ?

6. \_\_\_\_\_

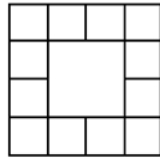
7. The lateral surface area of a cylinder is equal to the sum of the areas of its two bases. What is the ratio of the cylinder's height to its diameter? Express your answer as a common fraction.

7. \_\_\_\_\_

8. What is the base-3 sum of  $212_3 + 121_3$ ?

8. \_\_\_\_\_ base-3

9. How many distinct squares of any size can be traced along the lines of the grid below? 9. \_\_\_\_\_ squares



10. What is the 100<sup>th</sup> number in the sequence that begins: 9, -1, 4, -6, -1, -11, -6, -16, ... ? 10. \_\_\_\_\_

11. Parker is in pre-school learning to count by 5's and by 10's. He can count to 30 by 5's, but he sometimes skips ahead by 10 instead of 5. For example, he may count 10-15-25-30 or 5-10-15-25-30, or even just 10-20-30. How many different ways can Parker count from 0 to 30 if he always skips ahead either by 5 or by 10. 11. \_\_\_\_\_ ways



12. If garbage is collected at exactly 9am on the first Tuesday of every month, what is the greatest number of hours that could pass between consecutive garbage collections? 12. \_\_\_\_\_ hours



13. At a gas station, pump 1 can dispense 15 gallons in 4 minutes. Pump 2 can dispense 14 gallons in 5 minutes. Working simultaneously, pumps 1 and 2 dispense a whole number of gallons in a whole number of minutes. What is the least positive number of gallons for which this is possible? 13. \_\_\_\_\_ gallons

14. The height of a bamboo plant is a two-digit whole number of inches. After increasing in height by exactly 50% each week for four weeks, the height of the plant is still a two-digit whole number of inches. After four weeks how many inches tall is the plant? 14. \_\_\_\_\_ inches

15. For how many minutes between 9:00am and 3:00pm is the sum of the digits to the left of the colon equal to the sum of the digits to the right of the colon on a standard digital clock? 15. \_\_\_\_\_ minutes

16. An employer is putting together a task force to study the efficiency of the work environment. He is asking for volunteers from his 10 employees. If the group must contain at least 3 employees and at most 7 employees, how many different combinations of employees are possible? 16. \_\_\_\_\_ combinations

17. Pentagon ABCDE has integral side lengths such that  $AB=2BC=3CD=4DE=5EA$ . What is the least possible perimeter of pentagon ABCDE? 17. \_\_\_\_\_ units

18. What is the measure in degrees of the acute angle formed by the hour hand and minute hand when a standard clock reads 4:36? 18. \_\_\_\_\_ degrees

19. A box containing only chocolates and chocolate-covered caramels has an equal number of each, but they are indistinguishable on the outside. The probability of selecting a chocolate, eating it, and then selecting a caramel is  $\frac{3}{11}$ . How many candies are in the box initially? 19. \_\_\_\_\_ candies



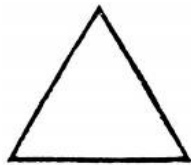
20. In a set of ten consecutive integers, the product of the first two is equal to the sum of the next eight integers. What is the least possible sum of all ten integers? 20. \_\_\_\_\_

21. In quadrilateral ABCD, the diagonals intersect at X. If  $AX:CX = 1:2$  and  $BX:DX = 4:5$ , what is the ratio of the area of triangle AXB to CXD? Express your answer as a common fraction in simplest form. 21. \_\_\_\_\_

22. A right rectangular prism has integer edge lengths and a space diagonal that is 7cm long. What is its volume? 22. \_\_\_\_\_ cm<sup>3</sup>

23. Martha's new "Light 'N Fit" cookies have 25% less sugar than her original cookies, but her "Decadent Ultra" cookies have 25% more sugar than her original recipe. Each cookie is the same size. Martha makes one batch of each type (original, light, and decadent) of cookie using exactly the same amount of sugar in each batch, and each batch contains a whole number of cookies. What is the fewest number of cookies she could have baked altogether? 23. \_\_\_\_\_ cookies

24. Angela draws an equilateral triangle and measures its area. 24. \_\_\_\_\_ sq. inches



Becky draws an equilateral triangle that is three times the area of Angela's. If the sides of Becky's triangle are 4 inches longer than the sides of Angela's triangle, what is the area of Angela's triangle? Express your answer in simplest radical form.

