MATHCOUNTS®

2009
■ National Competition ■
Sprint Round
Problems 1–30

Name _____

State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

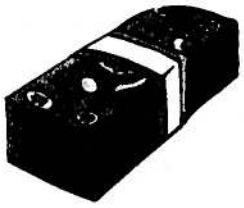
This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. Arika worked 40 hours and earned \$500. LaTisha worked 50 hours and earned \$600. How many more cents per hour did Arika earn than LaTisha?

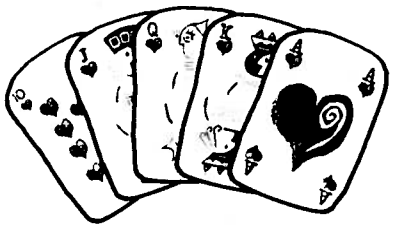
1. _____ cents



2. There are 200 students enrolled at Memorial Middle School. Seventy of the students are in band and 95 are in chorus. If only 150 students are in band and/or chorus, how many students are in both band and chorus?

2. _____ students

3. Dennis was playing FreeCell. If he completed one game every five minutes and won exactly 80% of the games he played, how many minutes did he stay at the computer to win exactly 60 games?



3. _____ minutes

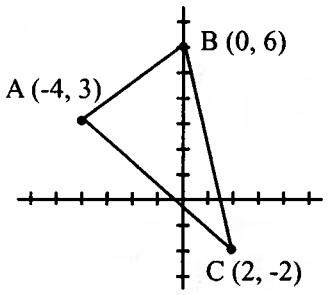
4. Abe, Bobby, Charles, Devin and Edwin are the participants in a race. How many different 1st–2nd–3rd place outcomes are possible if there are no ties? Two different outcomes to include are Bobby–Devin–Edwin and Devin–Bobby–Edwin.

4. _____ outcomes

5. What is the sum of all integer values of x such that $\frac{3}{x}$ is greater than $\frac{1}{3}$ and less than $\frac{3}{4}$?

5. _____

6. What is the area, in square units, of triangle ABC?



6. _____ sq units

7. The mean of a sequence of r numbers is 10. When the sequence is extended to $r + 2$ terms to include the numbers 20 and 30, the mean of the sequence increases to 12. What is the value of r?

7. _____

8. Both a and b are positive integers and $b > 1$. When a^b is the greatest possible value less than 399, what is the sum of a and b?

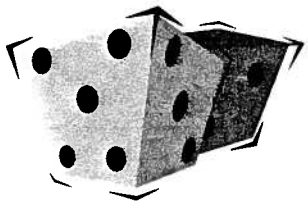
8. _____

9. The mean of five positive integers is 1.5 times the median of the five integers. If the five integers are 8, 36, 18, 62 and x, and no integer is larger than 100, what is the value of x?

9. _____

10. A car travels the 120 miles from A to B at 60 miles per hour, and then returns to A on the same road. If the average rate of the round trip is 45 miles per hour, what is the rate, in miles per hour, of the car traveling back from B to A?

10. _____ mph



11. Mike and Ike each roll two standard six-sided dice. What is the probability that the positive difference between their sums is greater than eight? Express your answer as a common fraction.

11. _____

12. A subset S of the set of integers from 50 to 100, inclusive, has the property that no two distinct elements of S sum to 130. What is the maximum possible number of elements in S?

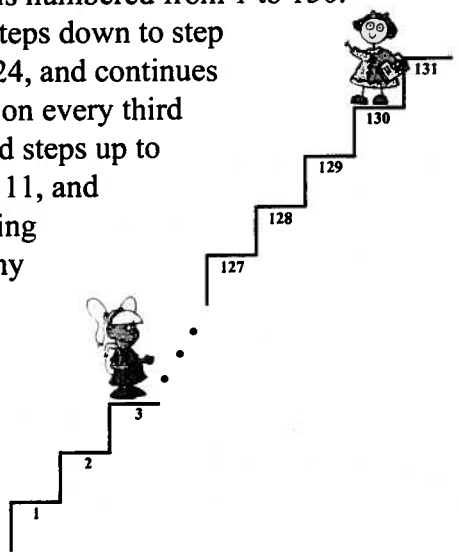
12. _____ elements

13. How many even divisors does 7! have?

13. _____ even divisors

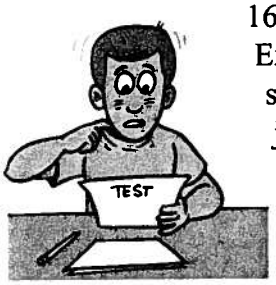
14. A stadium staircase with 150 steps is numbered from 1 to 150. Jan starts on step number 130 and steps down to step number 127, then to step number 124, and continues downward to step 13 stepping only on every third step. Jen starts on step number 3 and steps up to step number 7, then to step number 11, and continues upward to step 139 stepping only on every fourth step. How many steps were stepped on by both Jan and Jen (not necessarily at the same time)?

