

Unit: Limits

Module: Evaluating Limits

## An Overview of Limits

### key concepts:

- The **limit** is the range value that a function approaches as you get closer to a particular domain value.
- An **indeterminate form** is a mathematically meaningless expression.

### Limits that stay crunchy in milk

**example**  
Evaluate  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$ .


$\lim_{x \rightarrow 2} \frac{0}{0}$  **indeterminate form**

Direct substitution would result in  $\frac{0}{0}$ .

✓ Look for a way to cancel terms.

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2}$$

$$= \lim_{x \rightarrow 2} (x + 2) \quad \text{Cancel.}$$

$$= 4$$


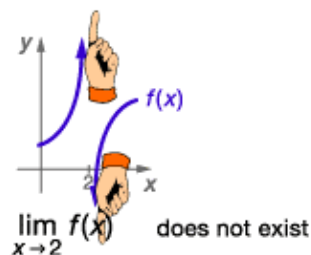
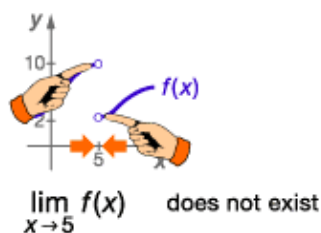
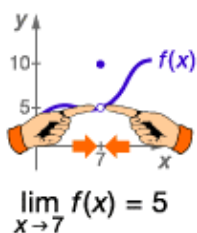
This **limit** involves an unusual variable.

Remember to use direct substitution as a first step in evaluating limits. In this case, direct substitution produces the familiar **indeterminate form** of  $0/0$ .

Proceed by factoring the numerator, which is a difference of two squares.

Use cancellation to simplify the limit expression and then apply direct substitution to arrive at the result.

### Existence of the limit



The existence of limits can be demonstrated graphically. On the far left, the graph shows that near  $x = 7$  the function is approaching the same value from both the left and the right. The limit exists and equals that value, even though the function takes on a different value at  $x = 7$ .

On the near left, the graph approaches different values on either side of  $x = 5$ . Since the two one-sided limits have different values, the limit of the function does not exist.

Here is an example of a function that is approaching very large values from the one side and very small values from the other. Therefore the limit does not exist.