25. When Elizabeth runs to the park it takes her 16 minutes to get there. When she walks, it takes her 48 minutes. If Elizabeth runs and walks for equal amounts of time on her way to the park, how many minutes will it take for her to get there? 25. \_\_\_\_\_ minutes



26. There are 21 white blocks and  $n$  red blocks in a bag. The probability of randomly drawing a red block from the bag is  $(21 + n)\%$ . If there are more red blocks than white, how many blocks are in the bag altogether? 26. \_\_\_\_\_ blocks

27. In the two circles shown below the area of the non-shaded region is twice as large as the shaded region. Find the ratio of radius of the larger circle to the radius of the small circle. Express your answer as a fraction in simplest radical form.

27. \_\_\_\_\_

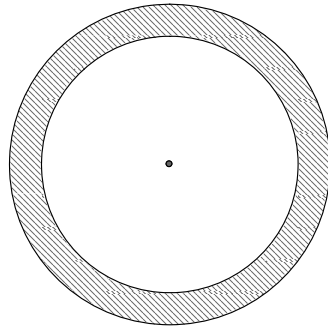


Figure 27

28. Perpendicular lines graphed on the coordinate plane have slopes  $a$  and  $b$ . If  $a + b = 4$ , what is  $|a - b|$ ? Express your answer in simplest radical form.

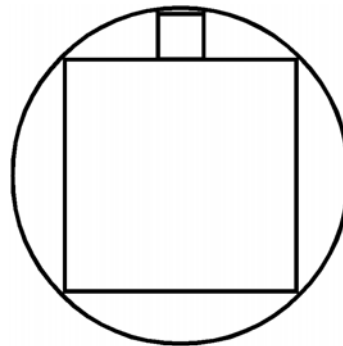
28. \_\_\_\_\_

29. What is the ratio of the least positive integer that has at least 10 odd factors to the least positive integer that has exactly 10 odd factors? Express your answer as a common fraction.

29. \_\_\_\_\_

30. A square is inscribed in a circle. A second square is drawn as shown, with two vertices on the circle and two vertices on one of the sides of the larger square. What is the ratio of the area of the smaller square to the area of the larger one? Express your answer as a common fraction

30. \_\_\_\_\_



---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Target Round**  
**Problems 1 and 2**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

This round of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record your final answers in the designated space on the problem sheet. All answers must be complete and legible. This round assumes the use of calculators, and calculations may be done on scratch paper, but no other aids are allowed.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

---

Total Correct	Scorer's Initials

1. A "word" is any sequence of letters. How many words with four letters which use only A, B and C can we form using at most 1 A, at most 2 B's, and at most 3 C's?

1. \_\_\_\_\_ words

2. A cube has each edge trisected. A triangular pyramid is cut away from each corner with vertices at the nearest trisected point. If the length of one of the sides of the cube is 18, what is volume of the figure remaining after the triangular pyramid's are removed?

2. \_\_\_\_\_ cu. units



---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Target Round**  
**Problems 3 and 4**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

---

Total Correct	Scorer's Initials

3. Including the endpoints, how many points  $(x, y)$ , where at least one of the coordinates is an integer, does the segment connecting  $(8, 12)$  and  $(56, 84)$  pass through?

