MATHCOUNTS®

2008
Aops Competition
Sprint Round
Problems 1-30

Name ______________________________
School _____________________________
State _______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Extra note: Answers will be posted later in the week. After finishing the test, please PM me your answers. A solutions thread will also be created sometime later.

1. Alice walks into a store where all items have a 20% discount. She has a coupon that lets her take an additional 30% off the discounted price. What is the total discount she receives? Give your answer as a percent.

1.______________%

2. Bob has six red socks, eight green socks, and ten blue socks. If Bob is in a dark room with no light and cannot distinguish between socks of different colors, what is the minimum number of socks Bob needs to take so that he is guaranteed at least three socks of one color?

2.___________ socks

3. Equilateral triangle ABC has side length of 4 cm. Find the length of the radius of its circumscribed circle in centimeters. Give your answer as a common fraction in simplest radical form.

3.______________cm

4. If $25^{x+2}=3125$, find 16^{x}.

4.________________

5. Two standard six-faced dice are rolled. Christine wins if the product of the two numbers rolled is odd or a multiple of three, otherwise Dean wins. What is the probability that Dean wins? Express your answer as a common fraction.

5.________________

6. Given that the greatest common factor of two positive integers is 10 and that their product is 600, find the least common multiple of the two integers.

6.________________

7. Find the smallest positive integer n such that n+125 and n+201 are both perfect squares.

7.________________

8. You are standing at point A, which is 1000 feet from point B. You choose a random direction and walk 1000 feet in that direction. What is the probability that you end up within 1000 feet of B? Express your answer as a common fraction.

8.________________

9. How many digits does the number $25^{10} * 16^{6}$ have?

9.________________

10. In right triangle ABC with $\angle B = 90^{\circ}$, AB has length 3 inches and BC is 4 inches. Construct the altitude from B to AC, and let the foot of this altitude be D. Find the length of DC in feet. Express your answer as a common fraction.

10.____________ feet

11. Earl puts one red and one blue marble into a box. In another box he places two red marbles. He then forgets which is which and randomly reaches into one of the boxes and takes out a red marble. What is the probability that the other marble in that box is blue? Express your answer as a common fraction.

11.________________

12. How many 3-digit numbers containing distinct digits from the set (0,1,2,3,4) are divisible by 3? Note: The first digit may not be 0.

12.________ numbers

13. Find the remainder when 7^{109} is divided by 16.

13.________________

14. Moving horizontally, vertically, or diagonally, how many ways can you start at A and end at C in the diagram while making the fewest moves possible? A move is defined as going from one letter to an adjacent letter. Diagonal moves can only be made to the nearest letter.

```
C  C  C  C  C
C  B  B  B  C
C  B  A  B  C
C  B  B  B  C
C  C  C  C  C
```

14.__________ ways

15. In trapezoid ABCD, AB is parallel to CD and BC is perpendicular to CD. Given that BC is 4 inches, AC is $2\sqrt{13}$ inches, and AD has an integer length, find the area of trapezoid ABCD in square inches.

15.____________ in^2

16. A rectangular prism, whose edges are all natural numbers, is constructed of unit cubes. The sum of the length, width, and height of this prism is 16. The prism's surface is painted blue. What is the greatest number of the original unit cubes that could not have paint on any faces?

16.____________ cubes

17. If a, b, and c are real numbers and satisfy the system of equations ac+b=1 and abc=-6, find the sum of all possible values of ac.

17.________________

18. The numbers 1, 2, 3,…, 2008 are written on a board. An operation is performed according to this rule: two numbers x and y on the board are erased and replaced by the number x+y-1. This operation is repeated until only one number N is left on the board. What is the value of N?

18.________________

19. Consider the sequence of k_i such that $k_1=1$, $k_2=2$, $k_3=2$, and $k_n=\left\lceil \sqrt{n} \right\rceil$ where $\lceil \ \rceil$ denotes the ceiling function. Find the sum of the first 29 terms of this sequence.

19.________________

20. George rotates a water bottle that is half full so that the water inside forms a cylinder with height 4 inches and radius 2 inches. The rotation of the water also creates a whirlpool in the shape of a right cone with height 2 inches and radius 2 inches. Since the cone is made of air, the shape of the water is a cylinder with a removed cone. Find the surface area of this figure in square inches. Express your answer as a simplified radical in terms of pi.

20.______________ in^2

21. If a+b=3 and $a^3+b^3=18$, find 1/a+1/b.

21.________________

22. Jessica plays a game where she walks around the entire circumference of a random circle. Given that she walks no more than 25 meters, find the sum of all distinct positive integral values for the area of all such circles, assuming that it must be an integer in square meters. Express your answer in square centimeters.

22.______________ cm^2

23. Quadrilateral ABCD has AB=2 inches, BC=6 inches, and DC=3 inches. It is possible to extend CD and AB to meet at point E with EA=3 inches, and DE=2 inches. Find the sum of the numerical values of the area of ABCD and the length of AD. Express your answer as a common fraction in simplest radical form.

