Unit: Computational Techniques **Module:** The Chain Rule

Using the Chain Rule

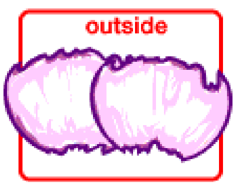
key concepts:

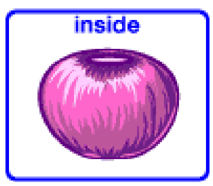
- The **chain rule** states that if $f(x) = g(h(x))$, where g and h are differentiable functions, then f is differentiable and $f'(x) = g'(h(x)) \cdot h'(x)$.
- Some functions are actually combinations of other functions, such as products or quotients. To differentiate these functions, it may be necessary to use several computational techniques and to use some more than once.

The chain rule in action

Suppose $f(x) = (3x^3 + 6x^2 - 5x)^9$. Find $f'(x)$.

$f(x) = (\text{BLOP})^9$ composite function





$$f'(x) = 9(3x^3 + 6x^2 - 5x)^8 \bullet (9x^2 + 12x - 5)$$

the derivative of the outside — the derivative of the inside (blop)

The **chain rule** is a shortcut for finding the derivative of a **composite function**.

To use the chain rule, consider the inside function as a single "blop."

Take the derivative of the outside function. Then replace the "blop" with what it was originally. Finally, multiply the entire expression by the derivative of the inside (or "blop").

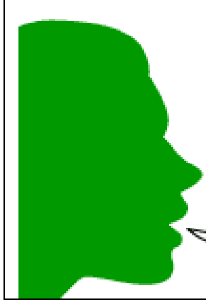
Using more than one rule together

Suppose $f(x) = (2x + 1)^4 (x^3 - 1)$. Find $f'(x)$.

$f(x) = (2x + 1)^4 (x^3 - 1)$

$f'(x) = (2x + 1)^4 \bullet (3x^2) + (x^3 - 1) \bullet [4(2x + 1)^3 \bullet (2)]$

$f'(x) = (2x + 1)^4 (3x^2) + 8(x^3 - 1)(2x + 1)^3$



Notice:
While using the **product rule** for this problem you will have to use the **chain rule**.

The product rule:
$p'(x) = f(x)g'(x) + g(x)f'(x)$

the first times the derivative of the second plus the second times the derivative of the first

Sometimes you will need to use the chain rule in the middle of some other rule, such as the product rule or quotient rule. Here is one example.

Work the problem like you normally would, chanting through the product rule. If the product rule requires you to take the derivative of a composite function, then use the chain rule to find that derivative.

It is also possible to use the chain rule more than once on a single problem. Expect to do this when taking the derivative of a function that is the composition of more than two other functions.