3. \_\_\_\_\_

4. How many four digit numbers can be formed if the final three digits have a sum of 11?

4. \_\_\_\_\_ numbers

---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Target Round**  
**Problems 5 and 6**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

---

Total Correct	Scorer's Initials

5. Three tetrahedral dice have faces numbered 1 – 4. If the Dice are rolled and all visible numbers are added together, what is the probability that the sum is greater than 24? Express your answer as a common fraction.

5. \_\_\_\_\_

6. The three triangles pictured are equilateral and all share a common vertex at the center of the circle. What is the ratio of the combined area of the three triangles to the area of the circle? Express your answer as a decimal to the nearest hundredth.

6. \_\_\_\_\_

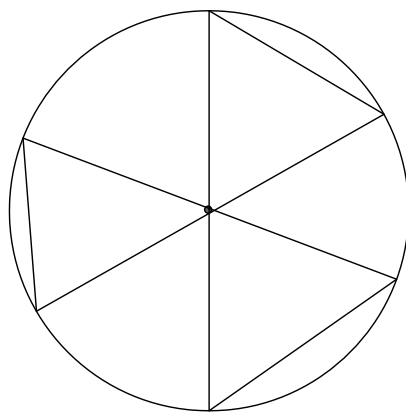


Figure 6

---

---

# MATHCOUNTS®

---

■ Batterson #1 ■  
Target Round  
Problems 7 and 8

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

---

Total Correct	Scorer's Initials

7. Using all of the letters of the alphabet all possible “words” are listed starting with all one letter words listed in alphabetical order, then all two letter words listed in alphabetical order, and so on. What is the 2010<sup>th</sup> word listed? 7. \_\_\_\_\_

The list begins A, B, C...Z, AA, AB, AC...ZZ, AAA, AAB...

8. The number 482 is called a “product number” because the product of two of the digits is equal to the other digit. What is the probability that a randomly selected three-digit product number will contain the digit 1? Express your answer as a common fraction. 8. \_\_\_\_\_

---

---

# MATHCOUNTS<sup>®</sup>

---

■ **Batterson #1** ■  
**Team Round**  
**Problems 1 – 10**

---

School \_\_\_\_\_

Chapter \_\_\_\_\_

Team Members \_\_\_\_\_, Captain

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

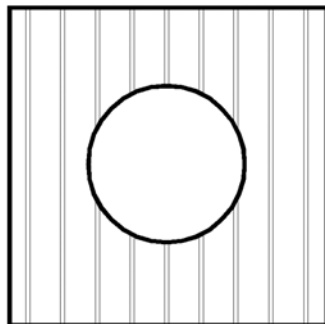
**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

---

Total Correct	Scorer's Initials

1. The point  $(10, 0)$  is rotated  $120^\circ$  counter clockwise around the origin to the point  $(x, y)$ . What is the product of  $x$  and  $y$ ? Express your answer in simplest radical form. 1. \_\_\_\_\_
  
2. If  $1 \leq a \leq 12$  and  $1 \leq b \leq 49$  for how many ordered pairs of integers,  $(a, b)$  is  $\sqrt{2a + \sqrt{b}}$  an integer? 2. \_\_\_\_\_ pairs
  
3. How many ordered pairs of integers  $(x, y)$  satisfy the equation  $y = \frac{14x-1}{x-1}$ ? 3. \_\_\_\_\_ pairs
  
4. A square of side length  $K$  is composed of  $K^2$  unit squares. exactly one-third of the unit squares are colored red and then one-fourth of the remaining squares are colored blue. What is the sum of the 3 smallest values of  $K$  for which this is possible? 4. \_\_\_\_\_
  
5. A circular swimming pool with a 6m radius is centered within a square deck whose sides are 24 meters long. Iggy the inchworm wants to walk from the center of one side of the deck to the center of the opposite side of the deck without getting wet. What is the length in meters of the shortest path he can take to get from one side of the deck to the other. Express your answer as a decimal rounded to the nearest hundredth. 5. \_\_\_\_\_ meters





