MATHCOUNTS®

2003
■ National Competition ■
Sprint Round
Problems 1–30

Name _____

School _____

State _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of 30 problems. You will have 40 minutes to complete. You are not allowed to use calculators, books or any other aids during this round. Calculator wrist watches should be given to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only your final answer in the answer blanks. If you complete the problems before time is called, use the remaining time to check your answers.

| Total Correct	Scorer's Initials

1. What is the positive difference between 120% of 30 and 130% of 20?

1. _____

2. Jack and Jill take a total of $80 to an amusement park. Including admission, Jack spends $24 and Jill spends $20. At the end of the day, Jack has twice as much money as Jill. How many dollars did Jill have at the end of the day?

2. _____

3. A quilter uses two different fabrics (white and black) in the pattern of squares and isosceles right triangles shown here. What fraction of the total area of the pattern shown here is the black fabric? Express your answer as a common fraction.
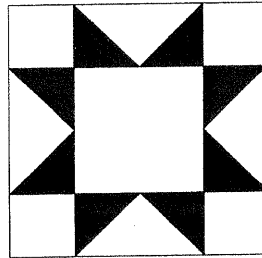
3. _____

4. What is the sum of the seven smallest distinct positive integer multiples of 9?

4. _____

5. The first three stages of a pattern are shown below, in which each line segment represents a toothpick. If the pattern continues such that at each successive stage three toothpicks are added to the previous arrangement, how many toothpicks are necessary to create the arrangement for the 250th stage?

5. _____
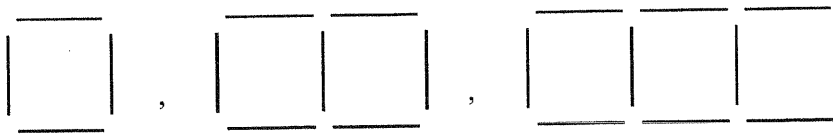
6. The arithmetic mean of three numbers x, y and z is 24. The arithmetic mean of x, $2y$ and $z - 7$ is 34. What is the arithmetic mean of x and z ? Express your answer as a decimal to the nearest tenth.

6. _____

7. Using each of the digits exactly once, place the digits 3, 4, 5 and 8, one in each box below, to make the sum as great as possible. What is the greatest possible sum? Express your answer as a common fraction.

$$\frac{\Box}{\Box} + \frac{\Box}{\Box}$$

7. _____

8. What is the greatest three-digit number "abc" such that 4, a, b forms a geometric sequence and b, c, 5 forms an arithmetic sequence?

8. _____

9. Twenty-four circles are drawn inside a rectangular region as shown, such that every circle is tangent to its neighbor circles and the rectangle is tangent to 16 of the circles. What is the ratio of the total area of the 24 circles to the area of the rectangle? Express your answer as a common fraction in terms of π .
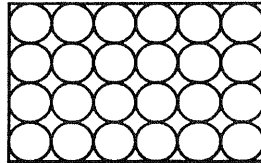
9. _____

10. In a horse race game on a computer, Secretariat, Man-Of-War, Affirmed and Citation finished in first through fourth places (not necessarily in that order), with no ties. Man-Of-War finished second or fourth. Affirmed did not win the race. Citation or Secretariat finished third. Man-Of-War beat Secretariat. What is the name of the horse that finished fourth?

10. _____

11. A triangle has vertices at $(-3, 2)$, $(6, -2)$ and $(3, 5)$. How many
    square units are in the area of the triangle? Express your answer as
    a decimal to the nearest tenth.

11. _____

12. Chris' running schedule dictates that he never runs more than
    two days in a row, nor does he go more than two days in
    a row without running. In two weeks, what is the
    positive difference between the fewest number of
    days and the greatest number of days that he could
    have run?

12. _____

13. The first term of a given sequence is 1, and each successive
    term is the sum of all the previous terms of the sequence. What
    is the value of the first term which exceeds 5000?

