MATHCOUNTS®

2009
■ National Competition ■
Target Round
Problems 1 and 2

Name _____

State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record only final answers in the designated blanks on the problem sheet. All answers must be complete, legible and simplified to lowest terms. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed.

Total Correct	Scorer's Initials

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1. Clark bought a new cape. The cape was reduced from the original price by 20% and then a 5% sales tax was added. If he received $14.72 in change from a $50 bill, what was the original price of the cape?

1. $ _____

2. The volume of a cube is numerically equal to six times the sum of the lengths of its edges. What is the volume, in cubic units, of the cube? Express your answer in simplest radical form.

2. _____ cu units

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Problems 3 and 4

Name _____

State _____

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General Motors Foundation * Lockheed Martin
3M Foundation * Texas Instruments Incorporated

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3. Dr. Math's four-digit house number ABCD contains no zeroes and can be split into two different two-digit primes "AB" and "CD" where the digits A, B, C and D are not necessarily distinct. If each of the two-digit primes is less than 40, how many such house numbers are possible?

3. _____ house
.numbers

4. In the base-three number system, $a = 2012_3$ and $b = 201_3$. What is the product ab expressed in the base-three number system?

4. _____ base 3

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Target Round
Problems 5 and 6

Name _____

State _____

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5. Four distinct points on line n (A, B, C, D) and five distinct points on line m (E, F, G, H, I) are chosen to maximize the number of intersections in the shaded region among the 20 unique line segments connecting one of the four points on n to one of the five points on m. What is the maximum number of intersections in the shaded region, excluding the intersections at points on lines m or n?

6. Given that $n > 1$, what is the smallest positive integer n whose divisors have a product of n^6?

6. _____

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2009
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Target Round
Problems 7 and 8

Name _____

State _____

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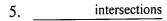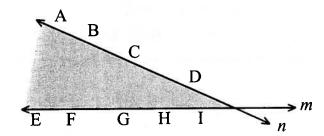General Motors Foundation * Lockheed Martin
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7. A tank holds 100 gallons of a thoroughly mixed solution which is 50% alcohol. Twenty-five gallons are drained from the tank and replaced with a solution that is 20% alcohol. The solution in the tank is thoroughly mixed. This procedure is repeated twice more. What percentage of the final solution is alcohol? Express your answer to the nearest whole number.

7. _____ %

8. A right square pyramid with base edges of length $8\sqrt{2}$ units each and slant edges of length 10 units each is cut by a plane that is parallel to its base and 3 units above its base. What is the volume, in cubic units, of the new pyramid that is cut off by this plane?

8. _____ cu units

10 units

$8\sqrt{2}$ units