6. How many whole numbers less than 1,000,000,000 use only the digits 0, 1, and 2 and have no two consecutive digits that are the same? 6. \_\_\_\_\_ numbers
7. The first term of a geometric sequence is 7 and the 5<sup>th</sup> term is 8. The seventh term can be expressed in the simplest radical form  $\frac{a\sqrt{b}}{c}$ . Find  $a + b + c$ . 7. \_\_\_\_\_
8. A large amount of planting is to be done on a farm. Al and Bob together can do the job in 10 hours. Al and Carl together can do the same job in 12 hours. Bob and Carl together can do the same job in 15 hours. Each worker has a constant rate of work that is unaffected by whoever else is working. How many hours would it take to do the same job if Al, Bob and Carl work together? 8. \_\_\_\_\_ hours
9. The ratio of the legs of a right triangle is 1:2. The perimeter of the triangle is 12 cm. What is the length of the hypotenuse of the right triangle? Express your answer in simplest radical form. 9. \_\_\_\_\_ cm
10. In a certain habitat, animals live in groups of two and groups of three. Each time a new animal arrives at the habitat, it randomly chooses a group. If the chosen group has two animals in it, the new animal joins the group forming a group of three animals. If the chosen group has three animals in it, the new animal takes one of the animals away from the group of three and forms a separate group of two animals. If initially there are five animals in the habitat, and new animals arrive one at a time, what is the probability that the fourth new animal joins a group of two animals? Express your answer as a common fraction. 10. \_\_\_\_\_

---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Sprint Round**  
**Solutions**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books, or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

---

Total Correct	Scorer's Initials

1. 1. 80
2. 2. 42
3. 3. 6
4. 4.  $\frac{3}{8}$
5. 5. 21
6. 6. 24
7. 7.  $\frac{1}{2}$
8. 8. 1110
9. 9. 18
10. 10. -246
11. 11. 13
12. 12. 840
13. 13. 131
14. 14. 81
15. 15. 20
16. 16. 912
17. 17. 137
18. 18. 78
19. 19. 12
20. 20. 5
21. 21.  $\frac{2}{5}$
22. 22. 36

23.

23. 47

24.

24.  $6 + 4\sqrt{3}$

25.

25. 24

26.

26. 70

27.

27.  $\frac{\sqrt{6}}{2}$

28.

28.  $2\sqrt{5}$

29.

29.  $\frac{7}{9}$

30.

30.  $\frac{1}{25}$

---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Target Round**  
**Solutions**

---

Name \_\_\_\_\_

Grade : \_\_\_\_\_ Teacher \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

This round of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record your final answers in the designated space on the problem sheet. All answers must be complete and legible. This round assumes the use of calculators, and calculations may be done on scratch paper, but no other aids are allowed.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

---

Total Correct	Scorer's Initials

1.

1. 38

2.

2. 5544

3.

3. 97

4.

4. 621

5.

5.  $\frac{5}{32}$

6.

6. 0.41

7.

7. BYH

8.

8.  $\frac{1}{2}$

---

---

# MATHCOUNTS®

---

■ **Batterson #1** ■  
**Team Round**  
**Solutions**

---

School \_\_\_\_\_

Chapter \_\_\_\_\_

Team Members \_\_\_\_\_, Captain

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk during this section of the competition. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. All answers must be complete, legible, and simplified to lowest terms. The team captain must record the team's official answers on his/her own problem sheet, which is the only sheet that will be scored.

---

Total Correct	Scorer's Initials

1.

$$1. -25\sqrt{3}$$

2.

$$2. 12$$

3.

$$3. 4$$

4.

$$4. 36$$

5.

$$5. 27.07$$

6.

$$6. 1023$$

7.

$$7. 37$$

8.

$$8. 8$$

9.

$$9. 9\sqrt{5} - 15$$

10.

$$10. \frac{5}{9}$$