GLENCOE MATHEMATICS

Algebra 1

Chapter 9
Resource Masters

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Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

Study Guide and Intervention Workbook	0-07-827753-1
Study Guide and Intervention Workbook (Spanish)	0-07-827754-X
Skills Practice Workbook	0-07-827747-7
Skills Practice Workbook (Spanish)	0-07-827749-3
Practice Workbook	0-07-827748-5
Practice Workbook (Spanish)	0-07-827750-7

ANSWERS FOR WORKBOOKS The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

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Algebra 1
Chapter 9 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 9 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 9 Resource Masters* includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Algebra 1 TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 9-1. Encourage them to add these pages to their Algebra Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Study Guide and Intervention

Each lesson in *Algebra 1* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics

One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 9 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Algebra 1. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of algebra concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and quantitative-comparison questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 520–521. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

9 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
composite number kahm·PAH·zeht		
factored form		
factoring		
factoring by grouping		
greatest common factor (GCF)		

(continued on the next page)

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Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
perfect square trinomial (try·NOH·mee·uhl)		
prime factorization (FAK·tuh·ruh·ZAY·shuhn)		
prime number		
prime polynomial		
Square Root Property		
Zero Product Property		

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9-1 Study Guide and Intervention

Factors and Greatest Common Factors

Prime Factorization When two or more numbers are multiplied, each number is called a **factor** of the product.

	Definition	Example
Prime Number	A prime number is a whole number, greater than 1, whose only factors are 1 and itself.	5
Composite Number	A composite number is a whole number, greater than 1, that has more than two factors.	10
Prime Factorization	Prime factorization occurs when a whole number is expressed as a product of factors that are all prime numbers.	$45 = 3^2 \cdot 5$

Example 1 **Factor each number. Then classify each number as *prime* or *composite*.**

a. 28

To find the factors of 28, list all pairs of whole numbers whose product is 28.

$1 \times 28 \quad 2 \times 14 \quad 4 \times 7$

Therefore, the factors of 28 are 1, 2, 4, 7, 14, and 28. Since 28 has more than 2 factors, it is a composite number.

b. 31

To find the factors of 31, list all pairs of whole numbers whose product is 31.

1×31

Therefore, the factors of 31 are 1 and 31. Since the only factors of 31 are itself and 1, it is a prime number.

Example 2 **Find the prime factorization of 200.**

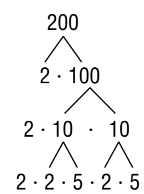
Method 1

$200 = 2 \cdot 100$
$= 2 \cdot 2 \cdot 50$
$= 2 \cdot 2 \cdot 2 \cdot 25$
$= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

All the factors in the last row are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Method 2

Use a factor tree.



All of the factors in each last branch of the factor tree are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Exercises

Find the factors of each number. Then classify the number as *prime* or *composite*.

1. 41

2. 121

3. 90

4. 2865

Find the prime factorization of each integer.

5. 600

6. 175

7. -150

Factor each monomial completely.

8. $32x^2$

9. $18m^2n$

10. $49a^3b^2$

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9-1 Study Guide and Intervention *(continued)*

Factors and Greatest Common Factors

Greatest Common Factor

Greatest Common Factor (GCF)	
Integers	the greatest number that is a factor of all the integers
Monomials	the product of their common factors when each monomial is expressed in factored form

If two or more integers or monomials have no common prime factors, their GCF is 1 and the integers or monomials are said to be **relatively prime**.

Example **Find the GCF of each set of monomials.**

a. 12 and 18

$12 = 2 \cdot 2 \cdot 3$ Factor each number.

$18 = 2 \cdot 3 \cdot 3$ Circle the common prime factors, if any.

The GCF of 12 and 18 is $2 \cdot 3$ or 6.

b. $16xy^2z^2$ and $72xyz^3$

$16xy^2z^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y \cdot z \cdot z$

$72xyz^3 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot z \cdot z \cdot z$

The GCF of $16xy^2z^2$ and $72xyz^3$ is $2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot z \cdot z$ or $8xyz^2$.

Exercises

Find the GCF of each set of monomials.

1. 12, 48

2. 18, 42

3. 64, 80

4. 32, 54

5. 27, 32

6. 44, 100

7. 45, 15

8. 169, 13

9. 20, 440

10. $49x$, $343x^2$

11. $4a^7b$, $28ab$

12. $96y$, $12x$, $8y$

13. $12a$, $18abc$

14. $28y^2$, $35xy$, $49x^2yz$

15. $2m^2n$, $12mn^2$, $18mn$

16. $12x^2$, $32x^2yz$, $60xy^2$

17. $18a^3b^2$, $36a^3b^2$

18. $15mn^2$, $30m^3n^2$, $90m^3$

19. $2x^2y$, $9x^2y^3$, $18xy^2$

20. a^4b, $8a^3b^2$

21. ab^2, $5a^4b^2$, $10b^3$

9-1 Skills Practice

Factors and Greatest Common Factors

Find the factors of each number. Then classify each number as *prime* or *composite*.

1. 10

2. 31

3. 16

4. 52

5. 38

6. 105

Find the prime factorization of each integer.

7. -16

8. 20

9. 24

10. 36

11. 112

12. -72

Factor each monomial completely.

13. $10a^4$

14. $-27x^3y^2$

15. $28pq^2$

16. $44m^2ns^3$

Find the GCF of each set of monomials.

17. 12, 18

18. 20, 27

19. 30, 48

20. 24, 81

21. 20, 36, 64

22. 42, 60, 78

23. $16c$, $21b^2d$

24. $18a$, $48a^4$

25. $32xyz$, $48xy^4$

26. $12m^3n^2$, $44mn^3$

9-1 Practice

Factors and Greatest Common Factors

Find the factors of each number. Then classify each number as *prime* or *composite*.

1. 18 **2.** 37 **3.** 48

4. 116 **5.** 138 **6.** 211

Find the prime factorization of each integer.

7. 52 **8.** -96 **9.** 108

10. 225 **11.** 286 **12.** -384

Factor each monomial completely.

13. $30d^5$ **14.** $-72mn$

15. $81b^2c^3$ **16.** $145abc^3$

17. $168pq^2r$ **18.** $-121x^2yz^2$

Find the GCF of each set of monomials.

19. 18, 49 **20.** 18, 45, 63 **21.** 16, 24, 48

22. 12, 30, 114 **23.** 9, 27, 77 **24.** 24, 72, 108

25. $24fg^5$, $56f^3g$ **26.** $72r^2s^2$, $36rs^3$ **27.** $15a^2b$, $35ab^2$

28. $28m^3n^2$, $45pq^2$ **29.** $40xy^2$, $56x^3y^2$, $124x^2y^3$ **30.** $88c^3d$, $40c^2d^2$, $32c^2d$

GEOMETRY For Exercises 31 and 32, use the following information.
The area of a rectangle is 84 square inches. Its length and width are both whole numbers.

31. What is the minimum perimeter of the rectangle?

32. What is the maximum perimeter of the rectangle?

RENOVATION For Exercises 33 and 34, use the following information.
Ms. Baxter wants to tile a wall to serve as a splashguard above a basin in the basement. She plans to use equal-sized tiles to cover an area that measures 48 inches by 36 inches.

33. What is the maximum-size square tile Ms. Baxter can use and not have to cut any of the tiles?

34. How many tiles of this size will she need?

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9-1 Reading to Learn Mathematics

Factors and Greatest Common Factors

Pre-Activity **How are prime numbers related to the search for extraterrestrial life?**

Read the introduction to Lesson 9-1 at the top of page 474 in your textbook.

If each "beep" counts as one, what are the first two prime numbers?

Lesson 9-1

Reading the Lesson

1. Every whole number greater than 1 is either composite or _____________.

2. Complete each statement.

a. In the prime factorization of a whole number, each factor is a _____________ number.

b. In the prime factorization of a negative integer, all the factors are prime except the factor _____________.

3. Explain why the monomial $5x^2y$ is *not* in factored form.

4. Explain the steps used below to find the greatest common factor (GCF) of 84 and 120.

$84 = 2 \cdot 2 \cdot 3 \cdot 7$

$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

Common prime factors: 2, 2, 3

$2 \cdot 2 \cdot 3 = 12$

Helping You Remember

5. How can the two words that make up the term *prime factorization* help you remember what the term means?

9-1 Enrichment

Finding the GCF by Euclid's Algorithm

Finding the greatest common factor of two large numbers can take a long time using prime factorizations. This method can be avoided by using Euclid's Algorithm as shown in the following example.

Example **Find the GCF of 209 and 532.**

Divide the greater number, 532, by the lesser, 209.

$$
\begin{array}{l}
209\overline{)532} \quad \text{quotient } 2 \\
\quad\ \underline{418} \\
\quad\ 114\overline{)209} \quad \text{quotient } 1 \\
\qquad\quad \underline{114} \\
\qquad\quad 95\overline{)114} \quad \text{quotient } 1 \\
\qquad\qquad \underline{95} \\
\qquad\qquad \mathbf{19}\overline{)95} \quad \text{quotient } 5 \\
\qquad\qquad\quad \underline{95} \\
\qquad\qquad\quad 0
\end{array}
$$

Divide the remainder into the divisor above. Repeat this process until the remainder is zero. The last nonzero remainder is the GCF.

The divisor, 19, is the GCF of 209 and 532.

Suppose the GCF of two numbers is found to be 1. Then the numbers are said to be **relatively prime**.

Find the GCF of each group of numbers by using Euclid's Algorithm.

1. 187; 578

2. 1802; 106

3. 161; 943

4. 215; 1849

5. 1325; 3498

6. 3484; 5963

7. 33,583; 4257

8. 453; 484

9. 95; 209; 589

10. 518; 407; 851

11. $17a^2x^2z$; $1615axz^2$

12. $752cf^3$; $893c^3f^3$

13. $979r^2s^2$; $495rs^3$, $154r^3s^3$

14. $360x^5y^7$; $328xy$; $568x^3y^3$

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9-2 Study Guide and Intervention

Factoring Using the Distributive Property

Factor by Using the Distributive Property The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1)$ $= 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1)$ $= 6y(2x + 1)$

Example 1 **Use the Distributive Property to factor $12mn + 80m^2$.**

Find the GCF of $12mn$ and $80m^2$.

$12mn = 2 \cdot 2 \cdot 3 \cdot m \cdot n$

$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$

$\text{GCF} = 2 \cdot 2 \cdot m$ or $4m$

Write each term as the product of the GCF and its remaining factors.

$$\begin{aligned} 12mn + 80m^2 &= 4m(3 \cdot n) + 4m(2 \cdot 2 \cdot 5 \cdot m) \\ &= 4m(3n) + 4m(20m) \\ &= 4m(3n + 20m) \end{aligned}$$

Thus $12mn + 80m^2 = 4m(3n + 20m)$.

Example 2 **Factor $6ax + 3ay + 2bx + by$ by grouping.**

$$\begin{aligned} &6ax + 3ay + 2bx + by \\ &= (6ax + 3ay) + (2bx + by) \\ &= 3a(2x + y) + b(2x + y) \\ &= (3a + b)(2x + y) \end{aligned}$$

Check using the FOIL method.

$$\begin{aligned} &(3a + b)(2x + y) \\ &= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y) \\ &= 6ax + 3ay + 2bx + by \checkmark \end{aligned}$$

Exercises

Factor each polynomial.

1. $24x + 48y$

2. $30mn^2 + m^2n - 6n$

3. $q^4 - 18q^3 + 22q$

4. $9x^2 - 3x$

5. $4m + 6n - 8mn$

6. $45s^3 - 15s^2$

7. $14c^3 - 42c^5 - 49c^4$

8. $55p^2 - 11p^4 + 44p^5$

9. $14y^3 - 28y^2 + y$

10. $4x + 12x^2 + 16x^3$

11. $4a^2b + 28ab^2 + 7ab$

12. $6y + 12x - 8z$

13. $x^2 + 2x + x + 2$

14. $6y^2 - 4y + 3y - 2$

15. $4m^2 + 4mn + 3mn + 3n^2$

16. $12ax + 3xz + 4ay + yz$

17. $12a^2 + 3a - 8a - 2$

18. $xa + ya + x + y$

Lesson 9-2

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9-2 Study Guide and Intervention *(continued)*

Factoring Using the Distributive Property

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal 0.

Example **Solve $9x^2 + x = 0$. Then check the solutions.**

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$	Original equation
$x(9x + 1) = 0$	Factor the GCF of $9x^2 + x$, which is x.
$x = 0$ or $9x + 1 = 0$	Zero Product Property
$x = 0$ $\quad x = -\frac{1}{9}$	Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

CHECK Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$9x^2 + x = 0$

$9(0)^2 + 0 = 0$

$0 = 0$ ✓

$9x^2 + x = 0$

$9\left(-\frac{1}{9}\right)^2 + \left(-\frac{1}{9}\right) = 0$

$\frac{1}{9} + \left(-\frac{1}{9}\right) = 0$

$0 = 0$ ✓

Exercises

Solve each equation. Check your solutions.

1. $x(x + 3) = 0$

2. $3m(m - 4) = 0$

3. $(r - 3)(r + 2) = 0$

4. $3x(2x - 1) = 0$

5. $(4m + 8)(m - 3) = 0$

6. $5s^2 = 25s$

7. $(4c + 2)(2c - 7) = 0$

8. $5p - 15p^2 = 0$

9. $4y^2 = 28y$

10. $12x^2 = -6x$

11. $(4a + 3)(8a + 7) = 0$

12. $8y = 12y^2$

13. $x^2 = -2x$

14. $(6y - 4)(y + 3) = 0$

15. $4m^2 = 4m$

16. $12x = 3x^2$

17. $12a^2 = -3a$

18. $(12a + 4)(3a - 1) = 0$

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9-2 Skills Practice

Factoring Using the Distributive Property

Factor each polynomial.

1. $7x + 49$

2. $8m - 6$

3. $5a^2 - 15$

4. $10q - 25q^2$

5. $8ax - 56a$

6. $81r + 48rs$

7. $t^2h + 3t$

8. $a^2b^2 + a$

9. $x + x^2y + x^3y^2$

10. $3p^2q^2 + 6pq + p$

11. $4a^2b^2 + 16ab + 12a$

12. $10m^3n^3 - 2mn^2 + 14mn$

13. $x^2 + 3x + x + 3$

14. $b^2 - 2b + 3b - 6$

15. $2s^2 + 2s + 3s + 3$

16. $2a^2 - 4a + a - 2$

17. $6t^2 - 4t - 3t + 2$

18. $9x^2 - 3xy + 6x - 2y$

Solve each equation. Check your solutions.

19. $x(x - 8) = 0$

20. $b(b + 12) = 0$

21. $(m - 3)(m + 5) = 0$

22. $(a - 9)(2a + 1) = 0$

23. $x^2 - 5x = 0$

24. $y^2 + 3y = 0$

25. $3a^2 = 6a$

26. $2x^2 = 3x$

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9-2 Practice

Factoring Using the Distributive Property

Factor each polynomial.

1. $64 - 40ab$ **2.** $4d^2 + 16$ **3.** $6r^2s - 3rs^2$

4. $15cd + 30c^2d^2$ **5.** $32a^2 + 24b^2$ **6.** $36xy^2 - 48x^2y$

7. $30x^3y + 35x^2y^2$ **8.** $9c^3d^2 - 6cd^3$ **9.** $75b^2c^3 + 60bc^3$

10. $8p^2q^2 - 24pq^3 + 16pq$ **11.** $5x^3y^2 + 10x^2y + 25x$ **12.** $9ax^3 + 18bx^2 + 24cx$

13. $x^2 + 4x + 2x + 8$ **14.** $2a^2 + 3a + 6a + 9$ **15.** $4b^2 - 12b + 2b - 6$

16. $6xy - 8x + 15y - 20$ **17.** $-6mn + 4m + 18n - 12$ **18.** $12a^2 - 15ab - 16a + 20b$

Solve each equation. Check your solutions.

19. $x(x - 32) = 0$ **20.** $4b(b + 4) = 0$ **21.** $(y - 3)(y + 2) = 0$

22. $(a + 6)(3a - 7) = 0$ **23.** $(2y + 5)(y - 4) = 0$ **24.** $(4y + 8)(3y - 4) = 0$

25. $2z^2 + 20z = 0$ **26.** $8p^2 - 4p = 0$ **27.** $9x^2 = 27x$

28. $18x^2 = 15x$ **29.** $14x^2 = -21x$ **30.** $8x^2 = -26x$

LANDSCAPING For Exercises 31 and 32, use the following information.

A landscaping company has been commissioned to design a triangular flower bed for a mall entrance. The final dimensions of the flower bed have not been determined, but the company knows that the height will be two feet less than the base. The area of the flower bed can be represented by the equation $A = \frac{1}{2}b^2 - b$.

31. Write this equation in factored form.

32. Suppose the base of the flower bed is 16 feet. What will be its area?

33. PHYSICAL SCIENCE Mr. Alim's science class launched a toy rocket from ground level with an initial upward velocity of 60 feet per second. The height h of the rocket in feet above the ground after t seconds is modeled by the equation $h = 60t - 16t^2$. How long was the rocket in the air before it returned to the ground?

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9-2 Reading to Learn Mathematics

Factoring Using the Distributive Property

Pre-Activity **How can you determine how long a baseball will remain in the air?**

Read the introduction to Lesson 9-2 at the top of page 481 in your textbook.

In the formula $h = 151t - 16t^2$, what does the number 151 represent?

Reading the Lesson

1. Factoring a polynomial means to find its completely factored form.

a. The expression $x(6x - 9)$ is a factored form of the polynomial $6x^2 - 9x$. Why is this *not* its completely factored form?

b. Provide an example of a completely factored polynomial.

c. Provide an example of a polynomial that is not completely factored.

2. The polynomial $5ab + 5b^2 + 3a + 6b$ can be rewritten as $5b(a + b) + 3(a + 2b)$. Does this indicate that the original polynomial can be factored by grouping? Explain.

3. The polynomial $3x^2 - 3xy + 2x - 2y$ can be rewritten as $3x(x - y) + 2(x - y)$. Does this indicate that the original polynomial can be factored by grouping? Explain.

Helping You Remember

4. How would you explain to a classmate when it is possible to use the Zero Product Property to solve an equation?

Lesson 9-2

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9-2 Enrichment

Perfect, Excessive, Defective, and Amicable Numbers

A **perfect number** is the sum of all of its factors except itself. Here is an example:

$28 = 1 + 2 + 4 + 7 + 14$

There are very few perfect numbers. Most numbers are either *excessive* or *defective*.

An **excessive number** is greater than the sum of all of its factors except itself.

A **defective number** is less than this sum.

Two numbers are **amicable** if the sum of the factors of the first number, except for the number itself, equals the second number, and vice versa.

Solve each problem.

1. Write the perfect numbers between 0 and 31.

2. Write the excessive numbers between 0 and 31.

3. Write the defective numbers between 0 and 31.

4. Show that 8128 is a perfect number.

5. The sum of the reciprocals of all the factors of a perfect number (including the number itself) equals 2. Show that this is true for the first two perfect numbers.

6. More than 1000 pairs of amicable numbers have been found. One member of the first pair is 220. Find the other member.

7. One member of the second pair of amicable numbers is 2620. Find the other member.

8. The Greek mathematician Euclid proved that the expression $2^{n-1}(2^n - 1)$ equals a perfect number if the expression inside the parentheses is prime. Use Euclid's expression with n equal to 5 to find the third perfect number.

9-3 Study Guide and Intervention

Factoring Trinomials: $x^2 + bx + c$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + n)$, where $m + n = b$ and $mn = c$.

Example 1 **Factor each trinomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $n = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$. Notice that $m + n$ is negative and mn is positive, so m and n are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $n = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 **Factor $x^2 + 6x - 16$.**

In this trinomial, $b = 6$ and $c = -16$. This means $m + n$ is positive and mn is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of −16	Sum of Factors
1, −16	−15
−1, 16	15
2, −8	−6
−2, 8	6

Therefore, $m = -2$ and $n = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each trinomial.

1. $x^2 + 4x + 3$

2. $m^2 + 12m + 32$

3. $r^2 - 3r + 2$

4. $x^2 - x - 6$

5. $x^2 - 4x - 21$

6. $x^2 - 22x + 121$

7. $c^2 - 4c - 12$

8. $p^2 - 16p + 64$

9. $9 - 10x + x^2$

10. $x^2 + 6x + 5$

11. $a^2 + 8a - 9$

12. $y^2 - 7y - 8$

13. $x^2 - 2x - 3$

14. $y^2 + 14y + 13$

15. $m^2 + 9m + 20$

16. $x^2 + 12x + 20$

17. $a^2 - 14a + 24$

18. $18 + 11y + y^2$

19. $x^2 + 2xy + y^2$

20. $a^2 - 4ab + 4b^2$

21. $x^2 + 6xy - 7y^2$

9-3 Study Guide and Intervention *(continued)*

Factoring Trinomials: $x^2 + bx + c$

Solve Equations by Factoring Factoring and the Zero Product Property from Lesson 9-2 can be used to solve many equations of the form $x^2 + bx + c = 0$.

Example 1 **Solve $x^2 + 6x = 7$. Check your solutions.**

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $\quad$ $x = -7$	Solve each equation.

The solution set is {1, −7}. Since $1^2 + 6 = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Example 2 **ROCKET LAUNCH A rocket is fired with an initial velocity of 2288 feet per second. How many seconds will it take for the rocket to hit the ground?**

The formula $h = vt - 16t^2$ gives the height h of the rocket after t seconds when the initial velocity v is given in feet per second.

$h = vt - 16t^2$	Formula
$0 = 2288t - 16t^2$	Substitute.
$0 = 16t(143 - t)$	Factor.
$16t = 0$ or $143 - t = 0$	Zero Product Property
$t = 0$ $\quad$ $t = 143$	Solve each equation.

The value $t = 0$ represents the time at launch. The rocket returns to the ground in 143 seconds, or a little less than 2.5 minutes after launch.

Exercises

Solve each equation. Check your solutions.

1. $x^2 - 4x + 3 = 0$
2. $y^2 - 5y + 4 = 0$
3. $m^2 + 10m + 9 = 0$
4. $x^2 = x + 2$
5. $x^2 - 4x = 5$
6. $x^2 - 12x + 36 = 0$
7. $c^2 - 8 = -7c$
8. $p^2 = 9p - 14$
9. $-9 - 8x + x^2 = 0$
10. $x^2 + 6 = 5x$
11. $a^2 = 11a - 18$
12. $y^2 - 8y + 15 = 0$
13. $x^2 = 24 - 10x$
14. $a^2 - 18a = -72$
15. $b^2 = 10b - 16$

Use the formula $h = vt - 16t^2$ to solve each problem.

16. **FOOTBALL** A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to return to the ground?

17. **BASEBALL** A ball is thrown up with an initial velocity of 32 feet per second. How many seconds will it take for the ball to return to the ground?

18. **ROCKET LAUNCH** If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high?

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9-3 Skills Practice

Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial.

1. $t^2 + 8t + 12$

2. $n^2 + 7n + 12$

3. $p^2 + 9p + 20$

4. $h^2 + 9h + 18$

5. $n^2 + 3n - 18$

6. $x^2 + 2x - 8$

7. $y^2 - 5y - 6$

8. $g^2 + 3g - 10$

9. $s^2 + 4s - 12$

10. $x^2 - x - 12$

11. $w^2 - w - 6$

12. $y^2 - 6y + 8$

13. $x^2 - 8x + 15$

14. $b^2 - 9b + 8$

15. $c^2 - 15c + 56$

16. $-4 - 3m + m^2$

Solve each equation. Check your solutions.

