MATHCOUNTS®

2006
■ **State Competition** ■
SPRINT ROUND
Problems 1–30

Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. For Field Day, the Poe M.S. students will be assigned to teams. No team will have more than eight students and each student will be on exactly one team. If there are 128 students at Poe M.S., what is the smallest number of teams that can be formed?

1. _____ teams

2. This graph shows the number of days in October that Carla studied for particular periods of time. For example, on five different days in October she studied for exactly one hour. On how many days in October did Carla study three or more hours?
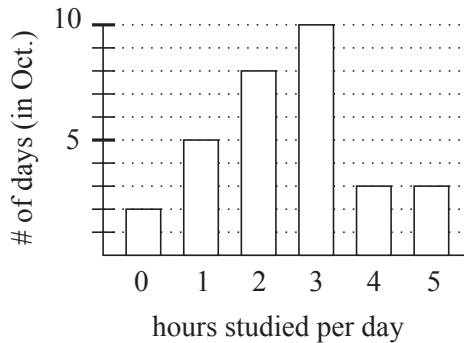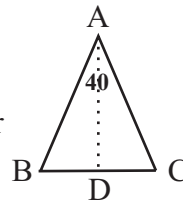

2. _____ days

3. Sasha has $3.20 in U.S. coins. She has the same number of quarters and nickels. What is the greatest number of quarters she could have?
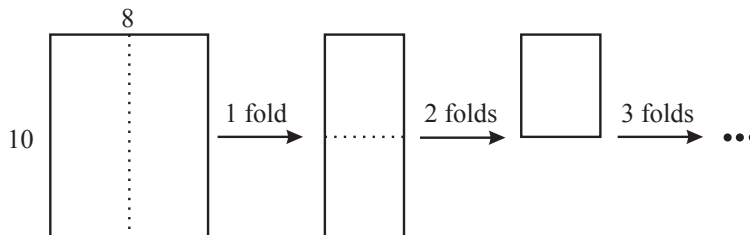
3. _____ quarters

4. In triangle ABC, point D is on segment BC, the measure of angle BAC is 40 degrees, and triangle ABD is a reflection of triangle ACD over segment AD. What is the measure of angle B?

4. _____ degrees

5. Lizzie folded a piece of 8-inch by 10-inch paper in half again and again (without ever unfolding it) until she ended up with a folded piece that measured 1 inch by 2.5 inches. How many times did she fold the paper in half?
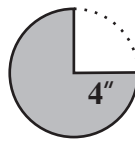

5. _____ times

6. Yesterday Allen's heart beat 100,000 times. On average, how many times did it beat in a six-hour period yesterday?

6. _____ times

7. A paper cone is to be made from a three-quarter circle having radius 4 inches (shaded). What is the length of the arc on the discarded quarter-circle (dotted portion)? Express your answer in terms of π.

7. _____ inches

8. The letters of the alphabet are given numeric values based on the two conditions below.
   • Only the numeric values of –2, –1, 0, 1 and 2 are used.
   • Starting with A and going through Z, a numeric value is assigned to each letter according to the following pattern:
   1, 2, 1, 0, –1, –2, –1, 0, 1, 2, 1, 0, –1, –2, –1, 0, ...
   Two complete cycles of the pattern are shown above. The letter A has a value of 1, B has a value of 2, F has a value of –2 and Z has a value of 2. What is the sum of the numeric values of the letters in the word "numeric?"

8. _____

9. What is the arithmetic mean (average) of the areas of all non-congruent rectangles with integer side lengths and perimeter 8 units? Express your answer as a decimal to the nearest tenth.

9. _____ sq units

10. What is the positive value of the expression $\sqrt{x^3 - 2^y}$ when $x = 5$ and $y = 2$?

10. _____

11. When the two-digit integer MM, with equal digits, is multiplied by the one-digit integer M, the result is the three-digit integer NPM. What is the greatest possible value of NPM?

11. _____

12. A 20-gallon container is filled halfway with a mixture that is 90% vinegar and 10% water. How many gallons of water must be added for the mixture to become 60% vinegar and 40% water?

12. _____ gallons

13. The area of the shaded region is 78 square inches. All angles are right angles and all measurements are given in inches. What is the perimeter of the non-shaded region?

10″

2″

10″

2″

4″
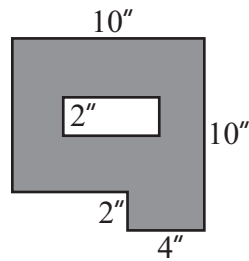
13. _____ inches

14. A robot moving forward at a constant speed takes 2.5 hours to travel 1 kilometer. Moving forward at this same constant speed, it takes the robot 90 seconds to travel the length of a particular hallway. How many meters long is the hallway?

14. _____ meters

15. A collection of five positive integers has mean 4.4, unique mode 3 and median 4. If an 8 is added to the collection, what is the new median? Express your answer as a decimal to the nearest tenth.

