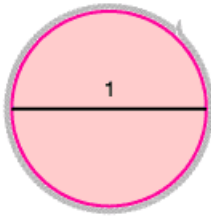


# The Number Pi


- Pi ( $\pi$ ) can be defined as the circumference of a circle whose diameter is 1.  $\pi$  is an irrational number, so its decimal expression never terminates or repeats.
- The area of a circle can be approximated by a rectangle.
- Trigonometric functions often involve the number pi. Differentiating such functions requires the use of the Chain Rule.


## The mystery of $\pi$

Consider a circle whose diameter is 1 unit.



$$\pi = 3.141592653\dots$$

 goes on forever

 has no repeating pattern

If the diameter of the circle is 1, then the circumference is  $\pi$ .

$$\text{In general: } C = \pi d$$

$$C = 2\pi r \quad d = 2r$$

3.1415926535897932384626433832795028841971693993751...

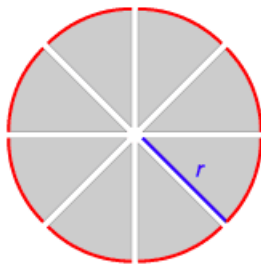
Consider a circle with a diameter of length 1. If you measure the circumference of the circle, you will find that it is approximately 3.14 times as long as the diameter. This ratio holds for any circle you draw. Mathematicians have a special name for this constant number. They call it pi ( $\pi$ ).

$\pi$  is an **irrational number**. An irrational number is one that cannot be expressed as a fraction. As a result, the decimal expression of  $\pi$  does not terminate or repeat. The numbers go on forever without pattern.

## Using $\pi$ to find the area of a circle

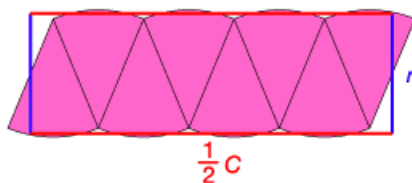
**Tip**

If you have a hard problem in math, do not do it. Break it up into easier problems instead.



$$A = bh$$

$$A = r\pi r \\ = \pi r^2$$



You can derive the area of a circle by cutting it up and rearranging the pieces. Half of the circumference makes up the width and the radius makes up the height. The more partitions you break the circle into the more the circle resembles a rectangle. If you cut the circle into an infinite number of slices, the circle would fit inside the rectangle exactly.

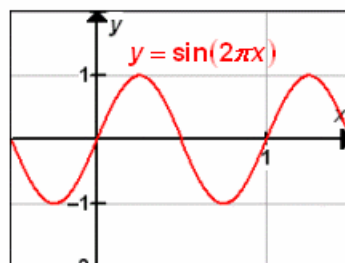
## Derivatives of Trigonometric Functions Involving $\pi$

Evaluate  $f'(x)$ , where  $f(x) = \sin(2\pi x)$ .

$$f(x) = \sin(2\pi x) \quad \text{Write the function.}$$

$$f'(x) = \cos(2\pi x) \cdot 2\pi \quad \text{Apply the Chain Rule.}$$

$$f'(x) = 2\pi \cos(2\pi x) \quad \text{Write in standard form.}$$



The number pi arises in trigonometry because there are  $2\pi$  radians in a circle. An example of a common trigonometric function is the function  $f(x)$  shown here.

To find the derivative of this function, apply the Chain Rule. The derivative of  $\sin(\text{blop})$  is  $\cos(\text{blop})$ , so the derivative of  $f(x)$  is  $\cos(\text{blop})$  times the derivative of the  $\text{blop}$ .

Remember, pi is just a number, so it should be treated like any other number. In this case, it is part of the coefficient that is multiplied by  $x$ .