Calculus Lecture Notes

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Unit: An Introduction to Derivatives **Module:** Using the Derivative

Equation of a Tangent Line

key concepts:

- To find the equation of a line tangent to a curve: take the derivative, evaluate the derivative at the point of tangency to find the slope, and substitute the slope and the point of tangency into the point-slope form of a line.
- To find where the line tangent to a curve is horizontal, set the derivative equal to zero and solve for x.

A calculus question

Let $f(x) = 5x^2 - x + 3$.

Q: What is the equation of the line tangent to $f(x)$ at the point $x = 1$?

The **derivative** of $f(x)$ at x_0 represents the slope of the tangent line at x_0.

$f'(x) = 10x - 1$ Plug in $x = 1$.

$f'(1) = 10(1) - 1$

$= \boxed{9}$ slope

$f(x) = 5x^2 - x + 3$

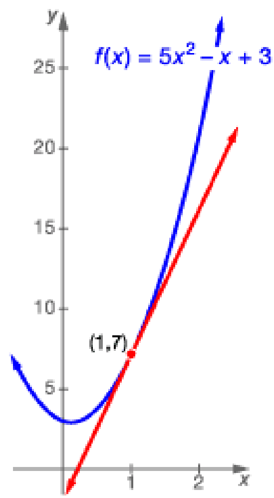
$f(1) = 5(1)^2 - (1) + 3$

$= 7$

The point of tangency is shared by both the line and the curve.

$y - y_1 = m(x - x_1)$ Plug into the point-slope form.

A: $y - 7 = 9(x - 1)$



To find the equation of a line tangent to a curve, start by taking the derivative.

Remember, the derivative evaluated at a point gives you the slope of the line tangent to the curve at that point.

Plug the point of tangency into the derivative. The result is the slope of the tangent line.

The equation of a line requires two pieces of information: the slope and a point on the line. Find the y-coordinate of the point of tangency by substituting the x-value into the function f.

Remember, the point of tangency is on the tangent line because it is the point where the line touches the curve.

Substitute the slope and the x- and y-coordinates into the point-slope form of a line to get the equation of the tangent line.

A follow–up question

Let $f(x) = 5x^2 - x + 3$. $f'(x) = 10x - 1$

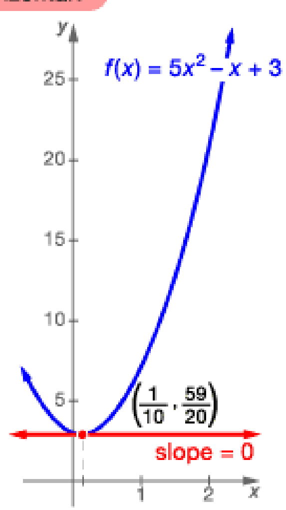
Q: At what x-value is the tangent line horizontal?

The slope of a horizontal line is equal to zero.

$f'(x) = 10x - 1$

$0 = 10x - 1$ Set the derivative equal to zero.

A: $x = \frac{1}{10}$



Horizontal tangents lead to many applications of calculus.

The derivative is a function machine that produces slopes. If you want to know where the slope of the line tangent to the curve is zero, you must set the derivative equal to zero and find which x-value makes that statement true.