23.______________

24. A machine takes any number given to it and triples it. It then passes along the new value to another machine, which subtracts two from this new value. The final value is recorded and sent to the head of McRonalds. He then looks at it and is able to determine the beginning number. He squares the beginning number and gets 1089. Find the positive difference between the final number and the beginning number, given that the final number was negative.

24.______________

25. Each car of a five-car train must be painted a solid color. The only color choices are red, blue, and yellow. If each of these colors must be used for at least one car, in how many ways can this train be painted?

25.___________ways

26. Harry usually drives to work at a constant rate of 60 mph in t hours. If his speed increases by 10 mph and he drives 15 minutes less, he will be five miles away from his office. If he decreases his speed by 15 mph and drives another 45 minutes, he will have traveled 15 miles past his office. How long does it normally take Harry to drive to work? Give your answer in minutes. (MPH denotes Miles Per Hour).

26.__________minutes

27. Call a positive integer evil if it is 666 larger than the sum of its digits. How many three-digit evil integers are there?

27.______________

28. An ant is crawling on the surface of a rectangular box with sides 9, 10, and 11 centimeters. What is the smallest distance it must crawl to get from one corner to the opposite corner? Express your answer as a simplified radical in centimeters.

28.____________cm

29. In how many distinct ways is it possible to color the six faces of a cube using two black squares, two green squares, and two yellow squares? Rotations of one cube are considered to be non-distinct.

29.______________

30. For this problem, assume one unit in the coordinate plane is equivalent to one inch.

Fred's fist is moving along the line $y=x\sqrt{3}$ starting at the origin. Tommy's face is 4 inches away from the origin and is also located on this line as well as in the first quadrant. At the moment, Tommy's hand is located 2 inches vertically below his face. Fred's fist and Tommy's hand both travel at 2 centimeters per second. Using the approximation 1 inch=2.5 centimeters, find the least amount of time it will take for Tommy's hand to intercept Fred's fist, given that both his hand and Fred's fist continuously move in a straight line. Express your answer in simplest radical form as a common fraction in seconds.

30.________ seconds

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2008
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Target Round
Problems 1 and 2

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This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the next sheet. When told to do so, turn the page over and begin working. Record your final answer in the designated space on the problem sheet. All answers must be complete, legible, and simplified to lowest terms. Calculators are allowed, and calculations may also be done on scratch paper, but no other aids are permitted.

Extra note: Answers will be posted later in the week. After finishing the test, please PM me your answers. A solutions thread will also be created sometime later.

1. Find the largest possible area, in square meters, of a closed figure with perimeter 800 meters. Express your answer to the nearest whole number.

1.______________ m^2

2. Find $\sum_{n=1}^{20} n(21-n)$, or, equivalently, find 1*20+2*19+…+20*1.

2.________________

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2008
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Problems 3 and 4

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3. In solving the quadratic equation, $ax^2+bx+c=0$, Anne made an error by solving bx^2+cx+a. Due to this, she came up with the solutions $\frac{4\pm\sqrt{22}}{6}$. Bob also made an error in copying the equation, instead using the equation $ax^2+cx+b=0$ to come up with the solutions $-4\pm\sqrt{22}$. Chris solved the equation $cx^2+ax+b=0$ to find solutions $\frac{-1\pm\sqrt{193}}{16}$. What were the original solutions of the equation? Express your answer as an ordered pair (a,b), with $a<b$.

3.(________, ________)

4. The sum of the digits of a positive integer n is 100. The sum of the digits of 44n is 800. What is the sum of the digits of 3n?

4.________________

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Problems 5 and 6

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5. A circle is divided into three equal regions. Each one has a central angle of measure 120 degrees. Consider the circle inscribed within one of these sectors tangent to both radii, which are part of the perimeter of the sector, and to the circumference of the circle. Then, consider the circle inscribed between this circle and the center of the largest circle such that it touches both radii of the larger circle as well as the part of the circumference of the other inscribed circle. Find the ratio of the areas between this circle and one sector of the largest circle. Express your answer as a percent to the nearest ten-thousandth.

5.______________%

6. In order to win a game, Lani must roll two standard six-sided dice and have their product to be either even or a multiple of 3. Find the probability that she wins any two consecutive games, given that she stops playing after such an event happens and that she plays no more than six times. Express your answer in the form $\frac{p_1^a * p_2^b * p_3^c}{p_4^d}$, where a, b, c, and d are positive integers and p_i, for $1 \le i \le 4$, are prime numbers.

6.________________

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Problems 7 and 8

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7. While eating dinner at his friend’s house, Carl is told by his friend that the product of his three sisters’ ages is 224. He is also given that his friend’s older sister enjoys swimming. Given that only one of his friend’s sisters’ ages is a prime number and that his friend is 15 years old, find the sum of his sisters’ ages in months. Assume all his friends’ sisters have distinct integral ages greater than one.

7.__________ months

8. In quadrilateral ABCD, $\angle A = 60^\circ$ and angle trisectors AE and AF are drawn with E on BC and F on CD. It is given that AF=AE=AB and AF bisects CD. If it is possible to write the value of AF^2 in terms of AD and BF in the form $a*AD^2 \pm b*AD\sqrt{c*AD^2 - d*BF^2}$, where a,b,c, and d are positive integers, find the value of a*b*c*d.

8.______________