15. _____

16. A strip of paper consists of eight squares as shown. The strip is folded in half so that the right-most square (8) lands face-down and on top of the left-most square (1). Then, the new right-most square is again folded over on top of the new left-most square, and then again one more time, so the strip has been folded into a stack of single squares with the square labeled 1 still on the bottom. What number is in the square on the top of the stack?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
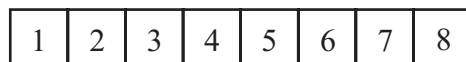
16. _____

17. Triangle ABC with vertices A(–2, 0), B(1, 4) and C(–3, 2) is reflected over the y-axis to form triangle A′B′C′. What is the length of a segment drawn from C to C′?

17. _____ units

18. Points A and B are located at $\frac{1}{8}$ and $\frac{1}{4}$, respectively, on a number line. What is the sum of the coordinates of the two points that trisect segment AB? Express your answer as a common fraction.

18. _____

19. In Mrs. Craig's statistics class there are twice as many 11th-grade students as there are 9th and 10th-grade students combined. There are twice as many 12th-grade students as 9th- grade students, and the number of 11th-grade students is ten times the number of 12th-grade students. If there are 32 students altogether in Mrs. Craig's statistics class, how many 12th-grade students are there?

19. _____ 12th-grade students

20. There is a point (x, y) in the first quadrant, and on the line $3x - 5y = 12$, for which the x-coordinate is three times the y-coordinate. What is the value of $x + y$ at that point?

20. _____

21. Four red candies and three green candies can be combined to make many different "flavors." Flavors are different if the percent red is different, so "3 red / 0 green" is the same flavor as "2 red / 0 green," and likewise "4 red / 2 green" is the same flavor as "2 red / 1 green." If a flavor is to be made using some or all of the seven candies, how many different flavors are possible?

21. _____ flavors

22. The sum of five different positive integers is 320. The sum of the greatest three integers in this set is 283. The sum of the greatest and least integers is 119. If x is the greatest integer in the set, what is the positive difference between the greatest possible value and least possible value for x?

22. _____

23. In regular pentagon PQRST, X is the midpoint of segment ST. What is the measure of angle XQS?

23. _____ degrees

24. In the game "Cover It Up" two standard six-faced dice are rolled and their sum determined. The player can then cover that number on the game board or any two numbers that have that sum. For example, if you roll a 3 and 5, you can cover the 8, the 1 and 7, the 2 and 6, or the 3 and 5. What is the probability of being able to cover the 9 on the first roll of the two dice? Express your answer as a common fraction.

24. _____

25. Jeremy made a Venn diagram showing the number of students in his class who own types of pets. There are 32 students in his class. In addition to the information in the Venn diagram, Jeremy knows half of the students have a dog, $\frac{3}{8}$ have a cat, six have some other pet and five have no pet at all. How many students have a cat, a dog and some other pet?
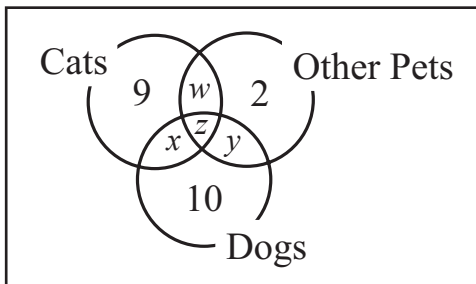
Cats ... 9 w 2 ... Other Pets
x z y
10
Dogs

26. The digits of a four-digit positive integer add up to 14. The sum of the two middle digits is nine, and the thousands digit minus the units digit is one. If the integer is divisible by 11, what is the integer?

26. _____

27. How many positive integers divisible by 4 can be formed using the digits 1, 2, 3 and 4, each at most once for each integer?
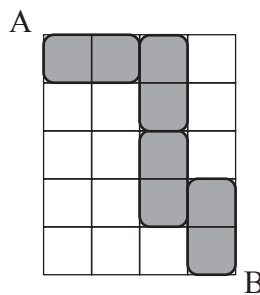
27. _____ integers

28. Derek's phone number, 336-7624, has the property that the three-digit prefix, 336, equals the product of the last four digits, $7 \times 6 \times 2 \times 4$. How many seven-digit phone numbers beginning with 336 have this property?

28. _____ phone numbers

29. Triangle ABC has vertices A(0, 0), B(0, 3) and C(5, 0). A point P inside the triangle is $\sqrt{10}$ units from point A and $\sqrt{13}$ units from point B. How many units is P from point C? Express your answer in simplest radical form.

29. _____ units

30. Matt will arrange four identical, dotless dominoes (shaded 1 by 2 rectangles) on the 5 by 4 grid to the right so that a path is formed from the upper left-hand corner A to the lower right-hand corner B. In a path, consecutive dominoes must touch at their sides and not just their corners. No domino may be placed diagonally; each domino covers exactly two of the unit squares shown on the grid. One arrangement is shown. How many distinct arrangements are possible, including the one shown?

A ... B

30. _____ arrangements