Unit: Parametric Equations and Polar Coordinates **Module:** Polar Functions and Slope

Calculus and the Rose Curve

key concepts:

- To find the **critical points** of a polar curve, take the derivative of *r* with respect to θ and set the derivative equal to zero.

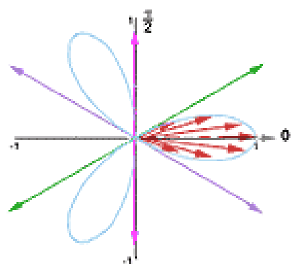
Finding maxima of the rose curve

Example! $r = \cos 3\theta$

The ends of the petal occur where *r* is maximum.

Take the derivative with respect to θ and solve for the locations where it is 0.

$$\frac{dr}{d\theta} = \frac{d\cos 3\theta}{d\theta} = -3\sin 3\theta$$



Set $\frac{dr}{d\theta} = 0$

$-3\sin 3\theta = 0$

$\sin 3\theta = 0$

$\frac{dr}{d\theta} = 0$ when $\theta = 0, \frac{\pi}{3}$ or $\frac{2\pi}{3}$

SUMMARY

$r = 0$ when

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

You can use calculus on polar equations a lot like you can use it on rectangular equations.

Suppose you wanted to find the maximum points on this rose curve. In rectangular coordinates, you would take the first derivative of *y* (the height) with respect to *x* and set that equation equal to zero. In polar coordinates you take the first derivative of *r* (the distance) with respect to θ and set that equation equal to zero.

Doing so will give you the angles where the curve reaches **critical points**.

Keep in mind that when dealing with polar coordinates the distance can be positive or negative. Polar coordinates are not unique, so be careful!

Example! $r = \cos 3\theta$

steps to sketching the the curve $r = \cos 3\theta$

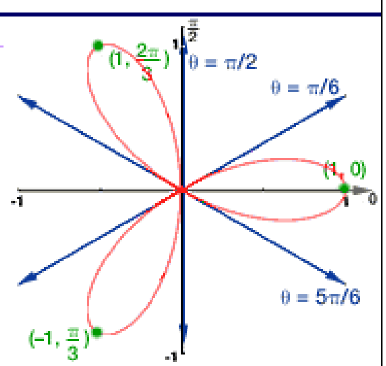
1. Draw the rays where $r = 0$.
2. Plot the points where $\frac{dr}{d\theta} = 0$.

when $\theta = 0$, $r = \cos 3\theta = \cos 3(0) = \cos 0 = 1$

when $\theta = \frac{\pi}{3}$, $r = \cos 3\theta = \cos 3(\frac{\pi}{3}) = \cos \pi = -1$

when $\theta = \frac{2\pi}{3}$, $r = \cos 3\theta = \cos 3(\frac{2\pi}{3}) = \cos 2\pi = 1$

3. Now fill in the curve.



SUMMARY

$r = 0$ when

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

$\frac{dr}{d\theta} = 0$ when

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$

Curve sketching with calculus can be very fun in polar coordinates.

Start by finding the rays (angles) where $r = 0$. These rays indicate the path the polar curve has to follow as it approaches the origin.

Notice that the polar graph is always tangent to the ray where $r = 0$ as well.

Next find the critical points by taking the derivative and setting it equal to zero.

Plot the resulting values. Then you can just fill in the curve.

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Unit: Parametric Equations and Polar Coordinates **Module:** Polar Functions and Slope

Finding the Slopes of Tangent Lines in Polar Form

key concepts:

- To convert from polar to Cartesian coordinates, use the formulas:
  $x = r\cos\theta,\quad y = r\sin\theta.$
- To find the slope of tangents to polar curves, use the formula $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$.

Finding the slope of tangent lines to the rose curve

Example! $r = f(\theta)$ **MISSION:** Find $\frac{dy}{dx}$.

$y = f(\theta)\sin\theta \quad x = f(\theta)\cos\theta$

fantasy math $\dfrac{dy}{d\theta} = \dfrac{dy}{dx}\cdot\dfrac{dx}{d\theta}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{d\theta} = \frac{d\,f(\theta)\sin\theta}{d\theta} = f(\theta)\cos\theta + f'(\theta)\sin\theta$$ Use the product rule.

$$\frac{dx}{d\theta} = \frac{d\,f(\theta)\cos\theta}{d\theta} = -f(\theta)\sin\theta + f'(\theta)\cos\theta$$ Use the product rule.

$$\boxed{\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}}$$

To find the slope of a tangent line on a polar curve, you need to find the rate of change of *y* with respect to *x*. Although it seems like you might have to change everything back to rectangular form, you can actually find the slope while in polar form by using the chain rule.

Start by finding the equations for *x* and *y* in terms of θ.

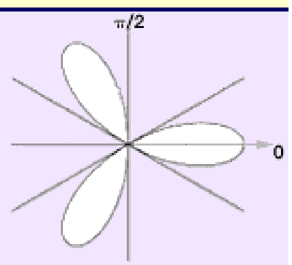
Then take the derivative of both equations with respect to θ.

Divide the derivative of *y* by the derivative of *x* and you get an equation for the slope by the chain rule.

Verifying the equation for dy/dx

Remember!

Example! The rose curve $r = \cos 3\theta$



Plug $\theta = \pi/6$ into dy/dx.

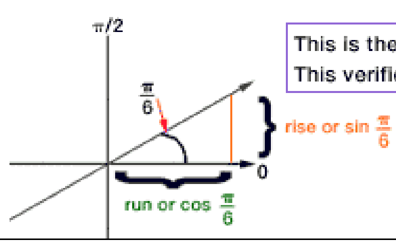
$$\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$= \frac{\cos 3\theta\cos\theta - 3\sin 3\theta\sin\theta}{-\cos 3\theta\sin\theta - 3\sin 3\theta\cos\theta}$$

$$= \frac{\cos[3(\pi/6)]\cos(\pi/6) - 3\sin[3(\pi/6)]\sin(\pi/6)}{-\cos[3(\pi/6)]\sin(\pi/6) - 3\sin[3(\pi/6)]\cos(\pi/6)}$$

$$= \frac{\cos(\pi/2)\cos(\pi/6) - 3\sin(\pi/2)\sin(\pi/6)}{-\cos(\pi/2)\sin(\pi/6) - 3\sin(\pi/2)\cos(\pi/6)}$$

$$= \frac{-3(1)\sin(\pi/6)}{-3(1)\cos(\pi/6)} = \tan(\pi/6) = \boxed{\frac{1}{\sqrt{3}}}$$



This is the same as $\frac{\text{rise}}{\text{run}}$ of the ray $\theta = \frac{\pi}{6}$. This verifies that the equation works.

$$\frac{\text{rise}}{\text{run}} = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \boxed{\tan(\pi/6)}$$

Consider the line tangent to the rose curve at π/6.

Start with the formula for *dy*/*dx*

Notice that $r = f(\theta)$. So plug the equation for *r* into the formula.

This value is the slope according to the formula.

You can double-check that slope by looking at the ray. Since you know the angle, you can find the rise and the run.

So the formula worked!

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