17. $x^2 - 6x + 8 = 0$

18. $b^2 - 7b + 12 = 0$

19. $m^2 + 5m + 6 = 0$

20. $d^2 + 7d + 10 = 0$

21. $y^2 - 2y - 24 = 0$

22. $p^2 - 3p = 18$

23. $h^2 + 2h = 35$

24. $a^2 + 14a = -45$

25. $n^2 - 36 = 5n$

26. $w^2 + 30 = 11w$

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9-3 Practice

Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial.

1. $a^2 + 10a + 24$

2. $h^2 + 12h + 27$

3. $x^2 + 14x + 33$

4. $g^2 - 2g - 63$

5. $w^2 + w - 56$

6. $y^2 + 4y - 60$

7. $b^2 + 4b - 32$

8. $n^2 - 3n - 28$

9. $c^2 + 4c - 45$

10. $z^2 - 11z + 30$

11. $d^2 - 16d + 63$

12. $x^2 - 11x + 24$

13. $q^2 - q - 56$

14. $x^2 - 6x - 55$

15. $32 + 18r + r^2$

16. $48 - 16g + g^2$

17. $j^2 - 9jk - 10k^2$

18. $m^2 - mv - 56v^2$

Solve each equation. Check your solutions.

19. $x^2 + 17x + 42 = 0$

20. $p^2 + 5p - 84 = 0$

21. $k^2 + 3k - 54 = 0$

22. $b^2 - 12b - 64 = 0$

23. $n^2 + 4n = 32$

24. $h^2 - 17h = -60$

25. $c^2 - 26c = 56$

26. $z^2 - 14z = 72$

27. $y^2 - 84 = 5y$

28. $80 + a^2 = 18a$

29. $u^2 = 16u + 36$

30. $17s + s^2 = -52$

31. Find all values of k so that the trinomial $x^2 + kx - 35$ can be factored using integers.

CONSTRUCTION For Exercises 32 and 33, use the following information.

A construction company is planning to pour concrete for a driveway. The length of the driveway is 16 feet longer than its width w.

32. Write an expression for the area of the driveway.

33. Find the dimensions of the driveway if it has an area of 260 square feet.

WEB DESIGN For Exercises 34 and 35, use the following information.

Janeel has a 10-inch by 12-inch photograph. She wants to scan the photograph, then reduce the result by the same amount in each dimension to post on her Web site. Janeel wants the area of the image to be one eighth that of the original photograph.

34. Write an equation to represent the area of the reduced image.

35. Find the dimensions of the reduced image.

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9-3 Reading to Learn Mathematics

Factoring Trinomials: $x^2 + bx + c$

Pre-Activity **How can factoring be used to find the dimensions of a garden?**

Read the introduction to Lesson 9-3 at the top of page 489 in your textbook.

- Why do you need to find two numbers whose *product* is 54?

- Why is the *sum* of these two numbers half the perimeter or 15?

Reading the Lesson

Tell what sum and product you want m and n to have to use the pattern $(x + m)(x + n)$ to factor the given trinomial.

1. $x^2 + 10x + 24$ sum: _____________ product: _____________

2. $x^2 - 12x + 20$ sum: _____________ product: _____________

3. $x^2 - 4x - 21$ sum: _____________ product: _____________

4. $x^2 + 6x - 16$ sum: _____________ product: _____________

5. To factor $x^2 - 18x + 32$, you can look for numbers with a product of 32 and a sum of -18. Explain why the numbers in the pair you are looking for must both be negative.

Helping You Remember

6. If you are using the pattern $(x + m)(x + n)$ to factor a trinomial of the form $x^2 + bx + c$, how can you use your knowledge of multiplying integers to help you remember whether m and n are positive or negative factors?

Lesson 9-3

9-3 Enrichment

Puzzling Primes

A prime number has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 25 prime numbers less than 100.

Prime numbers have interested mathematicians for centuries. They have tried to find expressions that will give all the prime numbers, or only prime numbers. In the 1700s, Euler discovered that the trinomial $x^2 + x + 41$ will yield prime numbers for values of x from 0 through 39.

2. Find the prime numbers generated by Euler's formula for x from 0 through 7.

3. Show that the trinomial $x^2 + x + 31$ will not give prime numbers for very many values of x.

4. Find the largest prime number generated by Euler's formula.

Goldbach's Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true, but no one has found a counterexample, either.

5. Show that Goldbach's Conjecture is true for the first 5 even numbers greater than 2.

6. Give a way that someone could disprove Goldbach's Conjecture.

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9-4 Study Guide and Intervention

Factoring Trinomials: $ax^2 + bx + c$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and n whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 **Factor $2x^2 + 15x + 18$.**

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers whose sum is 15 and whose product is $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors whose sum is 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + nx + c$, with $a = 2$, $m = 3$, $n = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 **Factor $3x^2 - 3x - 18$.**

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 whose sum is -1.

Factors of −6	Sum of Factors
1, −6	−5
−1, 6	5
−2, 3	1
2, −3	−1

Now use the pattern $(x + m)(x + n)$ with $m = 2$ and $n = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is $3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$

2. $3m^2 - 8m - 3$

3. $16r^2 - 8r + 1$

4. $6x^2 + 5x - 6$

5. $3x^2 + 2x - 8$

6. $18x^2 - 27x - 5$

7. $2a^2 + 5a + 3$

8. $18y^2 + 9y - 5$

9. $-4c^2 + 19c - 21$

10. $8x^2 - 4x - 24$

11. $28p^2 + 60p - 25$

12. $48x^2 + 22x - 15$

13. $3y^2 - 6y - 24$

14. $4x^2 + 26x - 48$

15. $8m^2 - 44m + 48$

16. $6x^2 - 7x + 18$

17. $2a^2 - 14a + 18$

18. $18 + 11y + 2y^2$

Lesson 9-4

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9-4 Study Guide and Intervention *(continued)*

Factoring Trinomials: $ax^2 + bx + c$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad$ $x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $8x^2 + 2x - 3 = 0$

2. $3n^2 - 2n - 5 = 0$

3. $2d^2 - 13d - 7 = 0$

4. $4x^2 = x + 3$

5. $3x^2 - 13x = 10$

6. $6x^2 - 11x - 10 = 0$

7. $2k^2 - 40 = -11k$

8. $2p^2 = -21p - 40$

9. $-7 - 18x + 9x^2 = 0$

10. $12x^2 - 15 = -8x$

11. $7a^2 = -65a - 18$

12. $16y^2 - 2y - 3 = 0$

13. $8x^2 + 5x = 3 + 7x$

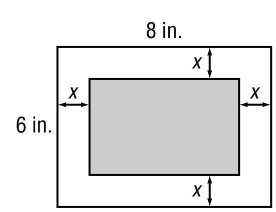
14. $4a^2 - 18a + 5 = 15$

15. $3b^2 - 18b = 10b - 49$

16. The difference of the squares of two consecutive odd integers is 24. Find the integers.

17. GEOMETRY The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?

18. GEOMETRY A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches.



9-4 Skills Practice

Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2x^2 + 5x + 2$

2. $3n^2 + 5n + 2$

3. $2s^2 + 9s - 5$

4. $3g^2 - 7g + 2$

5. $2t^2 - 11t + 15$

6. $2x^2 + 3x - 6$

7. $2y^2 + y - 1$

8. $4h^2 + 8h - 5$

9. $4x^2 - 3x - 3$

10. $4b^2 + 15b - 4$

11. $9p^2 + 6p - 8$

12. $6q^2 - 13q + 6$

13. $3a^2 + 30a + 63$

14. $10w^2 - 19w - 15$

Solve each equation. Check your solutions.

15. $2x^2 + 7x + 3 = 0$

16. $3w^2 + 14w + 8 = 0$

17. $3n^2 - 7n + 2 = 0$

18. $5d^2 - 22d + 8 = 0$

19. $6h^2 + 8h + 2 = 0$

20. $8p^2 - 16p = 10$

21. $9y^2 + 18y - 12 = 6y$

22. $4a^2 - 16a = -15$

23. $10b^2 - 15b = 8b - 12$

24. $6d^2 + 21d = 10d + 35$

Lesson 9-4

9-4 Practice

Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2b^2 + 10b + 12$

2. $3g^2 + 8g + 4$

3. $4x^2 + 4x - 3$

4. $8b^2 - 5b - 10$

5. $6m^2 + 7m - 3$

6. $10d^2 + 17d - 20$

7. $6a^2 - 17a + 12$

8. $8w^2 - 18w + 9$

9. $10x^2 - 9x + 6$

10. $15n^2 - n - 28$

11. $10x^2 + 21x - 10$

12. $9r^2 + 15r + 6$

13. $12y^2 - 4y - 5$

14. $14k^2 - 9k - 18$

15. $8z^2 + 20z - 48$

16. $12q^2 + 34q - 28$

17. $18h^2 + 15h - 18$

18. $12p^2 - 22p - 20$

Solve each equation. Check your solutions.

19. $3h^2 + 2h - 16 = 0$

20. $15n^2 - n = 2$

21. $8q^2 - 10q + 3 = 0$

22. $6b^2 - 5b = 4$

23. $10c^2 - 21c = -4c + 6$

24. $10g^2 + 10 = 29g$

25. $6y^2 = -7y - 2$

26. $9z^2 = -6z + 15$

27. $12k^2 + 15k = 16k + 20$

28. $12x^2 - 1 = -x$

29. $8a^2 - 16a = 6a - 12$

30. $18a^2 + 10a = -11a + 4$

31. DIVING Lauren dove into a swimming pool from a 15-foot-high diving board with an initial upward velocity of 8 feet per second. Find the time t in seconds it took Lauren to enter the water. Use the model for vertical motion given by the equation $h = -16t^2 + vt + s$, where h is height in feet, t is time in seconds, v is the initial upward velocity in feet per second, and s is the initial height in feet. (*Hint:* Let $h = 0$ represent the surface of the pool.)

32. BASEBALL Brad tossed a baseball in the air from a height of 6 feet with an initial upward velocity of 14 feet per second. Enrique caught the ball on its way down at a point 4 feet above the ground. How long was the ball in the air before Enrique caught it? Use the model of vertical motion from Exercise 31.

9-4 Reading to Learn Mathematics

Factoring Trinomials: $ax^2 + bx + c$

Pre-Activity **How can algebra tiles be used to factor $2x^2 + 7x + 6$?**

Read the introduction to Lesson 9-4 at the top of page 495 in your textbook.

- When you form the algebra tiles into a rectangle, what is the first step?
- What is the second step?

Reading the Lesson

1. Suppose you want to factor the trinomial $3x^2 + 14x + 8$.

a. What is the first step?

b. What is the second step?

c. Provide an explanation for the next two steps.

$(3x^2 + 2x) + (12x + 8)$

$x(3x + 2) + 4(3x + 2)$

d. Use the Distributive Property to rewrite the last expression in part c. You get

(_________ + _________)$(3x + 2)$.

2. Explain how you know that the trinomial $2x^2 - 7x + 4$ is a prime polynomial.

Helping You Remember

3. What are steps you could use to remember how to find the factors of a trinomial written in the form of $ax^2 + bx + c$?

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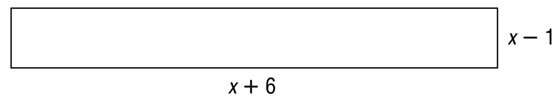
9-4 Enrichment

Area Models for Quadratic Trinomials

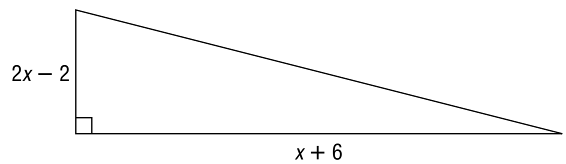
After you have factored a quadratic trinomial, you can use the factors to draw geometric models of the trinomial.

$x^2 + 5x - 6 = (x - 1)(x + 6)$

To draw a rectangular model, the value 2 was used for x so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is $x^2 + 5x - 6$.



To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.



$$x^2 + 5x - 6 = (x - 1)(x + 6) = \frac{1}{2}(2x - 2)(x + 6)$$

The area of the right triangle is also $x^2 + 5x - 6$.

Factor each trinomial. Then follow the directions to draw each model of the trinomial.

1. $x^2 + 2x - 3$ Use $x = 2$. Draw a rectangle in centimeters.

2. $3x^2 + 5x - 2$ Use $x = 1$. Draw a rectangle in centimeters.

3. $x^2 - 4x + 3$ Use $x = 4$. Draw two different right triangles in centimeters.

4. $9x^2 - 9x + 2$ Use $x = 2$. Draw two different right triangles. Use 0.5 centimeter for each unit.

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9-5 Study Guide and Intervention

Factoring Differences of Squares

Factor $a^2 - b^2$ The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each binomial.**

a. $n^2 - 64$

$n^2 - 64$
$= n^2 - 8^2$ Write in the form $a^2 - b^2$.
$= (n + 8)(n - 8)$ Factor.

b. $4m^2 - 81n^2$

$4m^2 - 81n^2$
$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.
$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$
$= 2(25a^2 - 36)$ Find the GCF.
$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$
$= 2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial
$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.
$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.
$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.
$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.
$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $x^2 - 81$

2. $m^2 - 100$

3. $16n^2 - 25$

4. $36x^2 - 100y^2$

5. $49x^2 - 32$

6. $16a^2 - 9b^2$

7. $225c^2 - a^2$

8. $72p^2 - 50$

9. $-2 + 2x^2$

10. $-81 + a^4$

11. $6 - 54a^2$

12. $8y^2 - 200$

13. $4x^3 - 100x$

14. $2y^4 - 32y^2$

15. $8m^3 - 128m$

16. $6x^2 - 25$

17. $2a^3 - 98ab^2$

18. $18y^2 - 72y^4$

19. $169x^3 - x$

20. $3a^4 - 3a^2$

21. $3x^4 + 6x^3 - 3x^2 - 6x$

9-5 Study Guide and Intervention *(continued)*

Factoring Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example **Solve each equation. Check your solutions.**

a. $x^2 - \frac{1}{25} = 0$

$x^2 - \frac{1}{25} = 0$	Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$	$x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$	Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$	Zero Product Property
$x = -\frac{1}{5}$ $\quad x = \frac{1}{5}$	Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $4x^3 = 9x$

$4x^3 = 9x$	Original equation
$4x^3 - 9x = 0$	Subtract $9x$ from each side.
$x(4x^2 - 9) = 0$	Find the GCF.
$x[(2x)^2 - 3^2] = 0$	$4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$
$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$	Factor the difference of squares.
$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$	Zero Product Property
$x = 0$ $\quad x = \frac{3}{2}$ $\quad x = -\frac{3}{2}$	Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $81x^2 = 49$

2. $36n^2 = 1$

3. $25d^2 - 100 = 0$

4. $\frac{1}{4}x^2 = 25$

5. $36 = \frac{1}{25}x^2$

6. $\frac{49}{100} - x^2 = 0$

7. $9x^3 = 25x$

8. $7a^3 = 175a$

9. $2m^3 = 32m$

10. $16y^3 = 25y$

11. $\frac{1}{64}x^2 = 49$

12. $4a^3 - 64a = 0$

13. $3b^3 - 27b = 0$

14. $\frac{9}{25}m^2 = 121$

15. $48n^3 = 147n$

9-5 Skills Practice

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $a^2 - 4$

2. $n^2 - 64$

3. $1 - 49c^2$

4. $-16 + p^2$

5. $k^2 + 25$

6. $36 - 100w^2$

7. $t^2 - 81u^2$

8. $4h^2 - 25g^2$

9. $64m^2 - 9y^2$

10. $4c^2 - 5d^2$

11. $-49r^2 + 4t^2$

12. $8x^2 - 72p^2$

13. $20q^2 - 5r^2$

14. $32a^2 - 50b^2$

Solve each equation by factoring. Check your solutions.

15. $16x^2 - 9 = 0$

16. $25p^2 - 16 = 0$

17. $36q^2 - 49 = 0$

18. $81 - 4b^2 = 0$

19. $16d^2 = 4$

20. $18a^2 = 8$

21. $s^2 - \frac{9}{25} = 0$

22. $k^2 - \frac{49}{64} = 0$

23. $\frac{1}{25}h^2 - 16 = 0$

24. $\frac{1}{16}y^2 = 81$

9-5 Practice

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $k^2 - 100$

2. $81 - r^2$

3. $16p^2 - 36$

4. $4x^2 + 25$

5. $144 - 9f^2$

6. $36g^2 - 49h^2$

7. $121m^2 - 144n^2$

8. $32 - 8y^2$

9. $24a^2 - 54b^2$

10. $32s^2 - 18u^2$

11. $9d^2 - 32$

12. $36z^3 - 9z$

13. $45q^3 - 20q$

14. $100b^3 - 36b$

15. $3t^4 - 48t^2$

Solve each equation by factoring. Check your solutions.

16. $4y^2 = 81$

17. $64p^2 = 9$

18. $98b^2 - 50 = 0$

19. $32 - 162k^2 = 0$

20. $s^2 - \frac{64}{121} = 0$

21. $\frac{16}{49} - v^2 = 0$

22. $\frac{1}{36}x^2 - 25 = 0$

23. $27h^3 = 48h$

24. $75g^3 = 147g$

25. EROSION A rock breaks loose from a cliff and plunges toward the ground 400 feet below. The distance d that the rock falls in t seconds is given by the equation $d = 16t^2$. How long does it take the rock to hit the ground?

26. FORENSICS Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $\frac{1}{24}s^2 = d$, where s is the speed of the car and d is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes?

9-5 Reading to Learn Mathematics

Factoring Differences of Squares

Pre-Activity **How can you determine a basketball player's hang time?**

Read the introduction to Lesson 9-5 at the top of page 501 in your textbook.

Suppose a player can jump 2 feet. Can you use the pattern for the difference of squares to solve the equation $4t^2 - 2 = 0$? Explain.

Reading the Lesson

1. Explain why each binomial is a difference of squares.

a. $4x^2 - 25$

b. $49a^2 - 64b^2$

2. Sometimes it is necessary to apply more than one technique when factoring, or to apply the same technique more than once.

a. What should you look for first when you are factoring a binomial?

b. Explain what is done in each step to factor $4x^4 - 64$.

$$4x^4 - 64$$
$$= 4(x^4 - 16)$$
$$= 4[(x^2)^2 - 4^2]$$
$$= 4(x^2 + 4)(x^2 - 4)$$
$$= 4(x^2 + 4)(x^2 - 2^2)$$
$$= 4(x^2 + 4)(x + 2)(x - 2)$$

3. Suppose you are solving the equation $16x^2 - 9 = 0$ and rewrite it as $(4x + 3)(4x - 3) = 0$. What would be your next steps in solving the equation?

Helping You Remember

4. How can you remember whether a binomial can be factored as a difference of squares?

Lesson 9-5

9-5 Enrichment

Factoring Trinomials of Fourth Degree

Some trinomials of the form $a^4 + a^2b^2 + b^4$ can be written as the difference of two squares and then factored.

Example **Factor $4x^4 - 37x^2y^2 + 9y^4$.**

Step 1 Find the square roots of the first and last terms.

$\sqrt{4x^4} = 2x^2$ $\quad\quad$ $\sqrt{9y^4} = 3y^2$

Step 2 Find twice the product of the square roots.

$2(2x^2)(3y^2) = 12x^2y^2$

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

$-37x^2y^2 = -12x^2y^2 - 25x^2y^2$

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned} 4x^4 - 37x^2y^2 + 9y^4 &= (4x^4 - 12x^2y^2 + 9y^4) - 25x^2y^2 \\ &= (2x^2 - 3y^2)^2 - 25x^2y^2 \\ &= [(2x^2 - 3y^2) + 5xy][(2x^2 - 3y^2) - 5xy] \\ &= (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2) \end{aligned}$$

Factor each trinomial.

1. $x^4 + x^2y^2 + y^4$

2. $x^4 + x^2 + 1$

3. $9a^4 - 15a^2 + 1$

4. $16a^4 - 17a^2 + 1$

5. $4a^4 - 13a^2 + 1$

6. $9a^4 + 26a^2b^2 + 25b^4$

7. $4x^4 - 21x^2y^2 + 9y^4$

8. $4a^4 - 29a^2c^2 + 25c^4$

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9-6 Study Guide and Intervention

Perfect Squares and Factoring

Lesson 9-6

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a + 16 = a^2 + 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2 = (4n - 3)^2$$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$	Original trinomial
$= 16x^2 + mx + nx + 15$	Write the pattern.
$= 16x^2 - 12x - 20x + 15$	$m = -12$ and $n = -20$
$= (16x^2 - 12x) - (20x - 15)$	Group terms.
$= 4x(4x - 3) - 5(4x - 3)$	Find the GCF.
$= (4x - 5)(4x - 3)$	Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $x^2 - 16x + 64$

2. $m^2 + 10m + 25$

3. $p^2 + 8p + 64$

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$

5. $x^2 + 22x + 121$

6. $81 + 18s + s^2$

7. $25c^2 - 10c - 1$

8. $169 - 26r + r^2$

9. $7x^2 - 9x + 2$

10. $16m^2 + 48m + 36$

11. $16 - 25a^2$

12. $b^2 - 16b + 256$

13. $36x^2 - 12x + 1$

14. $16a^2 - 40ab + 25b^2$

15. $8m^3 - 64m$

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9-6 Study Guide and Intervention *(continued)*

Perfect Squares and Factoring

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **square root property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $\mathbf{x^2 - 6x + 9 = 0}$

$x^2 - 6x + 9 = 0$ Original equation

$x^2 - 2(3x) + 3^2 = 0$ Recognize a perfect square trinomial.

$(x - 3)(x - 3) = 0$ Factor the perfect square trinomial.

$x - 3 = 0$ Set repeated factor equal to 0.

$x = 3$ Solve.

The solution set is $\{3\}$. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $\mathbf{(a - 5)^2 = 64}$

$(a - 5)^2 = 64$ Original equation

$a - 5 = \pm\sqrt{64}$ Square Root Property

$a - 5 = \pm 8$ $64 = 8 \cdot 8$

$a = 5 \pm 8$ Add 5 to each side.

$a = 5 + 8$ or $a = 5 - 8$ Separate into 2 equations.

$a = 13$ $a = -3$ Solve each equation.

The solution set is $\{-3, 13\}$. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $x^2 + 4x + 4 = 0$

2. $16n^2 + 16n + 4 = 0$

3. $25d^2 - 10d + 1 = 0$

4. $x^2 + 10x + 25 = 0$

5. $9x^2 - 6x + 1 = 0$

6. $x^2 + x + \frac{1}{4} = 0$

7. $25k^2 + 20k + 4 = 0$

8. $p^2 + 2p + 1 = 49$

9. $x^2 + 4x + 4 = 64$

10. $x^2 - 6x + 9 = 25$

11. $a^2 + 8a + 16 = 1$

12. $16y^2 + 8y + 1 = 0$

13. $(x + 3)^2 = 49$

14. $(y + 6)^2 = 1$

15. $(m - 7)^2 = 49$

16. $(2x + 1)^2 = 1$

17. $(4x + 3)^2 = 25$

18. $(3h - 2)^2 = 4$

19. $(x + 1)^2 = 7$

20. $(y - 3)^2 = 6$

21. $(m - 2)^2 = 5$

9-6 Skills Practice

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $c^2 - 6c + 9$

2. $r^2 + 4r + 4$

3. $g^2 - 14g + 49$

4. $2w^2 - 4w + 9$

5. $4d^2 - 4d + 1$

6. $9n^2 + 30n + 25$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $2x^2 - 72$

8. $6b^2 + 11b + 3$

9. $36t^2 - 24t + 4$

10. $4h^2 - 56$

11. $17a^2 - 24ac$

12. $q^2 - 14q + 36$

13. $y^2 + 24y + 144$

14. $6d^2 - 96$

15. $4k^2 + 12k + 9$

16. $6x^2 + 28x - 10$

Solve each equation. Check your solutions.