14. _____ steps



15. If r is a root of $x^2 + 2x - 15 = 0$, what is the greatest possible value of $\frac{r^3-1}{r^5+r^4-r^3-r^2}$? Express your answer as a common fraction.

15. _____



16. A group of juniors and seniors took a test. Exactly $\frac{3}{5}$ of the juniors and exactly $\frac{6}{7}$ of the seniors passed the test. If the number of juniors who passed the test was $\frac{2}{3}$ of the number of seniors who passed the test, what fraction of the entire group passed the test? Express your answer as a common fraction.

16. _____

17. The reduced value of the fraction $\frac{m}{n}$ is $\frac{1}{4}$. The reduced value of the fraction $\frac{m^2}{n}$ is 2. What is the value of $m + n$?

17. _____

18. What is the largest integer n such that 3^n is a factor of $1 \times 3 \times 5 \times \ldots \times 97 \times 99$?

18. _____

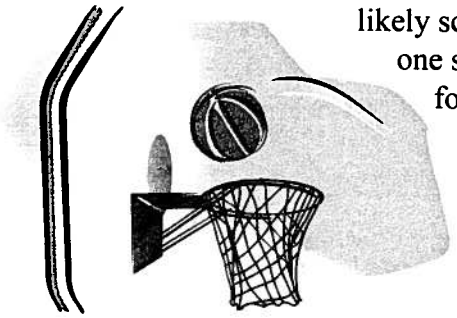
19. How many different positive three-digit integers can be formed using only the digits in the set {2, 3, 5, 5, 5, 6, 6} if no digit may be used more times than it appears in the given set of available digits?

19. _____ integers

20. What is the shortest distance, in units, between the circles $(x-9)^2 + (y-5)^2 = 6.25$ and $(x+6)^2 + (y+3)^2 = 49$? Express your answer as a decimal to the nearest tenth.

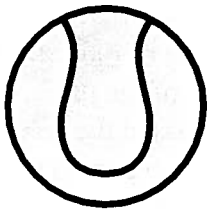
20. _____ units

21. When Sam shoots a basketball he makes 75% of his shots when he is 15 feet away from the basket and 25% of his shots when he is 20 feet away from the basket. He tries a new game: when he makes a shot he moves to the other distance, and when he misses a shot he keeps shooting from that same distance. He starts shooting from the 15-foot distance. What is the probability of the most likely scenario in which he makes exactly one shot from each distance in his first four shots? Express your answer as a common fraction.



21. _____

22. A tennis coach divides her 9-player squad into three 3-player groups with each player in only one group. How many different sets of three groups can be made?



22. _____ sets

23. Triangle ABC has vertices at A(5, 8), B(3, −2) and C(6, 1). The point D with coordinates (m, n) is chosen inside the triangle so that the three small triangles ABD, ACD and BCD all have equal areas. What is the value of $10m + n$?

23. _____

24. A positive integer is called n-primable if it is divisible by n and each of its digits is a one-digit prime number. How many 3-primable positive integers are there that are less than 1000?

24. _____ integers

25. The three side lengths of a particular triangle are 2, 5 and x units, and the area of the triangle is x square units. What is the value of x? Express your answer in simplest radical form.

25. _____

26. The letters A, B, C, D, E and F represent digits and ABC,DEF represents a positive six-digit integer. What is the number ABC,DEF if 4(ABC,DEF) = 3(DEF,ABC)?

26. _____

27. In triangle ABC, $m\angle A = 90°$, $m\angle B = 75°$, and BC = $\sqrt{3}$ units. What is the area of triangle ABC? Express your answer as a common fraction.

27. _____ sq units

28. What is the value of $\sqrt{15-6\sqrt{6}}+\sqrt{15+6\sqrt{6}}$?

28. _____

29. When $(x + 2y - z)^8$ is expanded, and the like terms are combined, what is the coefficient of the term $x^3y^2z^3$?

29. _____

30. In a trapezoid ABCD with AB parallel to CD, the diagonals AC and BD intersect at E. If the area of triangle ABE is 50 square units, and the area of triangle ADE is 20 square units, what is the area of trapezoid ABCD?

30. _____ sq units