Calculus Lecture Notes

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Unit: Applications of Integration **Module:** The Average Value of a Function

Finding the Average Value of a Function

key concepts:

- The average value of a function on an interval is the area under the curve divided by the length of the interval.
- Average value $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

consider: $f(x)=x^2$

Take all the numbers $f(x)$ as x ranges from 0 to 3.

Q: What is the average value?

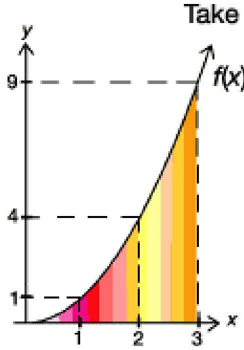
Normally to find an average, you would just add up all the values and divide by the number of values.

Since our universe is the set of real numbers, there are infinitely many values of x between 0 and 3. We could never write them all down to find an average.

Finding the average of infinitely many numbers

Q: What is the average value?

Take $f(x) = x^2$ as x ranges from 0 to 3.



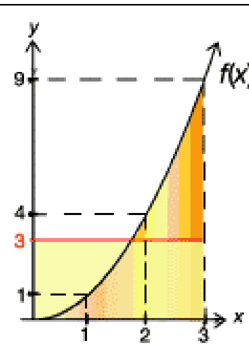
$$\frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{3-0}\int_0^3 x^2\,dx$$
$$= \frac{1}{3}\int_0^3 x^2\,dx$$
$$= \frac{1}{3}\cdot\frac{1}{3}x^3\Big|_0^3$$
$$= \frac{1}{9}x^3\Big|_0^3$$
$$= 3 - 0$$
$$= 3$$

A: The average value of f between 0 and 3 is 3.

It turns out the average value of a continuous function can be computed as an integral. To find the average value of a function, simply take the definite integral of the function across the desired interval and divide that result by the length of the interval.

Here you are finding the average on the interval [0,3]. The length of this interval is three. So to find the average value of the function on this interval, divide the definite integral evaluated from zero to three by the length of the interval, which is three.

So the average value of this function on the interval [0,3] is three.



The average value of a function has a graphical representation too.

If you draw a line at y equal to the average value, then the total area underneath the curve will fit underneath the line. In effect, the upper pieces will fill up the lower section exactly!

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