17. $x^2 - 18x + 81 = 0$

18. $4p^2 + 4p + 1 = 0$

19. $9g^2 - 12g + 4 = 0$

20. $y^2 - 16y + 64 = 81$

21. $4n^2 - 17 = 19$

22. $x^2 + 30x + 150 = -75$

23. $(k + 2)^2 = 16$

24. $(m - 4)^2 = 7$

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9-6 Practice

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $m^2 + 16m + 64$

2. $9s^2 - 6s + 1$

3. $4y^2 - 20y + 25$

4. $16p^2 + 24p + 9$

5. $25b^2 - 4b + 16$

6. $49k^2 - 56k + 16$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $3p^2 - 147$

8. $6x^2 + 11x - 35$

9. $50q^2 - 60q + 18$

10. $6t^3 - 14t^2 - 12t$

11. $6d^2 - 18$

12. $30k^2 + 38k + 12$

13. $15b^2 - 24bc$

14. $12h^2 - 60h + 75$

15. $9n^2 - 30n - 25$

16. $7u^2 - 28m^2$

17. $w^4 - 8w^2 - 9$

18. $16c^2 + 72cd + 81d^2$

Solve each equation. Check your solutions.

19. $4k^2 - 28k = -49$

20. $50b^2 + 20b + 2 = 0$

21. $\left(\frac{1}{2}t - 1\right)^2 = 0$

22. $g^2 + \frac{2}{3}g + \frac{1}{9} = 0$

23. $p^2 - \frac{6}{5}p + \frac{9}{25} = 0$

24. $x^2 + 12x + 36 = 25$

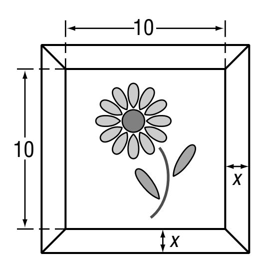
25. $y^2 - 8y + 16 = 64$

26. $(h + 9)^2 = 3$

27. $w^2 - 6w + 9 = 13$

28. GEOMETRY The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 1 inch gives the resulting circle an area of 100π square inches, what is the radius of the original circle?

29. PICTURE FRAMING Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 69 square inches. What is the width of the frame?



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9-6 Reading to Learn Mathematics

Perfect Squares and Factoring

Lesson 9-6

Pre-Activity **How can factoring be used to design a pavilion?**

Read the introduction to Lesson 9-6 at the top of page 508 in your textbook.

- On the left side of the equation $(8 + 2x)^2 = 144$, the number 8 in the expression $(8 + 2x)^2$ represents ______________________ and $2x$ represents twice ______________________.
- On the right side of the equation, the number 144 represents ______________________ in the center of the pavilion, plus the ____________ of the bricks surrounding the center mascot.

Reading the Lesson

1. Three conditions must be met if a trinomial can be factored as a ______________________. Complete the following sentences.

The first term of the trinomial $9x^2 - 6x + 1$ __________ (is/is not) a perfect square.

The last term of the trinomial, __________ (is/is not) a perfect square.

The ____________ is equal to $2(3x)(1)$.

The trinomial $9x^2 - 6x + 1$ __________ (is/is not) a ____________ trinomial.

2. Match each polynomial from the first column with a factoring technique in the second column. Some of the techniques may be used more than once. If none of the techniques can be used to factor the polynomial, write *none*.

a. $9x^2 - 64$

b. $9x^2 + 12x + 4$

c. $x^2 - 5x + 6$

d. $4x^2 + 13x + 9$

e. $9xy + 3y + 6x + 2$

f. $x^2 - 4x + 4$

g. $2x^2 - 16$

i. factor as $x^2 + bx + c$

ii. factor as $ax^2 + bx + c$

iii. difference of squares

iv. factoring by grouping

v. perfect square trinomial

vi. factor out the GCF

Helping You Remember

3. Sometimes it is easier to remember a set of instructions if you can state them in a short sentence or phrase. Summarize the conditions that must be met if a trinomial can be factored as a perfect square trinomial.

9-6 Enrichment

Squaring Numbers: A Shortcut

A shortcut helps you to square a positive two-digit number ending in 5. The method is developed using the idea that a two-digit number may be expressed as $10t + u$. Suppose $u = 5$.

$$(10t + 5)^2 = (10t + 5)(10t + 5)$$
$$= 100t^2 + 50t + 50t + 25$$
$$= 100t^2 + 100t + 25$$
$$(10t + 5)^2 = 100t(t + 1) + 25$$

In words, this formula says that the square of a two-digit number has $t(t + 1)$ in the hundreds place. Then 2 is the tens digit and 5 is the units digit.

Example **Using the formula for $(10t + 5)^2$, find 85^2.**

$$85^2 = 100 \cdot 8 \cdot (8 + 1) + 25$$
$$= 7200 + 25$$
$$= 7225$$

Shortcut: First think $8 \cdot 9 = 72$. Then write 25.

Thus, to square a number, such as 85, you can write the product of the tens digit and the next consecutive integer $t + 1$. Then write 25.

Find each of the following using the shortcut.

1. 15^2

2. 25^2

3. 35^2

4. 45^2

5. 55^2

6. 65^2

Solve each problem.

7. What is the tens digit in the square of 95?

8. What are the first two digits in the square of 75?

9. Any three-digit number can be written as $100a + 10b + c$. Square this expression to show that if the last digit of a three-digit number is 5 then the last two digits of the square of the number are 2 and 5.

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9 Chapter 9 Test, Form 1

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Find the prime factorization of 60.

A. $2 \cdot 5 \cdot 6$ **B.** $4 \cdot 15$ **C.** $2^2 \cdot 3 \cdot 5$ **D.** $6 \cdot 10$ **1.** ______

2. Factor $12x^3y$ completely.

A. $2 \cdot 3 \cdot x \cdot y$ **B.** $2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y$

C. $4 \cdot 3 \cdot x^3 \cdot y$ **D.** $12 \cdot x \cdot x \cdot x \cdot y$ **2.** ______

3. Find the GCF of $24a$ and $32b$.

A. 2 **B.** $6ab$ **C.** $4ab$ **D.** 8 **3.** ______

4. Which binomial is a factor of $2n^2 - 32n$?

A. $2n - 8$ **B.** $n + 16$ **C.** $n - 16$ **D.** $n + 4$ **4.** ______

5. Factor $xy + 3x - 2x^2$ completely.

A. $x(y + 3 - 2x)$ **B.** $(2x - 3y)(y + x)$

C. $x(y + 3) + 2x$ **D.** $y(x + 3x - 2x^2)$ **5.** ______

6. Solve $b(b + 17) = 0$.

A. $\left\{0, \frac{1}{17}\right\}$ **B.** $\{-17, 0\}$ **C.** $\{0, 17\}$ **D.** $\{17\}$ **6.** ______

For Questions 7–9, factor each trinomial.

7. $m^2 + 13m + 42$

A. $(m + 1)(m + 13)$ **B.** $(m + 6)(m + 7)$

C. $(m + 10)(m + 3)$ **D.** $(m - 6)(m - 7)$ **7.** ______

8. $y^2 - 8y + 15$

A. $(y - 2)(y - 6)$ **B.** $(y - 2)(y - 4)$

C. $(y - 1)(y - 15)$ **D.** $(y - 3)(y - 5)$ **8.** ______

9. $3m^2 + 14m - 5$

A. $(3m + 1)(m - 5)$ **B.** $(3m - 1)(m + 5)$

C. $(3m + 5)(m - 1)$ **D.** $(3m - 5)(m + 1)$ **9.** ______

10. Which binomial is a factor of $4x^2 - 13x + 3$?

A. $2x - 3$ **B.** $2x - 1$ **C.** $4x - 3$ **D.** $4x - 1$ **10.** ______

11. Solve $2x^2 - 5x - 3 = 0$.

A. $\left\{-\frac{1}{2}, 3\right\}$ **B.** $\left\{\frac{1}{2}, -3\right\}$ **C.** $\left\{\frac{1}{2}, 3\right\}$ **D.** $\left\{-\frac{1}{2}, -3\right\}$ **11.** ______

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9 Chapter 9 Test, Form 1 *(continued)*

For Questions 12 and 13, factor each polynomial, if possible. If the polynomial cannot be factored, choose *prime*.

12. $4m^2 - 25$

A. $(2m + 5)(2m + 5)$ **B.** $(2m + 5)(2m - 5)$
C. $(2m - 5)(2m - 5)$ **D.** prime **12.** ______

13. $x^2 + 16$

A. $(x + 4)(x + 4)$ **B.** $(x + 4)(x - 4)$
C. $(x - 4)(x - 4)$ **D.** prime **13.** ______

14. Solve $64y^2 = 25$ by factoring.

A. $\left\{\frac{8}{5}\right\}$ **B.** $\left\{\frac{5}{8}\right\}$ **C.** $\left\{-\frac{8}{5}, \frac{8}{5}\right\}$ **D.** $\left\{-\frac{5}{8}, \frac{5}{8}\right\}$ **14.** ______

15. Which trinomial is a perfect square trinomial?

A. $3x^2 - 6x + 9$ **B.** $x^2 + 10x + 25$
C. $x^2 + 8x - 16$ **D.** $x^2 + 12x - 36$ **15.** ______

16. Which binomial is a factor of $k^2 - 12k + 36$?

A. $k + 3$ **B.** $k + 4$ **C.** $k - 6$ **D.** $k - 12$ **16.** ______

17. Solve $x^2 - 16x + 64 = 0$.

A. $\{8\}$ **B.** $\{-8, 8\}$ **C.** $\{4\}$ **D.** $\{-4\}$ **17.** ______

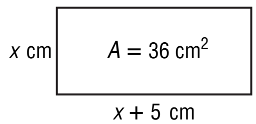
18. Solve $2x^2 + 12x = -18$.

A. $\{-3\}$ **B.** $\{3\}$ **C.** $\{-3, 3\}$ **D.** $\{-9\}$ **18.** ______

19. Solve $x^2 + 7x + 12 = 0$

A. $\{3, 4\}$ **B.** $\{-3, 4\}$ **C.** $\{-4, 3\}$ **D.** $\{-3, -4\}$ **19.** ______

20. **GEOMETRY** The length of a rectangle is 5 centimeters more than the width. The area of the rectangle is 36 square centimeters. What is the length?



A. 4 cm **B.** 9 cm **C.** 14 cm **D.** 26 cm **20.** ______

Bonus The sum of the squares of two consecutive odd integers is 74. Find the two integers. **B:** ____________

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the prime factorization of -116.

A. $-1 \cdot 2^2 \cdot 3^3$ **B.** $-4 \cdot 27$ **C.** $-1 \cdot 2^2 \cdot 29$ **D.** $-1 \cdot 2^4 \cdot 7$ **1.** ______

2. Factor $76u^3v^2$ completely.

A. $76 \cdot u \cdot u \cdot u \cdot v \cdot v$ **B.** $2 \cdot 19 \cdot u \cdot v$

C. $4 \cdot 19 \cdot u^3 \cdot v^2$ **D.** $2 \cdot 2 \cdot 19 \cdot u \cdot u \cdot u \cdot v \cdot v$ **2.** ______

3. Find the GCF of $45xy^2$ and $-60y$.

A. $5y^2$ **B.** $15y$ **C.** $180xy^2$ **D.** $30xy$ **3.** ______

4. Factor $24x^2y - 66xy^2 + 54x^2y^2$ completely.

A. $2xy(12x - 33y + 27xy)$ **B.** $6x^2y^2(4y - 11x + 9)$

C. $(4x^2 + 6y)(6x - 9y^2)$ **D.** $6xy(4x - 11y + 9xy)$ **4.** ______

Solve each equation.

5. $4x^2 - 16x = 0$

A. $\{4, -4\}$ **B.** $\{0, 2, -2\}$ **C.** $\{0, 4\}$ **D.** $\{16\}$ **5.** ______

6. $(3w + 4)(2w - 7) = 0$

A. $\left\{-\frac{3}{4}, \frac{2}{7}\right\}$ **B.** $\left\{\frac{3}{4}, -\frac{2}{7}\right\}$ **C.** $\left\{-\frac{4}{3}, \frac{7}{2}\right\}$ **D.** $\left\{\frac{4}{3}, -\frac{7}{2}\right\}$ **6.** ______

For Questions 7 and 8, factor each trinomial.

7. $x^2 - 10x + 9$

A. $(x - 1)(x - 9)$ **B.** $(x + 1)(x + 9)$

C. $(x - 1)(x + 9)$ **D.** $(x + 1)(x - 9)$ **7.** ______

8. $n^2 - 12n - 45$

A. $(n - 5)(n + 9)$ **B.** $(n - 3)(n + 15)$

C. $(n - 9)(n + 5)$ **D.** $(n + 3)(n - 15)$ **8.** ______

9. Solve $y^2 = 13y - 42$.

A. $\{-6, -7\}$ **B.** $\{6, 7\}$ **C.** $\{-6, 7\}$ **D.** $\{6, -7\}$ **9.** ______

10. Which binomial is a factor of $14a^2 - 15a + 4$?

A. $7a + 2$ **B.** $14a - 1$ **C.** $7a - 1$ **D.** $7a - 4$ **10.** ______

11. Factor $5x^2 - 13x + 6$.

A. $(x + 3)(5x - 2)$ **B.** $(x - 2)(5x - 3)$

C. $(x + 2)(5x + 3)$ **D.** $(x - 3)(5x + 2)$ **11.** ______

Chapter 9 Test, Form 2A *(continued)*

12. Solve $7x^2 - 20x = 3$.

A. $\left\{-\frac{1}{7}, 3\right\}$ **B.** $\left\{\frac{1}{7}, -3\right\}$ **C.** $\left\{-\frac{1}{7}, -3\right\}$ **D.** $\left\{\frac{1}{7}, 3\right\}$ **12.** ______

For Questions 13 and 14, factor each polynomial completely, if possible. If the polynomial cannot be factored, choose *prime*.

13. $121r^2 - 64t^2$

A. $(11r + 8t)(11r - 8t)$ **B.** $(11r - 8t)(11r - 8t)$

C. $(11r + 8t)(11r + 8t)$ **D.** prime **13.** ______

14. $3x^5 - 27x^3$

A. $3(x^2 - 3)(x^2 + 3)$ **B.** prime

C. $3x^3(x^2 - 9)$ **D.** $3x^3(x - 3)(x + 3)$ **14.** ______

15. Which trinomial is *not* a perfect square trinomial?

A. $4x^2 + 4x + 1$ **B.** $49x^2 - 56xy + 16y^2$

C. $x^2 + 10x - 25$ **D.** $25 - 30x + 9x^2$ **15.** ______

16. Which binomial is a factor of $6x^2 + 48x + 96$?

A. $x + 4$ **B.** $3x + 8$ **C.** $3x + 16$ **D.** $6x + 16$ **16.** ______

17. Solve $9x^2 = 16$.

A. $\left\{\frac{16}{9}\right\}$ **B.** $\left\{\pm\frac{4}{3}\right\}$ **C.** $\left\{\pm\frac{3}{4}\right\}$ **D.** $\left\{\pm\frac{16}{9}\right\}$ **17.** ______

18. **SOCCER** Julian kicked a soccer ball into the air with an initial upward velocity of 40 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 40t$, where t is the time in seconds after Julian kicked the ball. Find the time it takes the ball to reach 25 feet above the ground.

A. $2\frac{1}{2}$ s **B.** $\frac{15}{16}$ s **C.** $1\frac{1}{3}$ s **D.** $1\frac{1}{4}$ s **18.** ______

19. The product of two consecutive odd integers is 143. Find their sum.

A. −20 or 20 **B.** −28 or 28 **C.** −26 or 26 **D.** −24 or 24 **19.** ______

20. The length of a rectangle is twice the width. The area is 72 square centimeters. What is the length?

A. 48 cm **B.** 24 cm **C.** 12 cm **D.** 6 cm **20.** ______

Bonus Find the value of c that will make $9x^2 + 30x + c$ a perfect square trinomial. **B:** ______________

NAME ________________ DATE ________ PERIOD ______

9 Chapter 9 Test, Form 2B

SCORE ______

Assessment

Write the letter for the correct answer in the blank at the right of each question.

1. Find the prime factorization of 117.

A. $(-1)^2 \cdot 3 \cdot 13$ **B.** $2^2 \cdot 3 \cdot 13$ **C.** $3^3 \cdot 5$ **D.** $3^2 \cdot 13$ **1.** ______

2. Factor $126x^2y^3$ completely.

A. $7 \cdot 18 \cdot x^2 \cdot y^3$ **B.** $2 \cdot 3 \cdot 3 \cdot 7 \cdot x \cdot x \cdot y \cdot y \cdot y$

C. $126 \cdot x \cdot x \cdot y \cdot y \cdot y$ **D.** $2 \cdot 3 \cdot 7 \cdot x \cdot y$ **2.** ______

3. Find the GCF of $-42r^2$ and $63rs$.

A. $3r^2$ **B.** $21r$ **C.** $28rs$ **D.** $126r^2s$ **3.** ______

4. Factor $88a^2b^2 + 24a^2b - 32ab^2$ completely.

A. $8a^2b^2(11 + 3b - 4a)$ **B.** $2ab(44ab + 12a - 16b)$

C. $8ab(11ab + 3a - 4b)$ **D.** $(11a^2 + 8b)(8a - 4b^2)$ **4.** ______

Solve each equation.

5. $16x^2 - 64x = 0$

A. $\{-4, 4\}$ **B.** $\{0, 4\}$ **C.** $\{0, -2, 2\}$ **D.** $\{64\}$ **5.** ______

6. $(3m - 2)(9m + 5) = 0$

A. $\left\{\frac{3}{2}, \frac{9}{5}\right\}$ **B.** $\left\{\frac{2}{3}, -\frac{5}{9}\right\}$ **C.** $\left\{\frac{3}{2}, -\frac{9}{5}\right\}$ **D.** $\left\{-\frac{2}{3}, -\frac{5}{9}\right\}$ **6.** ______

For Questions 7 and 8, factor each trinomial.

7. $x^2 - 11x + 18$

A. $(x - 2)(x - 9)$ **B.** $(x + 2)(x - 9)$

C. $(x - 2)(x + 9)$ **D.** $(x + 2)(x + 9)$ **7.** ______

8. $p^2 + 6p - 40$

A. $(p + 2)(p - 20)$ **B.** $(p - 2)(p + 20)$

C. $(p + 4)(p - 10)$ **D.** $(p - 4)(p + 10)$ **8.** ______

9. Solve $y^2 = 15y - 56$.

A. $\{7, 8\}$ **B.** $\{-7, -8\}$ **C.** $\{-7, 8\}$ **D.** $\{7, -8\}$ **9.** ______

10. Which binomial is a factor of $6x^2 + x - 12$?

A. $3x - 3$ **B.** $3x - 4$ **C.** $6x + 3$ **D.** $x + 4$ **10.** ______

11. Factor $7x^2 - 16x + 4$.

A. $(x - 3)(7x + 5)$ **B.** $(x - 4)(7x + 1)$

C. $(x + 2)(7x - 4)$ **D.** $(x - 2)(7x - 2)$ **11.** ______

9 Chapter 9 Test, Form 2B *(continued)*

12. Solve $5x^2 + 13x = 6$.

A. $\left\{3, -\frac{2}{5}\right\}$ **B.** $\left\{3, \frac{2}{5}\right\}$ **C.** $\left\{-3, \frac{2}{5}\right\}$ **D.** $\left\{-3, -\frac{2}{5}\right\}$ **12.** ______

For Questions 13 and 14, factor each polynomial completely, if possible. If the polynomial cannot be factored, choose *prime*.

13. $196w^2 - 81z^2$

A. $(14w + 9z)(14w + 9z)$ **B.** $(14w - 9z)(14w - 9z)$
C. $(14w + 9z)(14w - 9z)$ **D.** prime **13.** ______

14. $8x^3 - 72xy^2$

A. $8x(x^2 - 9y^2)$ **B.** prime
C. $8(x^2 + 3y)(x - 3y)$ **D.** $8x(x + 3y)(x - 3y)$ **14.** ______

15. Which trinomial is *not* a perfect square trinomial?

A. $x^2 + 8x + 16$ **B.** $4x^2 - 20x + 25$
C. $9x^2 + 12x - 4$ **D.** $16 - 56x + 49x^2$ **15.** ______

16. Which binomial is a factor of $80y^2 - 120y + 45$?

A. $4y - 3$ **B.** $8y - 9$ **C.** $16y - 9$ **D.** $8y - 15$ **16.** ______

17. Solve $16x^2 = 25$.

A. $\left\{\frac{25}{16}\right\}$ **B.** $\left\{\pm\frac{5}{4}\right\}$ **C.** $\left\{\pm\frac{4}{5}\right\}$ **D.** $\left\{\pm\frac{25}{16}\right\}$ **17.** ______

18. GOLF Sisika hit a golf ball into the air with an initial upward velocity of 56 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 56t$, where t is the time in seconds after Sisika hit the ball. Find the time it takes the ball to reach 49 feet above the ground.

A. $3\frac{1}{2}$ s **B.** $\frac{7}{16}$ s **C.** $1\frac{3}{5}$ s **D.** $1\frac{3}{4}$ s **18.** ______

19. The product of two consecutive even integers is 224. Find their sum.

A. −30 or 30 **B.** −34 or 34 **C.** −26 or 26 **D.** −32 or 32 **19.** ______

20. The length of a rectangle is 5 times the width. The area is 125 square centimeters. What is the length?

A. 5 cm **B.** 25 cm **C.** 10 cm **D.** 30 cm **20.** ______

Bonus Factor $2n^4 - 10n^2 - 72$ completely. **B:** ____________

NAME ______________________ DATE ____________ PERIOD ________

9 Chapter 9 Test, Form 2C

SCORE ________

1. Find the factors of 48. Then classify 48 as *prime* or *composite*. **1.** ____________

2. Find the prime factorization of -96. **2.** ____________

Find the GCF of each set of monomials.

3. $12x^3y^2, 44xy^3$ **3.** ____________

4. $2ab, 5bc, 9ac$ **4.** ____________

Factor each polynomial.

5. $35a^3bc^2 - 45a^2b^2c$ **5.** ____________

6. $3xy - 4x + 6y - 8$ **6.** ____________

7. $t^2 - 11t + 24$ **7.** ____________

8. $n^2 + n - 42$ **8.** ____________

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

9. $10y^2 - 31y + 15$ **9.** ____________

10. $8n^2 - 36n + 40$ **10.** ____________

11. $36m^2 - 49$ **11.** ____________

12. $2x^4 - 18x^2$ **12.** ____________

13. $25w^2 - 60w + 36$ **13.** ____________

14. $9a^2 + 42a - 49$ **14.** ____________

Assessment

9 Chapter 9 Test, Form 2C *(continued)*

Solve each equation.

15. $(3n + 2)(n - 2) = 0$ **15.** ________

16. $16y^2 - 8y = 0$ **16.** ________

17. $x^2 = x + 110$ **17.** ________

18. $8n^2 + 4 = 12n$ **18.** ________

19. $36x^2 + 49 = 84x$ **19.** ________

20. $4y^2 + 16y + 7 = 0$ **20.** ________

For Questions 21 and 22, solve each equation by factoring.