13. _____

14. For what value of a is there a right triangle with sides $a + 1$, $6a$
    and $6a + 1$?

14. _____

15. On a standard six-faced die, the numbers on opposite faces
    always add to seven. Dena rolls a pair of fair, standard six-
    faced dice. She then takes the product of four numbers: the
    two numbers on the faces that are showing on top of the dice
    and the two numbers on the faces which are hidden on the bottom
    of the dice. What is the greatest possible product of these four
    numbers?

15. _____

16. A, B, C and D are distinct positive integers such that the product AB = 60, the product CD = 60 and A − B = C + D. What is the value of A?

16. _____

17. When a positive three-digit dividend with a units digit of 2 is divided by a positive one-digit divisor, the result is a whole-number quotient with a remainder of 1. How many distinct values are possible for this three-digit dividend?

17. _____

18. A cylinder's height equals the diameter of its base. What fraction of the total surface area of the cylinder is the total area of the two bases? Express your answer as a common fraction.

18. _____

19. In how many different arrangements can the digits 1, 2, 3, 4, 5, 6 be placed in the boxes below, one per box and without repetition, so that the numbers in each row decrease from left to right and the numbers in each column decrease from top to bottom?

19. _____
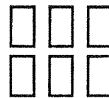
20. What is the sum of all integer values of n such that $\dfrac{20}{2n-1}$ is an integer?

20. _____

21. The sum of two fractions is $\frac{11}{12}$ and their product is $\frac{1}{6}$. What is the lesser of the two fractions? Express your answer as a common fraction.

21. _____

22. Dr. Dominguez charges each patient according to the length of the patient's visit, as indicated in the chart below. If she earned exactly $500 today, what is the positive difference between the fewest number and the greatest number of patients that she could have seen today?

22. _____

Length of visit	Cost
≤ 10 minutes	$15
$10-30$ minutes	$25
$30-60$ minutes	$40
$60-120$ minutes	$75

23. Brad bicycles from home at an average speed of 9 miles per hour until he gets a flat tire. With no way to fix the tire, Brad walks his bike back home by the same route, averaging 3 miles per hour. If the entire round trip of biking and walking took a total of 6 hours, what was Brad's average speed in miles per hour for the entire round trip? Express your answer as a decimal to the nearest tenth.

23. _____

24. Pipe A will fill a tank in 6 hours. Pipe B will fill the same tank in 4 hours. Pipe C will fill the tank in the same number of hours that it will take Pipes A and B working together to fill the tank. What fraction of the tank will be filled if all three of the pipes work together for one hour? Express your answer as a common fraction.

24. _____

25. The square with vertices $(-a, -a)$, $(a, -a)$, $(-a, a)$ and (a, a) is cut by the line $y = \frac{x}{2}$ into congruent quadrilaterals. What is the number of units in the perimeter of each quadrilateral? Express your answer in simplified radical form in terms of a.

25. _____

26. Sarah has four decks of 15 cards each. One deck is red, one is blue, one is yellow and one is green. Each deck's cards are numbered 1 through 15. Sara combines all four decks to form one large deck. When selected at random and without replacement, what is the probability that Sarah will first choose a red card with a prime number and then choose a card with a non-prime number on her second selection? Express your answer as a common fraction.

26. _____

27. Each page number of a 488-page book is printed one time in the book. The first page is page 1 and the last page is page 488. When printing all of the page numbers, how many more 4's are printed than 8's?

27. _____

28. Rectangle ABDE is inscribed in a circle. The lengths of segments AB and AE are 48 cm and 20 cm respectively. Point C is on the circle, and BC = CD. What is the number of centimeters in the perimeter of pentagon ABCDE? Express your answer in simplified radical form.
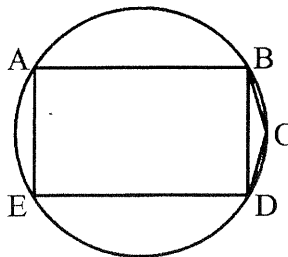
28. _____

29. Using three straight lines, what is the maximum number of regions into which the enclosed region can be divided?

29. _____

30. A fair, standard six-faced die is tossed eight times. The sequence of eight results is recorded to form an eight-digit number. What is the probability that the number formed is a multiple of eight? Express your answer as a common fraction.

30. _____