21. $49w^2 - 25 = 0$ **21.** ________

22. $5d^3 - 80d = 0$ **22.** ________

23. **BASEBALL** Tonisha hit a baseball into the air with an initial upward velocity of 48 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 48t + 2$, where t is the time in seconds after Tonisha hit the baseball. Find the time it takes the ball to reach 38 feet above the ground. **23.** ________

24. The area of a rectangular room is 238 square feet. The width is 3 feet less than the length. Find the dimensions of the room. **24.** ________

25. One number is 5 times another number. The product of the two numbers is 245. Find the two numbers. **25.** ________

Bonus Factor $v^2x^2 - 9x^2 + v^2n^2 - 9n^2$ completely. **B:** ________

NAME ______________________ DATE __________ PERIOD ______

9 Chapter 9 Test, Form 2D

SCORE ______

Assessment

1. Find the factors of 140. Then classify 140 as *prime* or *composite*. 1. ______

2. Find the prime factorization of −84. 2. ______

Find the GCF of each set of monomials.

3. $9x^2y^3$, $6xy^3$ 3. ______

4. $5ab$, $6ac$, $7bc$ 4. ______

Factor each polynomial.

5. $10x^2yz - 22x^3y^2z$ 5. ______

6. $2xy - 4x + 3y - 6$ 6. ______

7. $m^2 + 12m - 28$ 7. ______

8. $r^2 - r - 56$ 8. ______

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

9. $5t^2 + 17t - 12$ 9. ______

10. $6p^2 - 20p + 16$ 10. ______

11. $49a^2 - 169$ 11. ______

12. $3x^5 - 75x^3$ 12. ______

13. $81c^2 + 72c + 16$ 13. ______

14. $25x^2 + 70x - 49$ 14. ______

9 Chapter 9 Test, Form 2D *(continued)*

Solve each equation. Check your solutions.

15. $(x + 5)(4x - 3) = 0$ **15.** ________

16. $12b^2 - 8b = 0$ **16.** ________

17. $y^2 + 4y = 45$ **17.** ________

18. $9n^2 + 6n = 3$ **18.** ________

19. $25x^2 + 81 = 90x$ **19.** ________

20. $4b^2 - 8b - 5 = 0$ **20.** ________

For Questions 21 and 22, solve each equation by factoring. Check your solutions.

21. $64x^2 - 1 = 0$ **21.** ________

22. $5c^3 - 45c = 0$ **22.** ________

23. **VOLLEYBALL** Lanu hit a volleyball into the air with an initial upward velocity of 24 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 24t + 3$, where t is the time in seconds after Lanu hit the volleyball. Find the time it takes the ball to reach 12 feet above the ground. **23.** ________

24. The area of a rectangular room is 104 square feet. The length of the room is 5 feet longer than the width. Find the dimensions of the room. **24.** ________

25. One number is 7 times another number. The product of the two numbers is 252. Find the two numbers. **25.** ________

Bonus Find the value of c that will make $25x^2 - 40x + c$ a perfect square trinomial. **B:** ________

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9 Chapter 9 Test, Form 3

SCORE ________

1. Find the factors of 168. Then classify 168 as *prime* or *composite*. **1.** ________

2. Find the prime factorization of -1386. **2.** ________

Find the GCF of each set of monomials.

3. $276n^6y^2t$, $348n^3t^4$ **3.** ________

4. $132x^3y^2$, $126x^2z^3$, $15xyz^2$ **4.** ________

Factor each polynomial.

5. $12x^3y^2z - 24x^2y^3z + 16x^2y^3z^3$ **5.** ________

6. $4x^2y^2 - 9y^2 - 45 + 20x^2$ **6.** ________

7. $-x^2 + 5x + 24$ **7.** ________

8. $-18x + 2x^2 - 72$ **8.** ________

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

9. $10x^2 + 29x - 21$ **9.** ________

10. $3p^2 - 14p + 12$ **10.** ________

11. $(a + n)^2 - 9$ **11.** ________

12. $3x^3 - 24x^2y + 48xy^2$ **12.** ________

13. $3x^4 - 73x^2 - 50$ **13.** ________

14. $25x^2 + 20xy - 4y^2$ **14.** ________

Assessment

Chapter 9 Test, Form 3 *(continued)*

For Questions 15–19, solve each equation.

15. $6x^2 = 22x$ **15.** ________

16. $x^2 + 14x - 32 = 0$ **16.** ________

17. $x^2 + \frac{8x}{3} = -\frac{7}{9}$ **17.** ________

18. $(3x + 2)(x - 1) = x + 3$ **18.** ________

19. $a^2 - \frac{11}{2}a + \frac{121}{16} = 0$ **19.** ________

20. Solve $(2x - 3)^2 - 25 = 0$ by factoring. Check your solution. **20.** ________

21. Find all values of k so that $t^2 + kt - 8$ can be factored using integers. **21.** ________

22. Find an expression for c that will make $9x^2 + 12xy + c$ a perfect square trinomial. **22.** ________

23. The product of the least and greatest of three consecutive negative odd integers is 221. Find the three integers. **23.** ________

24. **TENNIS** Josefina hit a tennis ball into the air with an initial upward velocity of 16 feet per second. The height h in feet of the ball above the ground can be modeled by $h = -16t^2 + 16t + 3$, where t is the time in seconds after Josefina hit the tennis ball. Find the time it takes the ball to reach 7 feet above the ground. **24.** ________

25. **FIBERS** The basic breaking strength b in pounds for a natural fiber line is determined by the formula $900c^2 = b$, where c is the circumference of the line in inches. What circumference of natural line would have 100 pounds of breaking strength? **25.** ________

Bonus Solve the equation, and check your solutions.
$9t^3 + 15t^2 + t - 6 = (t + 3)(t - 2) - 3t^3$ **B:** ________

NAME ______________________________ DATE ____________ PERIOD ________

9 Chapter 9 Open-Ended Assessment

SCORE ________

Assessment

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Theo wants to build a rectangular deck on the front of his family's café, and he wants the deck to have an area of A square feet. If the length of the deck is b feet longer than the width, then the equation $A = x(x + b)$ gives the area of the deck in terms of the width x.

a. Explain the differences and similarities between $A = x(x + b)$ and $x^2 + bx - A$.

b. What must be true about $-A$ and b so that $x^2 + bx - A$ is factorable?

c. Choose values for A and b so that $x^2 + bx - A$ is factorable. Then, substitute your values into $x^2 + bx - A$ and factor the resulting polynomial.

d. Substitute the values you chose for A and b in part **c** into $A = x(x + b)$, and determine the area of the deck corresponding to these values and the dimensions of the deck.

2. A ball thrown horizontally has an initial upward velocity of 0 feet per second. The height h, in feet, of the ball above the ground is modeled by the equation $h = c - 16t^2$ where c is the height at which the ball is thrown and t is the time in seconds after the ball is thrown.

a. Compare possible values of c and h, and determine if h will ever be greater than c. Explain your reasoning.

b. Choose a value for c that is a perfect square, and determine how long it takes for the ball to hit the ground using your value for c.

c. Can a ball thrown horizontally at a height of 9 feet stay above the ground for more than 1 second? Explain your reasoning.

3. One way to factor a trinomial such as $x^2 - 2x - 3$ is to assign a value such as 10 to x and evaluate the expression.

$$\begin{aligned} x^2 - 2x - 3 &= 10^2 - 2(10) - 3 \\ &= 100 - 20 - 3 \\ &= 77 \end{aligned}$$

A factorization of 77 is 7×11. Since $x = 10$, $7 = x - 3$ and $11 = x + 1$. Multiply to see if $(x - 3)(x + 1) = x^2 - 2x - 3$.

a. Try the method above to factor $x^2 - 8x + 15$. Show your work and explain each step.

b. Try the method above to factor $2x^2 - 13x - 24$. (*Hint*: The factors will be of the form $(2x + a)(x - b)$. Show your work and explain each step.

c. Try the method above to factor $x^2 - 2x - 8$. Why is it difficult to find the correct factors for this trinomial using the method above?

d. Evaluate the expression in part **c** for $x = 5$. Then use the method above to find the factors. Explain each step. Remember in finding your factors that $x = 5$.

NAME ______________________________ DATE ____________ PERIOD ______

9

Chapter 9 Vocabulary Test/Review

SCORE ______

composite number	greatest common factor (GCF)	prime polynomial
factored form	perfect square trinomials	Square Root Property
factoring	prime factorization	Zero Product Property
factoring by grouping	prime number	

Write the letter of the term that best matches each statement or each expression.

______ **1.** $x^2 + 8x + 16$

______ **2.** You can apply this property to solve the equation $(x + 2)^2 = 16$.

______ **3.** $x^2 + 9x + 4$

______ **4.** a whole number that is expressed as the product of factors that are all prime numbers

______ **5.** You can often use this factoring technique if a polynomial has 4 or more terms.

______ **6.** the property you would use to solve the equation $(x + 2)(x - 2) = 0$

______ **7.** the number 14

______ **8.** a monomial that is expressed as the product of prime numbers and variables, with no variable having an exponent greater than 1

______ **9.** the number 5

a. composite number

b. factored form

c. factoring by grouping

d. perfect square trinomial

e. prime factorization

f. prime number

g. prime polynomial

h. Square Root Property

i. Zero Product Property

In your own words—
Define each term.

10. factoring

11. greatest common factor

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Quiz

SCORE ______

(Lessons 9–1 and 9–2)

1. Find the factors of 175. Then classify 175 as *prime* or *composite*. 1. ______
2. Find the prime factorization of −160. 2. ______
3. Factor $33a^3b^2$ completely. 3. ______

Find the GCF of each set of monomials.

4. 12, 90 4. ______
5. $20xy$, $48xy^2$ 5. ______

Factor each polynomial completely.

6. $48a^2b^2 - 12ab$ 6. ______
7. $6x^2y - 21y^2w + 24xw$ 7. ______
8. $xy - 2xz + 5y - 10z$ 8. ______

Solve each equation.

9. $(y + 4)(3y - 5) = 0$ 9. ______
10. $y^2 = -11y$ 10. ______

Assessment

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Quiz

SCORE ______

(Lesson 9–3)

Factor each trinomial completely.

1. $a^2 - 10a + 21$ 1. ______
2. $x^2 + 9x + 20$ 2. ______

For Questions 3 and 4, solve each equation.

3. $x^2 - 6x - 27 = 0$ 3. ______
4. $y^2 + 23y = 24$ 4. ______
5. Find two consecutive odd integers whose product is 195. 5. ______

NAME ______________________ DATE ____________ PERIOD ________

9 Chapter 9 Quiz

(Lessons 9–4 and 9–5)

SCORE ________

Factor each trinomial completely, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2x^2 + 7x + 3$ **2.** $6x^2 + x + 2$

Solve each equation.

3. $3n^2 + 6 = 11n$ **4.** $10x^2 + 11x - 6 = 0$

Factor each polynomial completely, if possible. If the polynomial cannot be factored write *prime*.

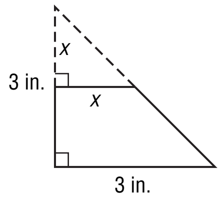
5. $a^2 - 25$ **6.** $49x^2 - 64y^2$

7. $x^3 + 3x^2 - 4x - 12$

For Questions 8 and 9, solve each equation by factoring.

8. $16x^2 = 81$ **9.** $\frac{4}{9}p^2 - 25 = 0$

10. Standardized Test Practice A corner is cut off a right triangle whose legs each measure 3 inches. The cut is x inches from the vertex and parallel to the opposite leg.



a. Write an equation in terms of x that represents the area A of the figure after the corner is removed.

b. What value of x will result in an area that is $\frac{3}{4}$ the area of the original triangle? Show how you arrived at your answer.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10a. ____________
b. ____________

NAME ______________________ DATE ____________ PERIOD ________

9 Chapter 9 Quiz

(Lesson 9–6)

SCORE ________

For Questions 1 and 2, determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $a^2 + 14a + 49$ **2.** $9z^2 - 3z + 1$

3. Factor $8m^3 - 24m^2 + 18m$ completely if possible. If it cannot be factored, write *prime*.

Solve each equation.

4. $16r^2 - 8r + 1 = 0$

5. $x^2 + 6x + 9 = 49$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

NAME ______________________ DATE __________ PERIOD ______

9

Chapter 9 Mid-Chapter Test

SCORE ______

(Lessons 9–1 through 9–3)

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

1. Find the prime factorization of 168.

A. $2^2 \cdot 3^2 \cdot 7$ **B.** $(-1)^2 \cdot 8 \cdot 21$ **C.** $2^3 \cdot 3 \cdot 7$ **D.** $2 \cdot 4 \cdot 21$ **1.** ______

2. Find the GCF of $72m^2n^2$ and $32m^3$.

A. $2m$ **B.** $288m^3n^2$ **C.** $4m^3$ **D.** $8m^2$ **2.** ______

3. Factor $75b^2c^3 + 60bc^6 - 35b^2c^4$ completely.

A. $5b^2c^6(15c^3 + 12b - 7c^2)$ **B.** $5bc^3(15b + 12c^3 - 7bc)$

C. $15bc(5bc^2 + 4c^5 - 7bc^3)$ **D.** $bc^3(75b + 60c^3 - 35bc)$ **3.** ______

4. Solve $3x(2x + 1) = 0$.

A. $\left\{-\frac{1}{2}, 0\right\}$ **B.** $\left\{\frac{1}{2}\right\}$ **C.** $\{-2, 0\}$ **D.** $\left\{0, \frac{1}{2}\right\}$ **4.** ______

5. Which binomial is a factor of $a^2 + 7a - 30$?

A. $a + 7$ **B.** $a - 6$ **C.** $3a + 10$ **D.** $a - 3$ **5.** ______

6. Factor $x^2 - 15x - 54$.

A. $(x - 18)(x + 3)$ **B.** $(x - 9)(x + 6)$

C. $(x - 3)(x - 18)$ **D.** $(x - 3)(x + 18)$ **6.** ______

7. Solve $y^2 - 28 = 3y$.

A. $\{0, 4\}$ **B.** $\{-5, 5\}$ **C.** $\{-4, 7\}$ **D.** $\{0, 3, -28\}$ **7.** ______

Part II

8. Find the factors of 136. Then classify 136 as *prime* or *composite*. **8.** ______________

9. CLASS SIZE Ms. Cedeño teaches three Algebra 1 classes. The enrollment in her classes is 28 students, 24 students, and 32 students. She wants the students to take the next quiz in groups, and she wants the group size to be the same in each of her three classes. What is the largest group size Ms. Cedeño can use and have all groups in each of the three classes be the same size? **9.** ______________

10. ______________

Factor each polynomial. **11.** ______________

10. $36xy^2 - 48x^2y$ **11.** $e^2 - 16e + 48$ **12.** $2xy - x + 4y - 2$ **12.** ______________

Solve each equation. **13.** ______________

13. $(3x - 7)(x + 5) = 0$ **14.** $w^2 = 16w$ **14.** ______________

15. $b^2 + 6b - 16 = 0$ **15.** ______________

Assessment

NAME ______________________ DATE ____________ PERIOD ______

Chapter 9 Cumulative Review

(Chapters 1–9)

1. Find the total price.
calculator: \$90 tax: 8%
(Lesson 3-7)

1. ____________

2. Solve $x - 2y = 6$ if the domain is $\{-2, 0, 2, 6, 8\}$.
(Lesson 4-4)

2. ____________

3. One section of a Brazilian shoreline receded 80 feet from 1985 to 1995. What was the rate of change for the shoreline?
(**Source:** *Time*) (Lesson 5-1)

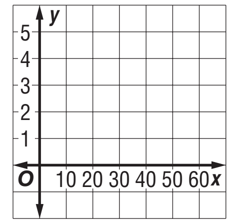
3. ____________

4. Solve $\frac{n}{7} < -6$. (Lesson 6-2)

4. ____________

5. Ayani needs to buy baseballs and baseball bats for his daughter's team. Baseballs cost \$5 each, and baseball bats cost \$80 each. Ayani doesn't want to spend more than \$350. Write an inequality that represents this situation and graph the solution set. (Lesson 6-6)

5. ____________



6. Determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions.
$3x - 4y = -12$ (Lesson 7-1)
$x + 4 = \frac{4}{3}y$

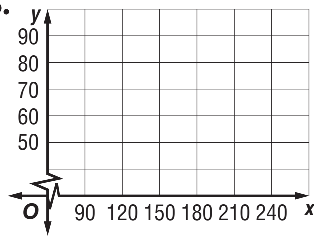
6. ____________

7. Use elimination to solve the system of equations.
$6x - 3y = 11$ (Lesson 7-3)
$6x + 3y = 17$

7. ____________

8. To enlist into the U.S. Army, a woman must have a height of 58 to 80 inches and a weight of 90 to 227 pounds. Graph the height and weight requirements for female enlistment into the army. (Lesson 7-5)

8.



9. Simplify $(-2x^2y^3)^4$. (Lesson 8-1)

9. ____________

10. Simplify $5(2y^2 + 3y - 2) + 8y(3y^2 + 4y - 2)$. (Lesson 8-6)

10. ____________

11. Simplify $(2x + 1)(x^2 - 3x - 4)$. (Lesson 8-7)

11. ____________

12. Find the prime factorization of 78. (Lesson 9-1)

12. ____________

13. Factor $12a^2b^2 - 16a^2b^3$. (Lesson 9-2)

13. ____________

14. Solve $5x^2 - 6x + 1 = 0$. (Lesson 9-4)

14. ____________

15. Factor $4x^2 - 49y^2$. (Lesson 9-5)

15. ____________

NAME ______________________ DATE ____________ PERIOD ______

9 Standardized Test Practice

(Chapters 1–9)

Assessment

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. Two dice are rolled. Find the odds that their sum is less than six. (Lesson 2-6)

A. 4 : 7 **B.** 10 : 26 **C.** 4 : 11 **D.** 10 : 36

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which is an equation for the line that passes through $(-2, 7)$ and $(3, -8)$? (Lesson 5-4)

E. $y = -3x + 1$ **F.** $y = -\frac{1}{3}x + 8$

G. $y = -\frac{1}{3}x + \frac{19}{3}$ **H.** $y = -3x + 13$

2. Ⓔ Ⓕ Ⓖ Ⓗ

3. Solve $24 + x < 18$. (Lesson 6-1)

A. $\{x \mid x < 42\}$ **B.** $\left\{x \,\middle|\, x < \frac{3}{4}\right\}$

C. $\{x \mid x > 6\}$ **D.** $\{x \mid x < -6\}$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Half the perimeter of a garden is 18 feet. The garden is 8 feet longer than it is wide. How wide is the garden? (Lesson 7-2)

E. 5 ft **F.** 40 ft^2 **G.** 36 ft **H.** 8 ft

4. Ⓔ Ⓕ Ⓖ Ⓗ

5. Arrange the terms of $4xy^2 - 5x^2y + 2y^3 + x^3$ so that the powers of x are in descending order. (Lesson 8-4)

A. $2y^3 + 4xy^2 - 5x^2y + x^3$ **B.** $-5x^2y + 4xy^2 + 2y^3 + x^3$

C. $x^3 - 5x^2y + 4xy^2 + 2y^3$ **D.** $x^3 + 2y^3 + 4xy^2 - 5x^2y$

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Find $(3g^2 - g + 2k) + (8k - 5g^2 + 7g)$. (Lesson 8-5)

E. $11g^2 - 6g + 9k$ **F.** $8g^2 - 8g - 6k$

G. $-6g^2 + 9g + 11k$ **H.** $-2g^2 + 6g + 10k$

6. Ⓔ Ⓕ Ⓖ Ⓗ

7. Find $(2x - 1)(3x + 2)$. (Lesson 8-7)

A. $5x^2 + 4x - 2$ **B.** $6x^2 + x - 2$

C. $6x^2 - 3x + 2$ **D.** $5x^2 + x + 1$

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Solve $8x^2 - 6x = 0$. (Lesson 9-2)

E. $\left\{0, \frac{3}{4}\right\}$ **F.** $\left\{0, \frac{4}{3}\right\}$ **G.** $\{6, 8\}$ **H.** $\left\{0, \frac{3}{4}, 2\right\}$

8. Ⓔ Ⓕ Ⓖ Ⓗ

9. Solve $p^2 - 10p = -21$. (Lesson 9-3)

A. $\{-3, 10\}$ **B.** $\{4, -7\}$ **C.** $\{5\}$ **D.** $\{3, 7\}$

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Each football game begins with a kickoff. The formula $h = -16t^2 + 64t$, where h is the height in feet of the football at t seconds, can be used to model a kickoff that is in the air for 4 seconds. At what times will the kickoff be 48 feet above the ground? (Lesson 9-4)

E. 1 s **F.** 1 s, 3 s

G. 2 s **H.** 1.5 s, 2.5 s

10. Ⓔ Ⓕ Ⓖ Ⓗ

Standardized Test Practice *(continued)*

Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

11. Solve $2(v + 3) - 26 = 7(1 - v)$. (Lesson 3-5)

12. Find the sum of the next three terms in the sequence. 2, 1, 4, 3, 6, 5, … . (Lesson 4-8)

11. ____ **12.** ____

13. Solve $5(x - 2) + 4x = 3(2x - 1)$. (Lesson 8-6)

14. Find the GCF of 56, 84, and 140. (Lesson 9-1)

13. ____ **14.** ____

Part 3: Quantitative Comparison

Instructions: Each question consists of two quantities in boxes, one in Column A and one in Column B. Compare the two quantities and fill in oval

Ⓐ if the quantity in column A is greater;
Ⓑ if the quantity in column B is greater;
Ⓒ if the quantities are equal; or
Ⓓ if the relationship cannot be determined from the information given.

	Column A	Column B	
15.	Given: $8t + 1 = 5t - 11$.		**15.** Ⓐ Ⓑ Ⓒ Ⓓ
	t	$-3t$	
	(Lessons 2-3 and 3-5)		
16.	the slope of the line that passes through $(-2, -8)$ and $(3, -5)$	the slope of any parallel line to the graph of $4x - 7y = 10$	**16.** Ⓐ Ⓑ Ⓒ Ⓓ
	(Lessons 5-1 and 5-6)		
17.	the value of k that makes $x^2 - 12x + k$ a perfect square trinomial	-36	**17.** Ⓐ Ⓑ Ⓒ Ⓓ
	(Lesson 9-6)		

NAME ______________________ DATE __________ PERIOD ______

9 Standardized Test Practice

Student Record Sheet *(Use with pages 520–521 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Question 12, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 ______________

11 ______________

12 ______________ (grid in)

13 ______________

14 ______________

15 ______________

16 ______________

17 ______________

12

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

18 Ⓐ Ⓑ Ⓒ Ⓓ
19 Ⓐ Ⓑ Ⓒ Ⓓ
20 Ⓐ Ⓑ Ⓒ Ⓓ
21 Ⓐ Ⓑ Ⓒ Ⓓ
22 Ⓐ Ⓑ Ⓒ Ⓓ

Part 4 *Open-Ended*

Record your answers for Question 23 on the back of this paper.

Answers

NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention

Factors and Greatest Common Factors

Prime Factorization When two or more numbers are multiplied, each number is called a **factor** of the product.

	Definition	Example
Prime Number	A prime number is a whole number, greater than 1, whose only factors are 1 and itself.	5
Composite Number	A composite number is a whole number, greater than 1, that has more than two factors.	10
Prime Factorization	Prime factorization occurs when a whole number is expressed as a product of factors that are all prime numbers.	$45 = 3^2 \cdot 5$

Example 1 **Factor each number. Then classify each number as *prime* or *composite*.**

a. 28

To find the factors of 28, list all pairs of whole numbers whose product is 28.

$1 \times 28 \quad 2 \times 14 \quad 4 \times 7$

Therefore, the factors of 28 are 1, 2, 4, 7, 14, and 28. Since 28 has more than 2 factors, it is a composite number.

b. 31

To find the factors of 31, list all pairs of whole numbers whose product is 31.

1×31

Therefore, the factors of 31 are 1 and 31. Since the only factors of 31 are itself and 1, it is a prime number.

Example 2 **Find the prime factorization of 200.**

Method 1

$200 = 2 \cdot 100$

$= 2 \cdot 2 \cdot 50$

$= 2 \cdot 2 \cdot 2 \cdot 25$

$= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

All the factors in the last row are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Method 2

Use a factor tree.

200

2 · 100

2 · 10 · 10

2 · 2 · 5 · 2 · 5

All of the factors in each last branch of the factor tree are prime, so the prime factorization of 200 is $2^3 \cdot 5^2$.

Exercises

Find the factors of each number. Then classify the number as *prime* or *composite*.

1. 41 **1, 41; prime**
2. 121 **1, 11, 121; composite**
3. 90 **1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90; composite**
4. 2865 **1, 3, 5, 15, 191, 573, 955, 2865; composite**

Find the prime factorization of each integer.

5. 600 $\mathbf{2^3 \cdot 3 \cdot 5^2}$
6. 175 $\mathbf{5^2 \cdot 7}$
7. -150 $\mathbf{-1 \cdot 2 \cdot 3 \cdot 5^2}$

Factor each monomial completely.

8. $32x^2$ $\mathbf{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x}$
9. $18m^2n$ $\mathbf{2 \cdot 3 \cdot 3 \cdot m \cdot m \cdot n}$
10. $49a^3b^2$ $\mathbf{7 \cdot 7 \cdot a \cdot a \cdot a \cdot b \cdot b}$

Lesson 9-1

NAME ______________________ DATE ________ PERIOD _____

9-1 Study Guide and Intervention *(continued)*

Factors and Greatest Common Factors

Greatest Common Factor

Greatest Common Factor (GCF)	
Integers	the greatest number that is a factor of all the integers
Monomials	the product of their common factors when each monomial is expressed in factored form

If two or more integers or monomials have no common prime factors, their GCF is 1 and the integers or monomials are said to be **relatively prime**.

Example **Find the GCF of each set of monomials.**

a. 12 and 18

$12 = (2) \cdot 2 \cdot (3)$ Factor each number.

$18 = (2) \cdot (3) \cdot 3$ Circle the common prime factors, if any.

The GCF of 12 and 18 is $2 \cdot 3$ or 6.

b. $16xy^2z^2$ and $72xyz^3$

$16xy^2z^2 = (2) \cdot (2) \cdot (2) \cdot 2 \cdot (x) \cdot (y) \cdot y \cdot (z) \cdot (z)$

$72xyz^3 = (2) \cdot (2) \cdot (2) \cdot 3 \cdot 3 \cdot (x) \cdot (y) \cdot (z) \cdot (z) \cdot z$

The GCF of $16xy^2z^2$ and $72xyz^3$ is $2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot z \cdot z$ or $8xyz^2$.

Exercises

Find the GCF of each set of monomials.

1. 12, 48 **12**
2. 18, 42 **6**
3. 64, 80 **16**
4. 32, 54 **2**
5. 27, 32 **1**
6. 44, 100 **4**
7. 45, 15 **15**
8. 169, 13 **13**
9. 20, 440 **20**
10. $49x$, $343x^2$ $\mathbf{49x}$
11. $4a^7b$, $28ab$ $\mathbf{4ab}$
12. $96y$, $12x$, $8y$ **4**
13. $12a$, $18abc$ $\mathbf{6a}$
14. $28y^2$, $35xy$, $49x^2yz$ $\mathbf{7y}$
15. $2m^2n$, $12mn^2$, $18mn$ $\mathbf{2mn}$
16. $12x^2$, $32x^2yz$, $60xy^2$ $\mathbf{4x}$
17. $18a^3b^2$, $36a^3b^2$ $\mathbf{18a^3b^2}$
18. $15mn^2$, $30m^3n^2$, $90m^3$ $\mathbf{15m}$
19. $2x^2y$, $9x^2y^3$, $18xy^2$ $\mathbf{xy}$
20. a^4b, $8a^3b^2$ $\mathbf{a^3b}$
21. ab^2, $5a^4b^2$, $10b^3$ $\mathbf{b^2}$

NAME ______________________ DATE ____________ PERIOD _____

9-1 Skills Practice

Factors and Greatest Common Factors

Find the factors of each number. Then classify each number as *prime* or *composite*.

1. 10 1, 2, 5, 10; composite

2. 31 1, 31; prime

3. 16 1, 2, 4, 8, 16; composite

4. 52 1, 2, 4, 13, 26, 52; composite

5. 38 1, 2, 19, 38; composite

6. 105 1, 3, 5, 7, 15, 21, 35, 105; composite

Find the prime factorization of each integer.

7. −16 $-1 \cdot 2^4$

8. 20 $2^2 \cdot 5$

9. 24 $2^3 \cdot 3$

10. 36 $2^2 \cdot 3^2$

11. 112 $2^4 \cdot 7$

12. −72 $-1 \cdot 2^3 \cdot 3^2$

Factor each monomial completely.

13. $10a^4$ $2 \cdot 5 \cdot a \cdot a \cdot a \cdot a$

14. $-27x^3y^2$ $-1 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$

15. $28pq^2$ $2 \cdot 2 \cdot 7 \cdot p \cdot q \cdot q$

16. $44m^2ns^3$ $2 \cdot 2 \cdot 11 \cdot m \cdot m \cdot n \cdot s \cdot s \cdot s$

Find the GCF of each set of monomials.

17. 12, 18 6

18. 20, 27 1

19. 30, 48 6

20. 24, 81 3

21. 20, 36, 64 4

22. 42, 60, 78 6

23. $16c$, $21b^2d$ 1

24. $18a$, $48a^4$ $6a$

25. $32xyz$, $48xy^4$ $16xy$

26. $12m^3n^2$, $44mn^3$ $4mn^2$

NAME ______________________ DATE ____________ PERIOD _____

9-1 Practice (Average)

Factors and Greatest Common Factors

Find the factors of each number. Then classify each number as *prime* or *composite*.

1. 18 1, 2, 3, 6, 9, 18; composite

2. 37 1, 37; prime

3. 48 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; composite

4. 116 1, 2, 4, 29, 58, 116; composite

5. 138 1, 2, 3, 6, 23, 46, 69, 138; composite

6. 211 1, 211; prime

Find the prime factorization of each integer.

7. 52 $2^2 \cdot 13$

8. −96 $-1 \cdot 2^5 \cdot 3$

9. 108 $2^2 \cdot 3^3$

10. 225 $3^2 \cdot 5^2$

11. 286 $2 \cdot 11 \cdot 13$

12. −384 $-1 \cdot 2^7 \cdot 3$

Factor each monomial completely.

13. $30d^5$ $2 \cdot 3 \cdot 5 \cdot d \cdot d \cdot d \cdot d \cdot d$

14. $-72mn$ $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot m \cdot n$

15. $81b^2c^3$ $3 \cdot 3 \cdot 3 \cdot 3 \cdot b \cdot b \cdot c \cdot c \cdot c$

16. $145abc^3$ $5 \cdot 29 \cdot a \cdot b \cdot c \cdot c \cdot c$

17. $168pq^2r$ $2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 \cdot p \cdot q \cdot q \cdot r$

18. $-121x^2yz^2$ $-1 \cdot 11 \cdot 11 \cdot x \cdot x \cdot y \cdot z \cdot z$

Find the GCF of each set of monomials.

19. 18, 49 1

20. 18, 45, 63 9

21. 16, 24, 48 8

22. 12, 30, 114 6

23. 9, 27, 77 1

24. 24, 72, 108 12

25. $24fg^5$, $56f^3g$ $8fg$

26. $72r^2s^2$, $36rs^3$ $36rs^2$

27. $15a^2b$, $35ab^2$ $5ab$

28. $28m^3n^2$, $45pq^2$ 1

29. $40xy^2$, $56x^3y^2$, $124x^2y^3$ $4xy^2$

30. $88c^3d$, $40c^2d^2$, $32c^2d$ $8c^2d$

GEOMETRY For Exercises 31 and 32, use the following information.
The area of a rectangle is 84 square inches. Its length and width are both whole numbers.

31. What is the minimum perimeter of the rectangle? 38 in.

32. What is the maximum perimeter of the rectangle? 170 in.

RENOVATION For Exercises 33 and 34, use the following information.
Ms. Baxter wants to tile a wall to serve as a splashguard above a basin in the basement. She plans to use equal-sized tiles to cover an area that measures 48 inches by 36 inches.

33. What is the maximum-size square tile Ms. Baxter can use and not have to cut any of the tiles? 12-in. square

34. How many tiles of this size will she need? 12

NAME ______________________ DATE __________ PERIOD _____

9-1 Reading to Learn Mathematics

Factors and Greatest Common Factors

Pre-Activity **How are prime numbers related to the search for extraterrestrial life?**

Read the introduction to Lesson 9-1 at the top of page 474 in your textbook.

If each "beep" counts as one, what are the first two prime numbers?

2 and 3

Reading the Lesson

1. Every whole number greater than 1 is either composite or **prime**.

2. Complete each statement.

a. In the prime factorization of a whole number, each factor is a **prime** number.

b. In the prime factorization of a negative integer, all the factors are prime except the factor **−1**.

3. Explain why the monomial $5x^2y$ is *not* in factored form.

The variable *x* has an exponent that is greater than 1.

4. Explain the steps used below to find the greatest common factor (GCF) of 84 and 120.

$84 = 2 \cdot 2 \cdot 3 \cdot 7$ **Write the prime factorization of 84.**

$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ **Write the prime factorization of 120.**

Common prime factors: 2, 2, 3 **Identify common prime factors of 84 and 120.**

$2 \cdot 2 \cdot 3 = 12$ **Multiply the common factors to find the GCF of 84 and 120.**

Helping You Remember

5. How can the two words that make up the term *prime factorization* help you remember what the term means?

Sample answer: The word *factorization* reminds you that the number must be expressed as a product of factors. The word *prime* reminds you that with the possible exception of −1, all of the factors used must be prime numbers.

Lesson 9-1

NAME ______________________ DATE __________ PERIOD _____

9-1 Enrichment

Finding the GCF by Euclid's Algorithm

Finding the greatest common factor of two large numbers can take a long time using prime factorizations. This method can be avoided by using Euclid's Algorithm as shown in the following example.

Example **Find the GCF of 209 and 532.**

Divide the greater number, 532, by the lesser, 209.

Divide the remainder into the divisor above. Repeat this process until the remainder is zero. The last nonzero remainder is the GCF.

$$532 = 2 \cdot 209 + 114,\quad 209 = 1 \cdot 114 + 95,\quad 114 = 1 \cdot 95 + 19,\quad 95 = 5 \cdot \mathbf{19} + 0$$

The divisor, 19, is the GCF of 209 and 532.

Suppose the GCF of two numbers is found to be 1. Then the numbers are said to be **relatively prime**.

Find the GCF of each group of numbers by using Euclid's Algorithm.

1. 187; 578 **17**

2. 1802; 106 **106**

3. 161; 943 **23**

4. 215; 1849 **43**

5. 1325; 3498 **53**

6. 3484; 5963 **67**

7. 33,583; 4257 **473**

8. 453; 484 **1 (Relatively Prime)**

9. 95; 209; 589 **19**

10. 518; 407; 851 **37**

11. $17a^2x^2z$; $1615axz^2$ **$17axz$**

12. $752cf^3$; $893c^3f^3$ **$47cf^3$**

13. $979r^2s^2$; $495rs^3$, $154r^3s^3$ **$11rs^2$**

14. $360x^5y^7$; $328xy$; $568x^3y^3$ **$8xy$**

NAME ______________ DATE ________ PERIOD _____

9-2 Study Guide and Intervention

Factoring Using the Distributive Property

Factor by Using the Distributive Property The Distributive Property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1) = 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1) = 6y(2x + 1)$

Example 1 **Use the Distributive Property to factor $12mn + 80m^2$.**

Find the GCF of $12mn$ and $80m^2$.

$12mn = 2 \cdot 2 \cdot 3 \cdot m \cdot n$

$80m^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot m \cdot m$

GCF $= 2 \cdot 2 \cdot m$ or $4m$

Write each term as the product of the GCF and its remaining factors.

$12mn + 80m^2 = 4m(3 \cdot n) + 4m(2 \cdot 2 \cdot 5 \cdot m)$
$= 4m(3n) + 4m(20m)$
$= 4m(3n + 20m)$

Thus $12mn + 80m^2 = 4m(3n + 20m)$.

Example 2 **Factor $6ax + 3ay + 2bx + by$ by grouping.**

$6ax + 3ay + 2bx + by$
$= (6ax + 3ay) + (2bx + by)$
$= 3a(2x + y) + b(2x + y)$
$= (3a + b)(2x + y)$

Check using the FOIL method.

$(3a + b)(2x + y)$
$= 3a(2x) + (3a)(y) + (b)(2x) + (b)(y)$
$= 6ax + 3ay + 2bx + by$ ✓

Exercises

Factor each polynomial.

1. $24x + 48y$ **$24(x + 2y)$**
2. $30mn^2 + m^2n - 6n$ **$n(30mn + m^2 - 6)$**
3. $q^4 - 18q^3 + 22q$ **$q(q^3 - 18q^2 + 22)$**
4. $9x^2 - 3x$ **$3x(3x - 1)$**
5. $4m + 6n - 8mn$ **$2(2m + 3n - 4mn)$**
6. $45s^3 - 15s^2$ **$15s^2(3s - 1)$**
7. $14c^3 - 42c^5 - 49c^4$ **$7c^3(2 - 6c^2 - 7c)$**
8. $55p^2 - 11p^4 + 44p^5$ **$11p^2(5 - p^2 + 4p^3)$**
9. $14y^3 - 28y^2 + y$ **$y(14y^2 - 28y + 1)$**
10. $4x + 12x^2 + 16x^3$ **$4x(1 + 3x + 4x^2)$**
11. $4a^2b + 28ab^2 + 7ab$ **$ab(4a + 28b + 7)$**
12. $6y + 12x - 8z$ **$2(3y + 6x - 4z)$**
13. $x^2 + 2x + x + 2$ **$(x + 1)(x + 2)$**
14. $6y^2 - 4y + 3y - 2$ **$(2y + 1)(3y - 2)$**
15. $4m^2 + 4mn + 3mn + 3n^2$ **$(4m + 3n)(m + n)$**
16. $12ax + 3xz + 4ay + yz$ **$(3x + y)(4a + z)$**
17. $12a^2 + 3a - 8a - 2$ **$(4a + 1)(3a - 2)$**
18. $xa + ya + x + y$ **$(x + y)(a + 1)$**

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9-2 Study Guide and Intervention *(continued)*

Factoring Using the Distributive Property

Solve Equations by Factoring The following property, along with factoring, can be used to solve certain equations.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal 0.

Example **Solve $9x^2 + x = 0$. Then check the solutions.**

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$ — Original equation

$x(9x + 1) = 0$ — Factor the GCF of $9x^2 + x$, which is x.

$x = 0$ or $9x + 1 = 0$ — Zero Product Property

$x = 0$ $\quad x = -\frac{1}{9}$ — Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

CHECK Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$9x^2 + x = 0$ $\qquad 9x^2 + x = 0$

$9(0)^2 + 0 = 0$ $\qquad 9\left(-\frac{1}{9}\right)^2 + \left(-\frac{1}{9}\right) = 0$

$0 = 0$ ✓ $\qquad \frac{1}{9} + \left(-\frac{1}{9}\right) = 0$

$0 = 0$ ✓

Exercises

Solve each equation. Check your solutions.

1. $x(x + 3) = 0$ **$\{0, -3\}$**
2. $3m(m - 4) = 0$ **$\{0, 4\}$**
3. $(r - 3)(r + 2) = 0$ **$\{-2, 3\}$**
4. $3x(2x - 1) = 0$ **$\left\{0, \frac{1}{2}\right\}$**
5. $(4m + 8)(m - 3) = 0$ **$\{-2, 3\}$**
6. $5s^2 = 25s$ **$\{0, 5\}$**
7. $(4c + 2)(2c - 7) = 0$ **$\left\{-\frac{1}{2}, \frac{7}{2}\right\}$**
8. $5p - 15p^2 = 0$ **$\left\{0, \frac{1}{3}\right\}$**
9. $4y^2 = 28y$ **$\{0, 7\}$**
10. $12x^2 = -6x$ **$\left\{-\frac{1}{2}, 0\right\}$**
11. $(4a + 3)(8a + 7) = 0$ **$\left\{-\frac{7}{8}, -\frac{3}{4}\right\}$**
12. $8y = 12y^2$ **$\left\{0, \frac{2}{3}\right\}$**
13. $x^2 = -2x$ **$\{-2, 0\}$**
14. $(6y - 4)(y + 3) = 0$ **$\left\{-3, \frac{2}{3}\right\}$**
15. $4m^2 = 4m$ **$\{0, 1\}$**
16. $12x = 3x^2$ **$\{0, 4\}$**
17. $12a^2 = -3a$ **$\left\{-\frac{1}{4}, 0\right\}$**
18. $(12a + 4)(3a - 1) = 0$ **$\left\{-\frac{1}{3}, \frac{1}{3}\right\}$**

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9-2 Skills Practice

Factoring Using the Distributive Property

Factor each polynomial.

1. $7x + 49$ **$7(x + 7)$**

2. $8m - 6$ **$2(4m - 3)$**

3. $5a^2 - 15$ **$5(a^2 - 3)$**

4. $10q - 25q^2$ **$5q(2 - 5q)$**

5. $8ax - 56a$ **$8a(x - 7)$**

6. $81r + 48rs$ **$3r(27 + 16s)$**

7. $t^2h + 3t$ **$t(th + 3)$**

8. $a^2b^2 + a$ **$a(ab^2 + 1)$**

9. $x + x^2y + x^3y^2$ **$x(1 + xy + x^2y^2)$**

10. $3p^2q^2 + 6pq + p$ **$p(3pq^2 + 6q + 1)$**

11. $4a^2b^2 + 16ab + 12a$ **$4a(ab^2 + 4b + 3)$**

12. $10m^3n^3 - 2mn^2 + 14mn$ **$2mn(5m^2n^2 - n + 7)$**

13. $x^2 + 3x + x + 3$ **$(x + 1)(x + 3)$**

14. $b^2 - 2b + 3b - 6$ **$(b + 3)(b - 2)$**

15. $2s^2 + 2s + 3s + 3$ **$(2s + 3)(s + 1)$**

16. $2a^2 - 4a + a - 2$ **$(2a + 1)(a - 2)$**

17. $6t^2 - 4t - 3t + 2$ **$(2t - 1)(3t - 2)$**

18. $9x^2 - 3xy + 6x - 2y$ **$(3x + 2)(3x - y)$**

Solve each equation. Check your solutions.

19. $x(x - 8) = 0$ **$\{0, 8\}$**

20. $b(b + 12) = 0$ **$\{-12, 0\}$**

21. $(m - 3)(m + 5) = 0$ **$\{-5, 3\}$**

22. $(a - 9)(2a + 1) = 0$ **$\left\{-\frac{1}{2}, 9\right\}$**

23. $x^2 - 5x = 0$ **$\{0, 5\}$**

24. $y^2 + 3y = 0$ **$\{-3, 0\}$**

25. $3a^2 = 6a$ **$\{0, 2\}$**

26. $2x^2 = 3x$ **$\left\{0, \frac{3}{2}\right\}$**

Lesson 9-2

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9-2 Practice (Average)

Factoring Using the Distributive Property

Factor each polynomial.

1. $64 - 40ab$ **$8(8 - 5ab)$**

2. $4d^2 + 16$ **$4(d^2 + 4)$**

3. $6r^2s - 3rs^2$ **$3rs(2r - s)$**

4. $15cd + 30c^2d^2$ **$15cd(1 + 2cd)$**

5. $32a^2 + 24b^2$ **$8(4a^2 + 3b^2)$**

6. $36xy^2 - 48x^2y$ **$12xy(3y - 4x)$**

7. $30x^3y + 35x^2y^2$ **$5x^2y(6x + 7y)$**

8. $9c^3d^2 - 6cd^3$ **$3cd^2(3c^2 - 2d)$**

9. $75b^2c^3 + 60bc^3$ **$15bc^3(5b + 4)$**

10. $8p^2q^2 - 24pq^3 + 16pq$ **$8pq(pq - 3q^2 + 2)$**

11. $5x^3y^2 + 10x^2y + 25x$ **$5x(x^2y^2 + 2xy + 5)$**

12. $9ax^3 + 18bx^2 + 24cx$ **$3x(3ax^2 + 6bx + 8c)$**

13. $x^2 + 4x + 2x + 8$ **$(x + 2)(x + 4)$**

14. $2a^2 + 3a + 6a + 9$ **$(a + 3)(2a + 3)$**

15. $4b^2 - 12b + 2b - 6$ **$(4b + 2)(b - 3)$**

16. $6xy - 8x + 15y - 20$ **$(2x + 5)(3y - 4)$**

17. $-6mn + 4m + 18n - 12$ **$(-2m + 6)(3n - 2)$**

18. $12a^2 - 15ab - 16a + 20b$ **$(3a - 4)(4a - 5b)$**

Solve each equation. Check your solutions.

19. $x(x - 32) = 0$ **$\{0, 32\}$**

20. $4b(b + 4) = 0$ **$\{-4, 0\}$**

21. $(y - 3)(y + 2) = 0$ **$\{-2, 3\}$**

22. $(a + 6)(3a - 7) = 0$ **$\left\{-6, \frac{7}{3}\right\}$**

23. $(2y + 5)(y - 4) = 0$ **$\left\{-\frac{5}{2}, 4\right\}$**

24. $(4y + 8)(3y - 4) = 0$ **$\left\{-2, \frac{4}{3}\right\}$**

25. $2z^2 + 20z = 0$ **$\{-10, 0\}$**

26. $8p^2 - 4p = 0$ **$\left\{0, \frac{1}{2}\right\}$**

27. $9x^2 = 27x$ **$\{0, 3\}$**

28. $18x^2 = 15x$ **$\left\{0, \frac{5}{6}\right\}$**

29. $14x^2 = -21x$ **$\left\{-\frac{3}{2}, 0\right\}$**

30. $8x^2 = -26x$ **$\left\{-\frac{13}{4}, 0\right\}$**

LANDSCAPING For Exercises 31 and 32, use the following information.

A landscaping company has been commissioned to design a triangular flower bed for a mall entrance. The final dimensions of the flower bed have not been determined, but the company knows that the height will be two feet less than the base. The area of the flower bed can be represented by the equation $A = \frac{1}{2}b^2 - b$.

31. Write this equation in factored form. **$A = b\left(\frac{1}{2}b - 1\right)$**

32. Suppose the base of the flower bed is 16 feet. What will be its area? **$112\ ft^2$**

33. PHYSICAL SCIENCE Mr. Alim's science class launched a toy rocket from ground level with an initial upward velocity of 60 feet per second. The height h of the rocket in feet above the ground after t seconds is modeled by the equation $h = 60t - 16t^2$. How long was the rocket in the air before it returned to the ground? **3.75 s**

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9-2 Reading to Learn Mathematics

Factoring Using the Distributive Property

Pre-Activity **How can you determine how long a baseball will remain in the air?**

Read the introduction to Lesson 9-2 at the top of page 481 in your textbook.

In the formula $h = 151t - 16t^2$, what does the number 151 represent?

the velocity of the baseball in feet per second at the instant it is thrown

Reading the Lesson

1. Factoring a polynomial means to find its completely factored form.
   a. The expression $x(6x - 9)$ is a factored form of the polynomial $6x^2 - 9x$. Why is this *not* its completely factored form?

      The numbers 6 and 9 have a common factor of 3, which needs to be factored out of the expression.

   b. Provide an example of a completely factored polynomial.

      Sample answer: $2b(5b + 4)$ is the completely factored form of $10b^2 + 8b$.

   c. Provide an example of a polynomial that is not completely factored.

      Sample answer: $b(10b + 8)$ is a factored form of $10b^2 + 8b$, but is not the completely factored form.

2. The polynomial $5ab + 5b^2 + 3a + 6b$ can be rewritten as $5b(a + b) + 3(a + 2b)$. Does this indicate that the original polynomial can be factored by grouping? Explain.

   No, because $5b(a + b)$ and $3(a + 2b)$ do not have a common factor other than 1.

3. The polynomial $3x^2 - 3xy + 2x - 2y$ can be rewritten as $3x(x - y) + 2(x - y)$. Does this indicate that the original polynomial can be factored by grouping? Explain.

   Yes, because $3x(x - y)$ and $2(x - y)$ have the common factor $(x - y)$.

Helping You Remember

4. How would you explain to a classmate when it is possible to use the Zero Product Property to solve an equation?

   The equation must have 0 on one side, and the other side must be the product of two expressions.

Lesson 9-2

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9-2 Enrichment

Perfect, Excessive, Defective, and Amicable Numbers

A **perfect number** is the sum of all of its factors except itself. Here is an example:

$28 = 1 + 2 + 4 + 7 + 14$

There are very few perfect numbers. Most numbers are either *excessive* or *defective*.

An **excessive number** is greater than the sum of all of its factors except itself.

A **defective number** is less than this sum.

Two numbers are **amicable** if the sum of the factors of the first number, except for the number itself, equals the second number, and vice versa.

Solve each problem.

1. Write the perfect numbers between 0 and 31.

   6, 28

2. Write the excessive numbers between 0 and 31.

   12, 18, 20, 24, 30

3. Write the defective numbers between 0 and 31.

   2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 25, 26, 27, 29

4. Show that 8128 is a perfect number. **8128 =**

   $1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 + 1016 + 2032 + 4064$

5. The sum of the reciprocals of all the factors of a perfect number (including the number itself) equals 2. Show that this is true for the first two perfect numbers.

   $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2$ $\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{56}{28} = 2$

6. More than 1000 pairs of amicable numbers have been found. One member of the first pair is 220. Find the other member.

   284

7. One member of the second pair of amicable numbers is 2620. Find the other member.

   2924

8. The Greek mathematician Euclid proved that the expression $2^{n-1}(2^n - 1)$ equals a perfect number if the expression inside the parentheses is prime. Use Euclid's expression with n equal to 5 to find the third perfect number.

   $2^4(2^5 - 1) = 496$

Answers

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9-3 Study Guide and Intervention

Factoring Trinomials: $x^2 + bx + c$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + n)$, where $m + n = b$ and $mn = c$.

Example 1 **Factor each trinomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $n = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$. Notice that $m + n$ is negative and mn is positive, so m and n are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $n = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 **Factor $x^2 + 6x - 16$.**

In this trinomial, $b = 6$ and $c = -16$. This means $m + n$ is positive and mn is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of −16	Sum of Factors
1, −16	−15
−1, 16	15
2, −8	−6
−2, 8	6

Therefore, $m = -2$ and $n = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each trinomial.

1. $x^2 + 4x + 3$ **$(x + 3)(x + 1)$**
2. $m^2 + 12m + 32$ **$(m + 4)(m + 8)$**
3. $r^2 - 3r + 2$ **$(r - 2)(r - 1)$**
4. $x^2 - x - 6$ **$(x - 3)(x + 2)$**
5. $x^2 - 4x - 21$ **$(x - 7)(x + 3)$**
6. $x^2 - 22x + 121$ **$(x - 11)(x - 11)$**
7. $c^2 - 4c - 12$ **$(c + 2)(c - 6)$**
8. $p^2 - 16p + 64$ **$(p - 8)(p - 8)$**
9. $9 - 10x + x^2$ **$(9 - x)(1 - x)$**
10. $x^2 + 6x + 5$ **$(x + 5)(x + 1)$**
11. $a^2 + 8a - 9$ **$(a - 1)(a + 9)$**
12. $y^2 - 7y - 8$ **$(y - 8)(y + 1)$**
13. $x^2 - 2x - 3$ **$(x - 3)(x + 1)$**
14. $y^2 + 14y + 13$ **$(y + 1)(y + 13)$**
15. $m^2 + 9m + 20$ **$(m + 4)(m + 5)$**
16. $x^2 + 12x + 20$ **$(x + 10)(x + 2)$**
17. $a^2 - 14a + 24$ **$(a - 2)(a - 12)$**
18. $18 + 11y + y^2$ **$(9 + y)(2 + y)$**
19. $x^2 + 2xy + y^2$ **$(x + y)(x + y)$**
20. $a^2 - 4ab + 4b^2$ **$(a - 2b)(a - 2b)$**
21. $x^2 + 6xy - 7y^2$ **$(x + 7y)(x - y)$**

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9-3 Study Guide and Intervention *(continued)*

Factoring Trinomials: $x^2 + bx + c$

Solve Equations by Factoring Factoring and the Zero Product Property from Lesson 9-2 can be used to solve many equations of the form $x^2 + bx + c = 0$.

Example 1 **Solve $x^2 + 6x = 7$. Check your solutions.**

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $\quad x = -7$	Solve each equation.

The solution set is {1, −7}. Since $1^2 + 6 = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Example 2 **ROCKET LAUNCH A rocket is fired with an initial velocity of 2288 feet per second. How many seconds will it take for the rocket to hit the ground?**

The formula $h = vt - 16t^2$ gives the height h of the rocket after t seconds when the initial velocity v is given in feet per second.

$h = vt - 16t^2$	Formula
$0 = 2288t - 16t^2$	Substitute.
$0 = 16t(143 - t)$	Factor.
$16t = 0$ or $143 - t = 0$	Zero Product Property
$t = 0$ $\quad t = 143$	Solve each equation.

The value $t = 0$ represents the time at launch. The rocket returns to the ground in 143 seconds, or a little less than 2.5 minutes after launch.

Exercises

Solve each equation. Check your solutions.

1. $x^2 - 4x + 3 = 0$ **{1, 3}**
2. $y^2 - 5y + 4 = 0$ **{1, 4}**
3. $m^2 + 10m + 9 = 0$ **{−1, −9}**
4. $x^2 = x + 2$ **{−1, 2}**
5. $x^2 - 4x = 5$ **{−1, 5}**
6. $x^2 - 12x + 36 = 0$ **{6}**
7. $c^2 - 8 = -7c$ **{−8, 1}**
8. $p^2 = 9p - 14$ **{2, 7}**
9. $-9 - 8x + x^2 = 0$ **{−1, 9}**
10. $x^2 + 6 = 5x$ **{2, 3}**
11. $a^2 = 11a - 18$ **{2, 9}**
12. $y^2 - 8y + 15 = 0$ **{3, 5}**
13. $x^2 = 24 - 10x$ **{−12, 2}**
14. $a^2 - 18a = -72$ **{6, 12}**
15. $b^2 = 10b - 16$ **{2, 8}**

Use the formula $h = vt - 16t^2$ to solve each problem.

16. **FOOTBALL** A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to return to the ground? **3 seconds**

17. **BASEBALL** A ball is thrown up with an initial velocity of 32 feet per second. How many seconds will it take for the ball to return to the ground? **2 seconds**

18. **ROCKET LAUNCH** If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high? **at 10 seconds and at 90 seconds**

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9-3 Skills Practice

Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial.

1. $t^2 + 8t + 12$ **(t + 2)(t + 6)**

2. $n^2 + 7n + 12$ **(n + 3)(n + 4)**

3. $p^2 + 9p + 20$ **(p + 5)(p + 4)**

4. $h^2 + 9h + 18$ **(h + 6)(h + 3)**

5. $n^2 + 3n - 18$ **(n + 6)(n − 3)**

6. $x^2 + 2x - 8$ **(x + 4)(x − 2)**

7. $y^2 - 5y - 6$ **(y + 1)(y − 6)**

8. $g^2 + 3g - 10$ **(g + 5)(g − 2)**

9. $s^2 + 4s - 12$ **(s − 2)(s + 6)**

10. $x^2 - x - 12$ **(x − 4)(x + 3)**

11. $w^2 - w - 6$ **(w − 3)(w + 2)**

12. $y^2 - 6y + 8$ **(y − 2)(y − 4)**

13. $x^2 - 8x + 15$ **(x − 5)(x − 3)**

14. $b^2 - 9b + 8$ **(b − 1)(b − 8)**

15. $c^2 - 15c + 56$ **(c − 7)(c − 8)**

16. $-4 - 3m + m^2$ **(m − 4)(m + 1)**

Solve each equation. Check your solutions.

17. $x^2 - 6x + 8 = 0$ **{2, 4}**

18. $b^2 - 7b + 12 = 0$ **{3, 4}**

19. $m^2 + 5m + 6 = 0$ **{−3, −2}**

20. $d^2 + 7d + 10 = 0$ **{−5, −2}**

21. $y^2 - 2y - 24 = 0$ **{−4, 6}**

22. $p^2 - 3p = 18$ **{−3, 6}**

23. $h^2 + 2h = 35$ **{−7, 5}**

24. $a^2 + 14a = -45$ **{−9, −5}**

25. $n^2 - 36 = 5n$ **{−4, 9}**

26. $w^2 + 30 = 11w$ **{5, 6}**

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9-3 Practice (Average)

Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial.

1. $a^2 + 10a + 24$ **(a + 4)(a + 6)**

2. $h^2 + 12h + 27$ **(h + 3)(h + 9)**

3. $x^2 + 14x + 33$ **(x + 11)(x + 3)**

4. $g^2 - 2g - 63$ **(g + 7)(g − 9)**

5. $w^2 + w - 56$ **(w + 8)(w − 7)**

6. $y^2 + 4y - 60$ **(y + 10)(y − 6)**

7. $b^2 + 4b - 32$ **(b − 4)(b + 8)**

8. $n^2 - 3n - 28$ **(n − 7)(n + 4)**

9. $c^2 + 4c - 45$ **(c − 5)(c + 9)**

10. $z^2 - 11z + 30$ **(z − 6)(z − 5)**

11. $d^2 - 16d + 63$ **(d − 9)(d − 7)**

12. $x^2 - 11x + 24$ **(x − 3)(x − 8)**

13. $q^2 - q - 56$ **(q − 8)(q + 7)**

14. $x^2 - 6x - 55$ **(x + 5)(x − 11)**

15. $32 + 18r + r^2$ **(r + 16)(r + 2)**

16. $48 - 16g + g^2$ **(g − 12)(g − 4)**

17. $j^2 - 9jk - 10k^2$ **(j − 10k)(j + k)**

18. $m^2 - mv - 56v^2$ **(m − 8v)(m + 7v)**

Solve each equation. Check your solutions.

19. $x^2 + 17x + 42 = 0$ **{−14, −3}**

20. $p^2 + 5p - 84 = 0$ **{−12, 7}**

21. $k^2 + 3k - 54 = 0$ **{−9, 6}**

22. $b^2 - 12b - 64 = 0$ **{−4, 16}**

23. $n^2 + 4n = 32$ **{−8, 4}**

24. $h^2 - 17h = -60$ **{5, 12}**

25. $c^2 - 26c = 56$ **{−2, 28}**

26. $z^2 - 14z = 72$ **{−4, 18}**

27. $y^2 - 84 = 5y$ **{−7, 12}**

28. $80 + a^2 = 18a$ **{8, 10}**

29. $u^2 = 16u + 36$ **{−2, 18}**

30. $17s + s^2 = -52$ **{−13, −4}**

31. Find all values of k so that the trinomial $x^2 + kx - 35$ can be factored using integers. **−34, −2, 2, 34**

CONSTRUCTION For Exercises 32 and 33, use the following information.

A construction company is planning to pour concrete for a driveway. The length of the driveway is 16 feet longer than its width w.

32. Write an expression for the area of the driveway. **w(w + 16) ft²**

33. Find the dimensions of the driveway if it has an area of 260 square feet. **10 ft by 26 ft**

WEB DESIGN For Exercises 34 and 35, use the following information.

Janeel has a 10-inch by 12-inch photograph. She wants to scan the photograph, then reduce the result by the same amount in each dimension to post on her Web site. Janeel wants the area of the image to be one eighth that of the original photograph.

34. Write an equation to represent the area of the reduced image. **(10 − x)(12 − x) = 15, or $x^2 - 22x + 105 = 0$**

35. Find the dimensions of the reduced image. **3 in. by 5 in.**

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9-3 Reading to Learn Mathematics

Factoring Trinomials: $x^2 + bx + c$

Pre-Activity **How can factoring be used to find the dimensions of a garden?**

Read the introduction to Lesson 9-3 at the top of page 489 in your textbook.

- Why do you need to find two numbers whose *product* is 54?

  The problem asks you to find the length and width of the garden. You know the area is 54 ft^2. Since you *multiply* length times width to find area, you need to find two numbers whose *product* is 54.

- Why is the *sum* of these two numbers half the perimeter or 15?

  You *add* to find the perimeter of the garden. Since you use the length twice and the width twice to find the perimeter, the *sum* of the length and the width is half the perimeter or 15.

Reading the Lesson

Tell what sum and product you want m and n to have to use the pattern $(x + m)(x + n)$ to factor the given trinomial.

1. $x^2 + 10x + 24$ sum: **10** product: **24**
2. $x^2 - 12x + 20$ sum: **−12** product: **20**
3. $x^2 - 4x - 21$ sum: **−4** product: **−21**
4. $x^2 + 6x - 16$ sum: **6** product: **−16**

5. To factor $x^2 - 18x + 32$, you can look for numbers with a product of 32 and a sum of −18. Explain why the numbers in the pair you are looking for must both be negative.

   To have a product of positive 32, the numbers must both be positive or both be negative. If both were positive, their sum would be positive instead of negative.

Helping You Remember

6. If you are using the pattern $(x + m)(x + n)$ to factor a trinomial of the form $x^2 + bx + c$, how can you use your knowledge of multiplying integers to help you remember whether m and n are positive or negative factors?

   Sample answer: Like signs multiplied together result in positive numbers and unlike signs result in negative numbers. So, if c is positive, the factors m and n both have the same sign as b. If c is negative, one of the factors is positive and the other is negative.

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9-3 Enrichment

Puzzling Primes

A prime number has only two factors, itself and 1. The number 6 is not prime because it has 2 and 3 as factors; 5 and 7 are prime. The number 1 is not considered to be prime.

1. Use a calculator to help you find the 25 prime numbers less than 100.

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Prime numbers have interested mathematicians for centuries. They have tried to find expressions that will give all the prime numbers, or only prime numbers. In the 1700s, Euler discovered that the trinomial $x^2 + x + 41$ will yield prime numbers for values of x from 0 through 39.

2. Find the prime numbers generated by Euler's formula for x from 0 through 7.

   41, 43, 47, 53, 61, 71, 83, 97

3. Show that the trinomial $x^2 + x + 31$ will not give prime numbers for very many values of x.

   It works for $x = 0, 2, 3, 5$, and 6 but not for $x = 1, 4$, and 7.

4. Find the largest prime number generated by Euler's formula.

   1601

Goldbach's Conjecture is that every nonzero even number greater than 2 can be written as the sum of two primes. No one has ever proved that this is always true, but no one has found a counterexample, either.

5. Show that Goldbach's Conjecture is true for the first 5 even numbers greater than 2.

   $4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7, 12 = 5 + 7$

6. Give a way that someone could disprove Goldbach's Conjecture.

   Find an even number that cannot be written as the sum of two primes.

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9-4 Study Guide and Intervention

Factoring Trinomials: $ax^2 + bx + c$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and n whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 **Factor $2x^2 + 15x + 18$.**

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers whose sum is 15 and whose product is $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors whose sum is 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + nx + c$, with $a = 2$, $m = 3$, $n = 12$, and $c = 18$.

$2x^2 + 15x + 18 = 2x^2 + 3x + 12x + 18$
$= (2x^2 + 3x) + (12x + 18)$
$= x(2x + 3) + 6(2x + 3)$
$= (x + 6)(2x + 3)$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 **Factor $3x^2 - 3x - 18$.**

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 whose sum is -1.

Factors of −6	Sum of Factors
1, −6	−5
−1, 6	5
−2, 3	1
2, −3	−1

Now use the pattern $(x + m)(x + n)$ with $m = 2$ and $n = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is $3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$ **$(2x + 1)(x - 2)$**
2. $3m^2 - 8m - 3$ **$(3m + 1)(m - 3)$**
3. $16r^2 - 8r + 1$ **$(4r - 1)(4r - 1)$**
4. $6x^2 + 5x - 6$ **$(2x + 3)(3x - 2)$**
5. $3x^2 + 2x - 8$ **$(3x - 4)(x + 2)$**
6. $18x^2 - 27x - 5$ **$(3x - 5)(6x + 1)$**
7. $2a^2 + 5a + 3$ **$(2a + 3)(a + 1)$**
8. $18y^2 + 9y - 5$ **$(6y + 5)(3y - 1)$**
9. $-4c^2 + 19c - 21$ **$(4c - 7)(3 - c)$**
10. $8x^2 - 4x - 24$ **$(4x - 8)(2x + 3)$**
11. $28p^2 + 60p - 25$ **$(2p + 5)(14p - 5)$**
12. $48x^2 + 22x - 15$ **$(6x + 5)(8x - 3)$**
13. $3y^2 - 6y - 24$ **$3(y + 2)(y - 4)$**
14. $4x^2 + 26x - 48$ **$2(x + 8)(2x - 3)$**
15. $8m^2 - 44m + 48$ **$4(2m - 3)(m - 4)$**
16. $6x^2 - 7x + 18$ **prime**
17. $2a^2 - 14a + 18$ **$2(a^2 - 7a + 9)$**
18. $18 + 11y + 2y^2$ **prime**

Lesson 9-4

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9-4 Study Guide and Intervention *(continued)*

Factoring Trinomials: $ax^2 + bx + c$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

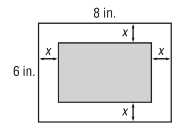
Solve each equation. Check your solutions.

1. $8x^2 + 2x - 3 = 0$ **$\left\{\frac{1}{2}, -\frac{3}{4}\right\}$**
2. $3n^2 - 2n - 5 = 0$ **$\left\{-1, \frac{5}{3}\right\}$**
3. $2d^2 - 13d - 7 = 0$ **$\left\{-\frac{1}{2}, 7\right\}$**
4. $4x^2 = x + 3$ **$\left\{1, -\frac{3}{4}\right\}$**
5. $3x^2 - 13x = 10$ **$\left\{-\frac{2}{3}, 5\right\}$**
6. $6x^2 - 11x - 10 = 0$ **$\left\{-\frac{2}{3}, \frac{5}{2}\right\}$**
7. $2k^2 - 40 = -11k$ **$\left\{-8, \frac{5}{2}\right\}$**
8. $2p^2 = -21p - 40$ **$\left\{-\frac{5}{2}, -8\right\}$**
9. $-7 - 18x + 9x^2 = 0$ **$\left\{\frac{7}{3}, -\frac{1}{3}\right\}$**
10. $12x^2 - 15 = -8x$ **$\left\{-\frac{3}{2}, \frac{5}{6}\right\}$**
11. $7a^2 = -65a - 18$ **$\left\{-\frac{2}{7}, -9\right\}$**
12. $16y^2 - 2y - 3 = 0$ **$\left\{\frac{1}{2}, -\frac{3}{8}\right\}$**
13. $8x^2 + 5x = 3 + 7x$ **$\left\{\frac{3}{4}, -\frac{1}{2}\right\}$**
14. $4a^2 - 18a + 5 = 15$ **$\left\{-\frac{1}{2}, 5\right\}$**
15. $3b^2 - 18b = 10b - 49$ **$\left\{\frac{7}{3}, 7\right\}$**

16. The difference of the squares of two consecutive odd integers is 24. Find the integers. **−5, −7 and 5, 7**

17. **GEOMETRY** The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions? **30 yd by 10 yd**

18. **GEOMETRY** A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches. **6 in. by 4 in.**



Answers

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9-4 Skills Practice

Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2x^2 + 5x + 2$ **$(x + 2)(2x + 1)$**

2. $3n^2 + 5n + 2$ **$(3n + 2)(n + 1)$**

3. $2s^2 + 9s - 5$ **$(s + 5)(2s - 1)$**

4. $3g^2 - 7g + 2$ **$(3g - 1)(g - 2)$**

5. $2t^2 - 11t + 15$ **$(t - 3)(2t - 5)$**

6. $2x^2 + 3x - 6$ **prime**

7. $2y^2 + y - 1$ **$(y + 1)(2y - 1)$**

8. $4h^2 + 8h - 5$ **$(2h + 5)(2h - 1)$**

9. $4x^2 - 3x - 3$ **prime**

10. $4b^2 + 15b - 4$ **$(4b - 1)(b + 4)$**

11. $9p^2 + 6p - 8$ **$(3p - 2)(3p + 4)$**

12. $6q^2 - 13q + 6$ **$(3q - 2)(2q - 3)$**

13. $3a^2 + 30a + 63$ **$3(a + 7)(a + 3)$**

14. $10w^2 - 19w - 15$ **$(2w - 5)(5w + 3)$**

Solve each equation. Check your solutions.

15. $2x^2 + 7x + 3 = 0$ $\left\{-3, -\frac{1}{2}\right\}$

16. $3w^2 + 14w + 8 = 0$ $\left\{-4, -\frac{2}{3}\right\}$

17. $3n^2 - 7n + 2 = 0$ $\left\{\frac{1}{3}, 2\right\}$

18. $5d^2 - 22d + 8 = 0$ $\left\{\frac{2}{5}, 4\right\}$

19. $6h^2 + 8h + 2 = 0$ $\left\{-1, -\frac{1}{3}\right\}$

20. $8p^2 - 16p = 10$ $\left\{-\frac{1}{2}, \frac{5}{2}\right\}$

21. $9y^2 + 18y - 12 = 6y$ $\left\{-2, \frac{2}{3}\right\}$

22. $4a^2 - 16a = -15$ $\left\{\frac{3}{2}, \frac{5}{2}\right\}$

23. $10b^2 - 15b = 8b - 12$ $\left\{\frac{4}{5}, \frac{3}{2}\right\}$

24. $6d^2 + 21d = 10d + 35$ $\left\{-\frac{7}{2}, \frac{5}{3}\right\}$

Lesson 9-4

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9-4 Practice (Average)

Factoring Trinomials: $ax^2 + bx + c$

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

1. $2b^2 + 10b + 12$ **$2(b + 2)(b + 3)$**

2. $3g^2 + 8g + 4$ **$(3g + 2)(g + 2)$**

3. $4x^2 + 4x - 3$ **$(2x + 3)(2x - 1)$**

4. $8b^2 - 5b - 10$ **prime**

5. $6m^2 + 7m - 3$ **$(3m - 1)(2m + 3)$**

6. $10d^2 + 17d - 20$ **$(5d - 4)(2d + 5)$**

7. $6a^2 - 17a + 12$ **$(3a - 4)(2a - 3)$**

8. $8w^2 - 18w + 9$ **$(4w - 3)(2w - 3)$**

9. $10x^2 - 9x + 6$ **prime**

10. $15n^2 - n - 28$ **$(5n - 7)(3n + 4)$**

11. $10x^2 + 21x - 10$ **$(2x + 5)(5x - 2)$**

12. $9r^2 + 15r + 6$ **$3(3r + 2)(r + 1)$**

13. $12y^2 - 4y - 5$ **$(2y + 1)(6y - 5)$**

14. $14k^2 - 9k - 18$ **$(2k - 3)(7k + 6)$**

15. $8z^2 + 20z - 48$ **$4(z + 4)(2z - 3)$**

16. $12q^2 + 34q - 28$ **$2(3q - 2)(2q + 7)$**

17. $18h^2 + 15h - 18$ **$3(2h + 3)(3h - 2)$**

18. $12p^2 - 22p - 20$ **$2(3p + 2)(2p - 5)$**

Solve each equation. Check your solutions.

19. $3h^2 + 2h - 16 = 0$ $\left\{-\frac{8}{3}, 2\right\}$

20. $15n^2 - n = 2$ $\left\{-\frac{1}{3}, \frac{2}{5}\right\}$

21. $8q^2 - 10q + 3 = 0$ $\left\{\frac{1}{2}, \frac{3}{4}\right\}$

22. $6b^2 - 5b = 4$ $\left\{-\frac{1}{2}, \frac{4}{3}\right\}$

23. $10c^2 - 21c = -4c + 6$ $\left\{-\frac{3}{10}, 2\right\}$

24. $10g^2 + 10 = 29g$ $\left\{\frac{2}{5}, \frac{5}{2}\right\}$

25. $6y^2 = -7y - 2$ $\left\{-\frac{2}{3}, -\frac{1}{2}\right\}$

26. $9z^2 = -6z + 15$ $\left\{-\frac{5}{3}, 1\right\}$

27. $12k^2 + 15k = 16k + 20$ $\left\{-\frac{5}{4}, \frac{4}{3}\right\}$

28. $12x^2 - 1 = -x$ $\left\{-\frac{1}{3}, \frac{1}{4}\right\}$

29. $8a^2 - 16a = 6a - 12$ $\left\{\frac{3}{4}, 2\right\}$

30. $18a^2 + 10a = -11a + 4$ $\left\{-\frac{4}{3}, \frac{1}{6}\right\}$

31. DIVING Lauren dove into a swimming pool from a 15-foot-high diving board with an initial upward velocity of 8 feet per second. Find the time t in seconds it took Lauren to enter the water. Use the model for vertical motion given by the equation $h = -16t^2 + vt + s$, where h is height in feet, t is time in seconds, v is the initial upward velocity in feet per second, and s is the initial height in feet. (*Hint:* Let $h = 0$ represent the surface of the pool.) **1.25 s**

32. BASEBALL Brad tossed a baseball in the air from a height of 6 feet with an initial upward velocity of 14 feet per second. Enrique caught the ball on its way down at a point 4 feet above the ground. How long was the ball in the air before Enrique caught it? Use the model of vertical motion from Exercise 31. **1 s**

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9-4 Reading to Learn Mathematics

Factoring Trinomials: $ax^2 + bx + c$

Pre-Activity **How can algebra tiles be used to factor $2x^2 + 7x + 6$?**

Read the introduction to Lesson 9-4 at the top of page 495 in your textbook.

- When you form the algebra tiles into a rectangle, what is the first step?

  Place the two x^2 tiles on the product mat and arrange the six 1 tiles into a rectangular array.

- What is the second step?

  Arrange the seven x tiles to complete the rectangle.

Reading the Lesson

1. Suppose you want to factor the trinomial $3x^2 + 14x + 8$.

a. What is the first step?

Find integers with a product of 24 and a sum of 14. The integers are 2 and 12.

b. What is the second step?

Rewrite the polynomial by breaking the middle term into two addends that use 2 and 12 as coefficients. You can use $3x^2 + 14x + 8 = 3x^2 + 2x + 12x + 8$.

c. Provide an explanation for the next two steps.

$(3x^2 + 2x) + (12x + 8)$ **Group terms with common factors.**

$x(3x + 2) + 4(3x + 2)$ **Factor the GCF from each grouping.**

d. Use the Distributive Property to rewrite the last expression in part c. You get

(**x** + **4**)$(3x + 2)$.

2. Explain how you know that the trinomial $2x^2 - 7x + 4$ is a prime polynomial.

To factor $2x^2 - 7x + 4$, you would need two negative integers whose product is 8 and whose sum is −7. There are no such negative integers. Therefore, $2x^2 - 7x + 4$ is prime.

Helping You Remember

3. What are steps you could use to remember how to find the factors of a trinomial written in the form of $ax^2 + bx + c$?

Sample answer: Look for two integers with a product equal to *ac* and a sum of *b*. Replace the middle term with a sum of *x* terms that have those two integers as coefficients. Then factor by grouping.

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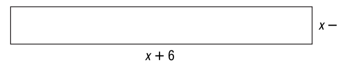
9-4 Enrichment

Area Models for Quadratic Trinomials

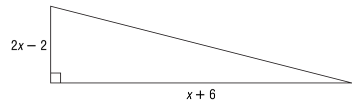
After you have factored a quadratic trinomial, you can use the factors to draw geometric models of the trinomial.

$x^2 + 5x - 6 = (x - 1)(x + 6)$

To draw a rectangular model, the value 2 was used for x so that the shorter side would have a length of 1. Then the drawing was done in centimeters. So, the area of the rectangle is $x^2 + 5x - 6$.



To draw a right triangle model, recall that the area of a triangle is one-half the base times the height. So, one of the sides must be twice as long as the shorter side of the rectangular model.



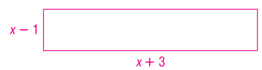
$x^2 + 5x - 6 = (x - 1)(x + 6)$

$= \frac{1}{2}(2x - 2)(x + 6)$

The area of the right triangle is also $x^2 + 5x - 6$.

Factor each trinomial. Then follow the directions to draw each model of the trinomial.

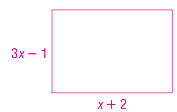
1. $x^2 + 2x - 3$ Use $x = 2$. Draw a rectangle in centimeters.



$(x + 3)(x - 1)$

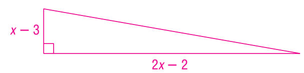
2. $3x^2 + 5x - 2$ Use $x = 1$. Draw a rectangle in centimeters.

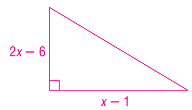
$(x + 2)(3x - 1)$



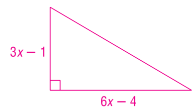
3. $x^2 - 4x + 3$ Use $x = 4$. Draw two different right triangles in centimeters.

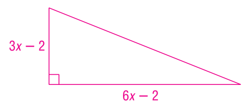
$(x - 1)(x - 3)$





4. $9x^2 - 9x + 2$ Use $x = 2$. Draw two different right triangles. Use 0.5 centimeter for each unit. **$(3x - 2)(3x - 1)$**





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9-5 Study Guide and Intervention

Factoring Differences of Squares

Factor $a^2 - b^2$ The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each binomial.**

a. $n^2 - 64$

$n^2 - 64$
$= n^2 - 8^2$ Write in the form $a^2 - b^2$.
$= (n + 8)(n - 8)$ Factor.

b. $4m^2 - 81n^2$

$4m^2 - 81n^2$
$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.
$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$
$= 2(25a^2 - 36)$ Find the GCF.
$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$
$= 2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial
$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.
$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.
$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.
$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.
$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $x^2 - 81$ **$(x + 9)(x - 9)$**
2. $m^2 - 100$ **$(m + 10)(m - 10)$**
3. $16n^2 - 25$ **$(4n - 5)(4n + 5)$**
4. $36x^2 - 100y^2$ **$(6x + 10y)(6x - 10y)$**
5. $49x^2 - 32$ **prime**
6. $16a^2 - 9b^2$ **$(4a - 3b)(4a + 3b)$**
7. $225c^2 - a^2$ **$(15c - a)(15c + a)$**
8. $72p^2 - 50$ **$2(6p + 5)(6p - 5)$**
9. $-2 + 2x^2$ **$2(x - 1)(x + 1)$**
10. $-81 + a^4$ **$(a - 3)(a + 3)(a^2 + 9)$**
11. $6 - 54a^2$ **$6(1 + 3a)(1 - 3a)$**
12. $8y^2 - 200$ **$8(y + 5)(y - 5)$**
13. $4x^3 - 100x$ **$4x(x + 5)(x - 5)$**
14. $2y^4 - 32y^2$ **$2y^2(y + 4)(y - 4)$**
15. $8m^3 - 128m$ **$8m(m + 4)(m - 4)$**
16. $6x^2 - 25$ **prime**
17. $2a^3 - 98ab^2$ **$2a(a - 7b)(a + 7b)$**
18. $18y^2 - 72y^4$ **$18y^2(1 - 2y)(1 + 2y)$**
19. $169x^3 - x$ **$x(13x + 1)(13x - 1)$**
20. $3a^4 - 3a^2$ **$3a^2(a + 1)(a - 1)$**
21. $3x^4 + 6x^3 - 3x^2 - 6x$ **$3x(x - 1)(x + 1)(x + 2)$**

Lesson 9-5

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9-5 Study Guide and Intervention *(continued)*

Factoring Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example **Solve each equation. Check your solutions.**

a. $x^2 - \frac{1}{25} = 0$

$x^2 - \frac{1}{25} = 0$ Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$ $x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$ Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$ Zero Product Property
$x = -\frac{1}{5}$ $x = \frac{1}{5}$ Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $4x^3 = 9x$

$4x^3 = 9x$ Original equation
$4x^3 - 9x = 0$ Subtract $9x$ from each side.
$x(4x^2 - 9) = 0$ Find the GCF.
$x[(2x)^2 - 3^2] = 0$ $4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$
$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$ Factor the difference of squares.
$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$ Zero Product Property
$x = 0$ $x = \frac{3}{2}$ $x = -\frac{3}{2}$ Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $81x^2 = 49$ **$\left\{\frac{7}{9}, -\frac{7}{9}\right\}$**
2. $36n^2 = 1$ **$\left\{-\frac{1}{6}, \frac{1}{6}\right\}$**
3. $25d^2 - 100 = 0$ **$\{2, -2\}$**
4. $\frac{1}{4}x^2 = 25$ **$\{10, -10\}$**
5. $36 = \frac{1}{25}x^2$ **$\{-30, 30\}$**
6. $\frac{49}{100} - x^2 = 0$ **$\left\{-\frac{7}{10}, \frac{7}{10}\right\}$**
7. $9x^3 = 25x$ **$\left\{0, -\frac{5}{3}, \frac{5}{3}\right\}$**
8. $7a^3 = 175a$ **$\{0, -5, 5\}$**
9. $2m^3 = 32m$ **$\{0, -4, 4\}$**
10. $16y^3 = 25y$ **$\left\{0, -\frac{5}{4}, \frac{5}{4}\right\}$**
11. $\frac{1}{64}x^2 = 49$ **$\{-56, 56\}$**
12. $4a^3 - 64a = 0$ **$\{0, -4, 4\}$**
13. $3b^3 - 27b = 0$ **$\{0, -3, 3\}$**
14. $\frac{9}{25}m^2 = 121$ **$\left\{-\frac{55}{3}, \frac{55}{3}\right\}$**
15. $48n^3 = 147n$ **$\left\{0, -\frac{7}{4}, \frac{7}{4}\right\}$**

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9-5 Skills Practice

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $a^2 - 4$
   $(a + 2)(a - 2)$
2. $n^2 - 64$
   $(n + 8)(n - 8)$
3. $1 - 49c^2$
   $(1 + 7c)(1 - 7c)$
4. $-16 + p^2$
   $(p + 4)(p - 4)$
5. $k^2 + 25$
   prime
6. $36 - 100w^2$
   $(6 - 10w)(6 + 10w)$
7. $t^2 - 81u^2$
   $(t + 9u)(t - 9u)$
8. $4h^2 - 25g^2$
   $(2h + 5g)(2h - 5g)$
9. $64m^2 - 9y^2$
   $(8m - 3y)(8m + 3y)$
10. $4c^2 - 5d^2$
   prime
11. $-49r^2 + 4t^2$
   $(2t + 7r)(2t - 7r)$
12. $8x^2 - 72p^2$
   $8(x + 3p)(x - 3p)$
13. $20q^2 - 5r^2$
   $5(2q + r)(2q - r)$
14. $32a^2 - 50b^2$
   $2(4a + 5b)(4a - 5b)$

Solve each equation by factoring. Check your solutions.

15. $16x^2 - 9 = 0$ **$\left\{\pm\frac{3}{4}\right\}$**
16. $25p^2 - 16 = 0$ **$\left\{\pm\frac{4}{5}\right\}$**
17. $36q^2 - 49 = 0$ **$\left\{\pm\frac{7}{6}\right\}$**
18. $81 - 4b^2 = 0$ **$\left\{\pm\frac{9}{2}\right\}$**
19. $16d^2 = 4$ **$\left\{\pm\frac{1}{2}\right\}$**
20. $18a^2 = 8$ **$\left\{\pm\frac{2}{3}\right\}$**
21. $s^2 - \frac{9}{25} = 0$ **$\left\{\pm\frac{3}{5}\right\}$**
22. $k^2 - \frac{49}{64} = 0$ **$\left\{\pm\frac{7}{8}\right\}$**
23. $\frac{1}{25}h^2 - 16 = 0$ **$\{\pm 20\}$**
24. $\frac{1}{16}y^2 = 81$ **$\{\pm 36\}$**

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9-5 Practice (Average)

Factoring Differences of Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

1. $k^2 - 100$
   $(k + 10)(k - 10)$
2. $81 - r^2$
   $(9 + r)(9 - r)$
3. $16p^2 - 36$
   $(4p + 6)(4p - 6)$
4. $4x^2 + 25$
   prime
5. $144 - 9f^2$
   $(12 + 3f)(12 - 3f)$
6. $36g^2 - 49h^2$
   $(6g + 7h)(6g - 7h)$
7. $121m^2 - 144n^2$
   $(11m - 12n)(11m + 12n)$
8. $32 - 8y^2$
   $8(2 - y)(2 + y)$
9. $24a^2 - 54b^2$
   $6(2a - 3b)(2a + 3b)$
10. $32s^2 - 18u^2$
   $2(4s - 3u)(4s + 3u)$
11. $9d^2 - 32$
   prime
12. $36z^3 - 9z$
   $9z(2z + 1)(2z - 1)$
13. $45q^3 - 20q$
   $5q(3q + 2)(3q - 2)$
14. $100b^3 - 36b$
   $4b(5b + 3)(5b - 3)$
15. $3t^4 - 48t^2$
   $3t^2(t + 4)(t - 4)$

Solve each equation by factoring. Check your solutions.

16. $4y^2 = 81$ **$\left\{\pm\frac{9}{2}\right\}$**
17. $64p^2 = 9$ **$\left\{\pm\frac{3}{8}\right\}$**
18. $98b^2 - 50 = 0$ **$\left\{\pm\frac{5}{7}\right\}$**
19. $32 - 162k^2 = 0$ **$\left\{\pm\frac{4}{9}\right\}$**
20. $s^2 - \frac{64}{121} = 0$ **$\left\{\pm\frac{8}{11}\right\}$**
21. $\frac{16}{49} - v^2 = 0$ **$\left\{\pm\frac{4}{7}\right\}$**
22. $\frac{1}{36}x^2 - 25 = 0$ **$\{\pm 30\}$**
23. $27h^3 = 48h$ **$\left\{\pm\frac{4}{3}, 0\right\}$**
24. $75g^3 = 147g$ **$\left\{\pm\frac{7}{5}, 0\right\}$**

25. **EROSION** A rock breaks loose from a cliff and plunges toward the ground 400 feet below. The distance d that the rock falls in t seconds is given by the equation $d = 16t^2$. How long does it take the rock to hit the ground? **5 s**

26. **FORENSICS** Mr. Cooper contested a speeding ticket given to him after he applied his brakes and skidded to a halt to avoid hitting another car. In traffic court, he argued that the length of the skid marks on the pavement, 150 feet, proved that he was driving under the posted speed limit of 65 miles per hour. The ticket cited his speed at 70 miles per hour. Use the formula $\frac{1}{24}s^2 = d$, where s is the speed of the car and d is the length of the skid marks, to determine Mr. Cooper's speed when he applied the brakes. Was Mr. Cooper correct in claiming that he was not speeding when he applied the brakes? **60 mi/h; yes**

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9-5 Reading to Learn Mathematics

Factoring Differences of Squares

Pre-Activity **How can you determine a basketball player's hang time?**

Read the introduction to Lesson 9-5 at the top of page 501 in your textbook.

Suppose a player can jump 2 feet. Can you use the pattern for the difference of squares to solve the equation $4t^2 - 2 = 0$? Explain.

No; 2 is not a perfect square.

Reading the Lesson

1. Explain why each binomial is a difference of squares.

a. $4x^2 - 25$

$4x^2$ is the square of $2x$, 25 is the square of 5, and the operation sign is a minus sign.

b. $49a^2 - 64b^2$

$49a^2$ is the square of $7a$, $64b^2$ is the square of $8b$, and the operation sign is a minus sign.

2. Sometimes it is necessary to apply more than one technique when factoring, or to apply the same technique more than once.

a. What should you look for first when you are factoring a binomial?

Look for a factor or factors common to the terms of the binomial.

b. Explain what is done in each step to factor $4x^4 - 64$.

$4x^4 - 64$	
$= 4(x^4 - 16)$	**Factor out the GCF.**
$= 4[(x^2)^2 - 4^2]$	**Write $x^4 - 16$ in difference of squares form.**
$= 4(x^2 + 4)(x^2 - 4)$	**Factor the difference of squares.**
$= 4(x^2 + 4)(x^2 - 2^2)$	**Write $x^2 - 4$ in difference of squares form.**
$= 4(x^2 + 4)(x + 2)(x - 2)$	**Factor the difference of squares.**

3. Suppose you are solving the equation $16x^2 - 9 = 0$ and rewrite it as $(4x + 3)(4x - 3) = 0$. What would be your next steps in solving the equation?

Set each factor equal to zero, then solve the resulting equations.

Helping You Remember

4. How can you remember whether a binomial can be factored as a difference of squares?

The operation sign must be a minus sign, and the expressions before and after the minus sign must be perfect squares.

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9-5 Enrichment

Factoring Trinomials of Fourth Degree

Some trinomials of the form $a^4 + a^2b^2 + b^4$ can be written as the difference of two squares and then factored.

Example **Factor $4x^4 - 37x^2y^2 + 9y^4$.**

Step 1 Find the square roots of the first and last terms.

$\sqrt{4x^4} = 2x^2$ $\sqrt{9y^4} = 3y^2$

Step 2 Find twice the product of the square roots.

$2(2x^2)(3y^2) = 12x^2y^2$

Step 3 Separate the middle term into two parts. One part is either your answer to Step 2 or its opposite. The other part should be the opposite of a perfect square.

$-37x^2y^2 = -12x^2y^2 - 25x^2y^2$

Step 4 Rewrite the trinomial as the difference of two squares and then factor.

$$\begin{aligned}4x^4 - 37x^2y^2 + 9y^4 &= (4x^4 - 12x^2y^2 + 9y^4) - 25x^2y^2\\ &= (2x^2 - 3y^2)^2 - 25x^2y^2\\ &= [(2x^2 - 3y^2) + 5xy][(2x^2 - 3y^2) - 5xy]\\ &= (2x^2 + 5xy - 3y^2)(2x^2 - 5xy - 3y^2)\end{aligned}$$

Factor each trinomial.

1. $x^4 + x^2y^2 + y^4$

$(x^2 + xy + y^2)(x^2 - xy + y^2)$

2. $x^4 + x^2 + 1$

$(x^2 + x + 1)(x^2 - x + 1)$

3. $9a^4 - 15a^2 + 1$

$(3a^2 + 3a - 1)(3a^2 - 3a - 1)$

4. $16a^4 - 17a^2 + 1$

$(4a - 1)(a + 1)(4a + 1)(a - 1)$

5. $4a^4 - 13a^2 + 1$

$(2a^2 + 3a - 1)(2a^2 - 3a - 1)$

6. $9a^4 + 26a^2b^2 + 25b^4$

$(3a^2 + 2ab + 5b^2)(3a^2 - 2ab + 5b^2)$

7. $4x^4 - 21x^2y^2 + 9y^4$

$(2x^2 + 3xy - 3y^2)(2x^2 - 3xy - 3y^2)$

8. $4a^4 - 29a^2c^2 + 25c^4$

$(2a + 5c)(a - c)(2a - 5c)(a + c)$

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9-6 Study Guide and Intervention

Perfect Squares and Factoring

Factor Perfect Square Trinomials

Perfect Square Trinomial	a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The patterns shown below can be used to factor perfect square trinomials.

Squaring a Binomial	Factoring a Perfect Square Trinomial
$(a + 4)^2 = a^2 + 2(a)(4) + 4^2$ $= a^2 + 8a + 16$	$a^2 + 8a + 16 = a^2 + 2(a)(4) + 4^2$ $= (a + 4)^2$
$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$ $= 4x^2 - 12x + 9$	$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ $= (2x - 3)^2$

Example 1 **Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.**

Since $16n^2 = (4n)(4n)$, the first term is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

Therefore, $16n^2 - 24n + 9$ is a perfect square trinomial.

$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2$
$= (4n - 3)^2$

Example 2 **Factor $16x^2 - 32x + 15$.**

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$ — Original trinomial
$= 16x^2 + mx + nx + 15$ — Write the pattern.
$= 16x^2 - 12x - 20x + 15$ — $m = -12$ and $n = -20$
$= (16x^2 - 12x) - (20x - 15)$ — Group terms.
$= 4x(4x - 3) - 5(4x - 3)$ — Find the GCF.
$= (4x - 5)(4x - 3)$ — Factor by grouping.

Therefore $16x^2 - 32x + 15 = (4x - 5)(4x - 3)$.

Exercises

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $x^2 - 16x + 64$ **yes; $(x - 8)(x - 8)$**
2. $m^2 + 10m + 25$ **yes; $(m + 5)(m + 5)$**
3. $p^2 + 8p + 64$ **no**

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

4. $98x^2 - 200y^2$ **$2(7x + 10y)(7x - 10y)$**
5. $x^2 + 22x + 121$ **$(x + 11)^2$**
6. $81 + 18s + s^2$ **$(9 + s)^2$**
7. $25c^2 - 10c - 1$ **prime**
8. $169 - 26r + r^2$ **$(13 - r)^2$**
9. $7x^2 - 9x + 2$ **$(7x - 2)(x - 1)$**
10. $16m^2 + 48m + 36$ **$4(2m + 3)^2$**
11. $16 - 25a^2$ **$(4 + 5a)(4 - 5a)$**
12. $b^2 - 16b + 256$ **prime**
13. $36x^2 - 12x + 1$ **$(6x - 1)^2$**
14. $16a^2 - 40ab + 25b^2$ **$(4a - 5b)^2$**
15. $8m^3 - 64m$ **$8m(m^2 - 8)$**

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9-6 Study Guide and Intervention *(continued)*

Perfect Squares and Factoring

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **square root property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example **Solve each equation. Check your solutions.**

a. $x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = 0$ — Original equation
$x^2 - 2(3x) + 3^2 = 0$ — Recognize a perfect square trinomial.
$(x - 3)(x - 3) = 0$ — Factor the perfect square trinomial.
$x - 3 = 0$ — Set repeated factor equal to 0.
$x = 3$ — Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $(a - 5)^2 = 64$

$(a - 5)^2 = 64$ — Original equation
$a - 5 = \pm\sqrt{64}$ — Square Root Property
$a - 5 = \pm 8$ — $64 = 8 \cdot 8$
$a = 5 \pm 8$ — Add 5 to each side.
$a = 5 + 8$ or $a = 5 - 8$ — Separate into 2 equations.
$a = 13$ $a = -3$ — Solve each equation.

The solution set is {−3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $x^2 + 4x + 4 = 0$ **{−2}**
2. $16n^2 + 16n + 4 = 0$ **$\left\{-\frac{1}{2}\right\}$**
3. $25d^2 - 10d + 1 = 0$ **$\left\{\frac{1}{5}\right\}$**
4. $x^2 + 10x + 25 = 0$ **{−5}**
5. $9x^2 - 6x + 1 = 0$ **$\left\{\frac{1}{3}\right\}$**
6. $x^2 + x + \frac{1}{4} = 0$ **$\left\{-\frac{1}{2}\right\}$**
7. $25k^2 + 20k + 4 = 0$ **$\left\{-\frac{2}{5}\right\}$**
8. $p^2 + 2p + 1 = 49$ **{−8, 6}**
9. $x^2 + 4x + 4 = 64$ **{−10, 6}**
10. $x^2 - 6x + 9 = 25$ **{−2, 8}**
11. $a^2 + 8a + 16 = 1$ **{−3, −5}**
12. $16y^2 + 8y + 1 = 0$ **$\left\{-\frac{1}{4}\right\}$**
13. $(x + 3)^2 = 49$ **{−10, 4}**
14. $(y + 6)^2 = 1$ **{−7, −5}**
15. $(m - 7)^2 = 49$ **{0, 14}**
16. $(2x + 1)^2 = 1$ **{−1, 0}**
17. $(4x + 3)^2 = 25$ **$\left\{-2, \frac{1}{2}\right\}$**
18. $(3h - 2)^2 = 4$ **$\left\{\frac{4}{3}, 0\right\}$**
19. $(x + 1)^2 = 7$ **$\{-1 \pm \sqrt{7}\}$**
20. $(y - 3)^2 = 6$ **$\{3 \pm \sqrt{6}\}$**
21. $(m - 2)^2 = 5$ **$\{2 \pm \sqrt{5}\}$**

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9-6 Skills Practice

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $c^2 - 6c + 9$ **yes; $(c - 3)^2$**

2. $r^2 + 4r + 4$ **yes; $(r + 2)^2$**

3. $g^2 - 14g + 49$ **yes; $(g - 7)^2$**

4. $2w^2 - 4w + 9$ **no**

5. $4d^2 - 4d + 1$ **yes; $(2d - 1)^2$**

6. $9n^2 + 30n + 25$ **yes; $(3n + 5)^2$**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $2x^2 - 72$ **$2(x + 6)(x - 6)$**

8. $6b^2 + 11b + 3$ **$(2b + 3)(3b + 1)$**

9. $36t^2 - 24t + 4$ **$4(3t - 1)^2$**

10. $4h^2 - 56$ **$4(h^2 - 14)$**

11. $17a^2 - 24ac$ **$a(17a - 24c)$**

12. $q^2 - 14q + 36$ **prime**

13. $y^2 + 24y + 144$ **$(y + 12)^2$**

14. $6d^2 - 96$ **$6(d - 4)(d + 4)$**

15. $4k^2 + 12k + 9$ **$(2k + 3)^2$**

16. $6x^2 + 28x - 10$ **$2(x + 5)(3x - 1)$**

Solve each equation. Check your solutions.

17. $x^2 - 18x + 81 = 0$ **$\{9\}$**

18. $4p^2 + 4p + 1 = 0$ **$\left\{-\frac{1}{2}\right\}$**

19. $9g^2 - 12g + 4 = 0$ **$\left\{\frac{2}{3}\right\}$**

20. $y^2 - 16y + 64 = 81$ **$\{-1, 17\}$**

21. $4n^2 - 17 = 19$ **$\{\pm 3\}$**

22. $x^2 + 30x + 150 = -75$ **$\{-15\}$**

23. $(k + 2)^2 = 16$ **$\{-6, 2\}$**

24. $(m - 4)^2 = 7$ **$\{4 \pm \sqrt{7}\}$**

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9-6 Practice (Average)

Perfect Squares and Factoring

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

1. $m^2 + 16m + 64$ **yes; $(m + 8)^2$**

2. $9s^2 - 6s + 1$ **yes; $(3s - 1)^2$**

3. $4y^2 - 20y + 25$ **yes; $(2y - 5)^2$**

4. $16p^2 + 24p + 9$ **yes; $(4p + 3)^2$**

5. $25b^2 - 4b + 16$ **no**

6. $49k^2 - 56k + 16$ **yes; $(7k - 4)^2$**

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

7. $3p^2 - 147$ **$3(p + 7)(p - 7)$**

8. $6x^2 + 11x - 35$ **$(2x + 7)(3x - 5)$**

9. $50q^2 - 60q + 18$ **$2(5q - 3)^2$**

10. $6t^3 - 14t^2 - 12t$ **$2t(3t + 2)(t - 3)$**

11. $6d^2 - 18$ **$6(d^2 - 3)$**

12. $30k^2 + 38k + 12$ **$2(5k + 3)(3k + 2)$**

13. $15b^2 - 24bc$ **$3b(5b - 8c)$**

14. $12h^2 - 60h + 75$ **$3(2h - 5)^2$**

15. $9n^2 - 30n - 25$ **prime**

16. $7u^2 - 28m^2$ **$7(u - 2m)(u + 2m)$**

17. $w^4 - 8w^2 - 9$ **$(w^2 + 1)(w + 3)(w - 3)$**

18. $16c^2 + 72cd + 81d^2$ **$(4c + 9d)^2$**

Solve each equation. Check your solutions.

19. $4k^2 - 28k = -49$ **$\left\{\frac{7}{2}\right\}$**

20. $50b^2 + 20b + 2 = 0$ **$\left\{-\frac{1}{5}\right\}$**

21. $\left(\frac{1}{2}t - 1\right)^2 = 0$ **$\{2\}$**

22. $g^2 + \frac{2}{3}g + \frac{1}{9} = 0$ **$\left\{-\frac{1}{3}\right\}$**

23. $p^2 - \frac{6}{5}p + \frac{9}{25} = 0$ **$\left\{\frac{3}{5}\right\}$**

24. $x^2 + 12x + 36 = 25$ **$\{-11, -1\}$**

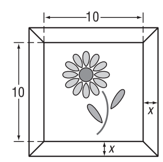
25. $y^2 - 8y + 16 = 64$ **$\{-4, 12\}$**

26. $(h + 9)^2 = 3$ **$\{-9 \pm \sqrt{3}\}$**

27. $w^2 - 6w + 9 = 13$ **$\{3 \pm \sqrt{13}\}$**

28. GEOMETRY The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. If increasing the radius of a circle by 1 inch gives the resulting circle an area of 100π square inches, what is the radius of the original circle? **9 in.**

29. PICTURE FRAMING Mikaela placed a frame around a print that measures 10 inches by 10 inches. The area of just the frame itself is 69 square inches. What is the width of the frame? **1.5 in.**



NAME ______________ DATE ______ PERIOD ____

9-6 Reading to Learn Mathematics

Perfect Squares and Factoring

Pre-Activity **How can factoring be used to design a pavilion?**

Read the introduction to Lesson 9-6 at the top of page 508 in your textbook.

- On the left side of the equation $(8 + 2x)^2 = 144$, the number 8 in the expression $(8 + 2x)^2$ represents **the length of the pavilion** and $2x$ represents twice **the width of the pavilion**.
- On the right side of the equation, the number 144 represents **the area of the school mascot** in the center of the pavilion, plus the **area** of the bricks surrounding the center mascot.

Reading the Lesson

1. Three conditions must be met if a trinomial can be factored as a **perfect square trinomial**. Complete the following sentences.

   The first term of the trinomial $9x^2 - 6x + 1$ **is** (is/is not) a perfect square.

   The last term of the trinomial, **is** (is/is not) a perfect square.

   The **middle term** is equal to $2(3x)(1)$.

   The trinomial $9x^2 - 6x + 1$ **is** (is/is not) a **perfect square** trinomial.

2. Match each polynomial from the first column with a factoring technique in the second column. Some of the techniques may be used more than once. If none of the techniques can be used to factor the polynomial, write *none*.

a. $9x^2 - 64$ **iii**	**i.** factor as $x^2 + bx + c$
b. $9x^2 + 12x + 4$ **v**	**ii.** factor as $ax^2 + bx + c$
c. $x^2 - 5x + 6$ **i**	**iii.** difference of squares
d. $4x^2 + 13x + 9$ **ii**	**iv.** factoring by grouping
e. $9xy + 3y + 6x + 2$ **iv**	**v.** perfect square trinomial
f. $x^2 - 4x + 4$ **v**	**vi.** factor out the GCF
g. $2x^2 - 16$ **vi**	

Helping You Remember

3. Sometimes it is easier to remember a set of instructions if you can state them in a short sentence or phrase. Summarize the conditions that must be met if a trinomial can be factored as a perfect square trinomial. **Sample answer: The first and last terms are perfect squares, and the middle term is twice the product of the square roots of the first and last terms.**

NAME ______________ DATE ______ PERIOD ____

9-6 Enrichment

Squaring Numbers: A Shortcut

A shortcut helps you to square a positive two-digit number ending in 5. The method is developed using the idea that a two-digit number may be expressed as $10t + u$. Suppose $u = 5$.

$$(10t + 5)^2 = (10t + 5)(10t + 5)$$
$$= 100t^2 + 50t + 50t + 25$$
$$= 100t^2 + 100t + 25$$
$$(10t + 5)^2 = 100t(t + 1) + 25$$

In words, this formula says that the square of a two-digit number has $t(t + 1)$ in the hundreds place. Then 2 is the tens digit and 5 is the units digit.

Example **Using the formula for $(10t + 5)^2$, find 85^2.**

$85^2 = 100 \cdot 8 \cdot (8 + 1) + 25$
$= 7200 + 25$
$= 7225$ Shortcut: First think $8 \cdot 9 = 72$. Then write 25.

Thus, to square a number, such as 85, you can write the product of the tens digit and the next consecutive integer $t + 1$. Then write 25.

Find each of the following using the shortcut.

1. 15^2 **225**	**2.** 25^2 **625**	**3.** 35^2 **1225**
4. 45^2 **2025**	**5.** 55^2 **3025**	**6.** 65^2 **4225**

Solve each problem.

7. What is the tens digit in the square of 95? **2**

8. What are the first two digits in the square of 75? **56**

9. Any three-digit number can be written as $100a + 10b + c$. Square this expression to show that if the last digit of a three-digit number is 5 then the last two digits of the square of the number are 2 and 5.
   $10{,}000a^2 + 2000ab + 200ac + 100b^2 + 20bc + c^2 =$
   $10{,}000a^2 + 2000ab + 1000a + 100b^2 + 100b + 25$
   The last two digits are not affected by the first five terms.

Chapter 9 Assessment Answer Key

Form 1
Page 559

1. C
2. B
3. D
4. C
5. A
6. B
7. B
8. D
9. B
10. D
11. A

Page 560

12. B
13. D
14. D
15. B
16. C
17. A
18. A
19. D
20. B

B: 5 and 7 or −5 and −7

Form 2A
Page 561

1. C
2. D
3. B
4. D
5. C
6. C
7. A
8. D
9. B
10. D
11. B

(continued on the next page)

Chapter 9 Assessment Answer Key

Form 2A *(continued)*
Page 562

12. A
13. A
14. D
15. C
16. A
17. B
18. D
19. D
20. C

B: 25

Form 2B
Page 563

1. D
2. B
3. B
4. C
5. B
6. B
7. A
8. D
9. A
10. B
11. D

Page 564

12. C
13. C
14. D
15. C
16. A
17. B
18. D
19. A
20. B

B: $2(n + 3)(n - 3)(n^2 + 4)$

Answers

Chapter 9 Assessment Answer Key

Form 2C

Page 565

1. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48; composite
2. $-1 \cdot 2^5 \cdot 3$
3. $4xy^2$
4. 1
5. $5a^2bc(7ac - 9b)$
6. $(3y - 4)(x + 2)$
7. $(t - 8)(t - 3)$
8. $(n + 7)(n - 6)$
9. $(5y - 3)(2y - 5)$
10. $4(2n - 5)(n - 2)$
11. $(6m + 7)(6m - 7)$
12. $2x^2(x + 3)(x - 3)$
13. $(5w - 6)^2$
14. prime

Page 566

15. $\left\{-\frac{2}{3}, 2\right\}$
16. $\left\{0, \frac{1}{2}\right\}$
17. $\{-10, 11\}$
18. $\left\{\frac{1}{2}, 1\right\}$
19. $\left\{\frac{7}{6}\right\}$
20. $\left\{-\frac{7}{2}, -\frac{1}{2}\right\}$
21. $\left\{-\frac{5}{7}, \frac{5}{7}\right\}$
22. $\{-4, 0, 4\}$
23. $1\frac{1}{2}$ s
24. width is 14 ft; length is 17 ft
25. -7 and -35 or 7 and 35

B: $(v + 3)(v - 3)(x^2 + n^2)$

Chapter 9 Assessment Answer Key

Form 2D

Page 567

1. 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, 140; composite
2. $-1 \cdot 2^2 \cdot 3 \cdot 7$
3. $3xy^3$
4. 1
5. $2x^2yz(5 - 11xy)$
6. $(y - 2)(2x + 3)$
7. $(m + 14)(m - 2)$
8. $(r - 8)(r + 7)$
9. $(5t - 3)(t + 4)$
10. $2(3p - 4)(p - 2)$
11. $(7a + 13)(7a - 13)$
12. $3x^3(x + 5)(x - 5)$
13. $(9c + 4)^2$
14. prime

Page 568

15. $\left\{-5, \frac{3}{4}\right\}$
16. $\left\{0, \frac{2}{3}\right\}$
17. $\{-9, 5\}$
18. $\left\{-1, \frac{1}{3}\right\}$
19. $\left\{\frac{9}{5}\right\}$
20. $\left\{-\frac{1}{2}, \frac{5}{2}\right\}$
21. $\left\{-\frac{1}{8}, \frac{1}{8}\right\}$
22. $\{-3, 0, 3\}$
23. $\frac{3}{4}$ s
24. width is 8 ft; length is 13 ft
25. −6 and −42 or 6 and 42

B: 16

Answers

Chapter 9 Assessment Answer Key

Form 3

Page 569

1. 1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168; composite
2. $-1 \cdot 2 \cdot 3^2 \cdot 7 \cdot 11$
3. $12n^3t$
4. $3x$
5. $4x^2y^2z(3x - 6y + 4yz^2)$
6. $(2x + 3)(2x - 3)(y^2 + 5)$
7. $-1(x - 8)(x + 3)$
8. $2(x - 12)(x + 3)$
9. $(5x - 3)(2x + 7)$
10. prime
11. $(a + n + 3)(a + n - 3)$
12. $3x(x - 4y)^2$
13. $(x + 5)(x - 5)(3x^2 + 2)$
14. prime

Page 570

15. $\left\{0, \frac{11}{3}\right\}$
16. $\{-16, 2\}$
17. $\left\{-\frac{7}{3}, -\frac{1}{3}\right\}$
18. $\left\{-1, \frac{5}{3}\right\}$
19. $\left\{\frac{11}{4}\right\}$
20. $\{-1, 4\}$
21. $-7, -2, 2, 7$
22. $4y^2$
23. $-17, -15, -13$
24. $\frac{1}{2}$ s
25. $\frac{1}{3}$ in.

B: $\left\{-\frac{7}{6}, 0\right\}$

Chapter 9 Assessment Answer Key

Page 571, Open-Ended Assessment
Scoring Rubric

Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *finding factors of integers, factoring polynomials,* and *using the Zero Product Property to solve equations.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 9 Assessment Answer Key

Page 571, Open-Ended Assessment
Sample Answers

In addition to the scoring rubric found on page A25, the following sample answers may be used as guidance in evaluating open-ended assessment items.

1a. The student should recognize that $A = x(x + b)$ is an equation and $x^2 + bx - A$ is a trinomial expression. When the equation is put into standard form, the trinomial expression is one side of the equation. For example, $A = x(x + b)$ is equivalent to $x^2 + bx - A = 0$.

1b. For $x^2 + bx - A$ to factor, there must be factors of $-A$ whose sum is b.

1c. Sample answer: $A = 144$, $b = 10$;
$x^2 + 10x - 144 = (x + 18)(x - 8)$

1d. Sample answer: $144 = x(x + 10)$;
144 sq ft; width is 8 ft, length is 18 ft

2a. The student should recognize that both c and h must be positive to be above ground, and since c is the starting height and h is equal to c minus $16t^2$ where t^2 is always positive, h will always be less than or equal to c.

2b. Sample answer: $c = 4$; $\frac{1}{2}$ s

2c. The student should explain that $c = 9$ and $h = 0$ yields a solution set of $\left\{-\frac{3}{4}, \frac{3}{4}\right\}$. Since we are considering only the time after the ball is thrown, the ball is only in the air for $\frac{3}{4}$ second. Thus, a ball thrown horizontally at a height of 9 feet cannot stay above the ground for more than 1 second.

3a. Sample answer: Choose a value for x and evaluate $x^2 - 8x + 15$.
$x^2 - 8x + 15 = 10^2 - 8(10) + 15 = 35$
A factorization of 35 is 5×7. Since $x = 10$, $5 = x - 5$ and $7 = x - 3$.
Check the product $(x - 5)(x - 3)$.
$(x - 5)(x - 3) = x^2 - 3x - 5x + 15$
$= x^2 - 8x + 15$

3b. Sample answer: Choose a value for x and evaluate $2x^2 - 13x - 24$.
$2x^2 - 13x - 24 = 2(10)^2 - 13(10) - 24$
$= 46$
A factorization of 46 is 2×23.
Since $x = 10$, $2 = x - 8$ and $23 = 2x + 3$.
Check the product $(x - 8)(2x + 3)$.
$(x - 8)(2x + 3) = 2x^2 + 3x - 16x - 24$
$= 2x^2 - 13x - 24$ ✔

3c. Sample answer: Choose a value for x and evaluate $x^2 - 2x - 8$.
$x^2 - 2x - 8 = 10^2 - 2(10) - 8 = 72$
A factorization of 72 is 6×12. Since $x = 10$, $6 = x - 4$ and $12 = x + 2$.
Check the product $(x - 4)(x + 2)$.
$(x - 4)(x + 2) = x^2 + 2x - 4x - 8$
$= x^2 - 2x - 8$
The student should explain that there are more factors for 72 than for the numbers in the previous parts, thus making the process more difficult in finding the correct factors.

3d. Sample answer: Evaluate $x^2 - 2x - 8$ for $x = 5$.
$x^2 - 2x - 8 = 5^2 - 2(5) - 8 = 7$
The only factorization of 7 is 1×7.
Since $x = 5$, $1 = x - 4$ and $7 = (x + 2)$.
Check the product $(x - 4)(x + 2)$.
$(x - 4)(x + 2) = x^2 + 2x - 4x - 8$
$= x^2 - 2x - 8$ ✔

Chapter 9 Assessment Answer Key

Vocabulary Test/Review
Page 572

1. d
2. h
3. g
4. e
5. c
6. i
7. a
8. b
9. f
10. Sample answer: Factoring means writing a number or polynomial as a product of factors.
11. Sample answer: The greatest common factor of two or more monomials is the product of their common factors.

Quiz (Lessons 9–1 and 9–2)
Page 573

1. 1, 5, 7, 25, 35, 175; composite
2. $-1 \cdot 2^5 \cdot 5$
3. $3 \cdot 11 \cdot a \cdot a \cdot a \cdot b \cdot b$
4. 6
5. $4xy$
6. $12ab(4ab - 1)$
7. $3(2x^2y - 7y^2w + 8xw)$
8. $(x + 5)(y - 2z)$
9. $\left\{-4, \frac{5}{3}\right\}$
10. $\{-11, 0\}$

Quiz (Lesson 9–3)
Page 573

1. $(a - 3)(a - 7)$
2. $(x + 4)(x + 5)$
3. $\{-3, 9\}$
4. $\{-24, 1\}$
5. −15 and −13 or 13 and 15

Quiz (Lessons 9–4 and 9–5)
Page 574

1. $(2x + 1)(x + 3)$
2. prime
3. $\left\{\frac{2}{3}, 3\right\}$
4. $\left\{-\frac{3}{2}, \frac{2}{5}\right\}$
5. $(a - 5)(a + 5)$
6. $(7x + 8y)(7x - 8y)$
7. $(x + 2)(x - 2)(x + 3)$
8. $\left\{\pm\frac{9}{4}\right\}$
9. $\left\{\pm\frac{15}{2}\right\}$

10a. $A = \frac{9}{2} - \frac{1}{2}x^2$

b. $\frac{3}{2}$

Quiz (Lesson 9–6)
Page 574

1. perfect square trinomial; $(a + 7)^2$
2. not a perfect square trinomial
3. $2m(2m - 3)^2$
4. $\left\{\frac{1}{4}\right\}$
5. $\{-10, 4\}$

Answers

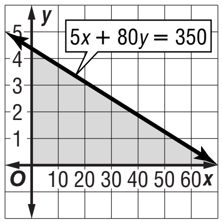
Chapter 9 Assessment Answer Key

Mid-Chapter Test
Page 575

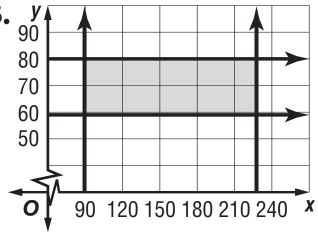
1. C
2. D
3. B
4. A
5. D
6. A
7. C
8. 1, 2, 4, 8, 17, 34, 68, 136; composite
9. 4 students
10. $12xy(3y - 4x)$
11. $(e - 4)(e - 12)$
12. $(2y - 1)(x + 2)$
13. $\left\{-5, \frac{7}{3}\right\}$
14. $\{0, 16\}$
15. $\{-8, 2\}$

Cumulative Review
Page 576

1. \$97.20
2. $\{(-2, -4), (0, -3), (2, -2), (6, 0), (8, 1)\}$
3. 8 ft per yr
4.
5. $5x + 80y \le 350$;



6. infinitely many
7. $\left(2\frac{1}{3}, 1\right)$
8.



9. $16x^8y^{12}$
10. $24y^3 + 42y^2 - y - 10$
11. $2x^3 - 5x^2 - 11x - 4$
12. $2 \cdot 3 \cdot 13$
13. $4a^2b^2(3 - 4b)$
14. $\left\{\frac{1}{5}, 1\right\}$
15. $(2x + 7y)(2x - 7y)$

Chapter 9 Assessment Answer Key

Standardized Test Practice

Page 577

1. (A) **(B)** (C) (D)

2. **(E)** (F) (G) (H)

3. (A) (B) (C) **(D)**

4. **(E)** (F) (G) (H)

5. (A) (B) **(C)** (D)

6. (E) (F) (G) **(H)**

7. (A) **(B)** (C) (D)

8. **(E)** (F) (G) (H)

9. (A) (B) (C) **(D)**

10. (E) **(F)** (G) (H)

Page 578

11. 3

12. 8710

13. 7/3

14. 28

15. (A) **(B)** (C) (D)

16. **(A)** (B) (C) (D)

17. **(A)** (B) (C) (D)

Answers