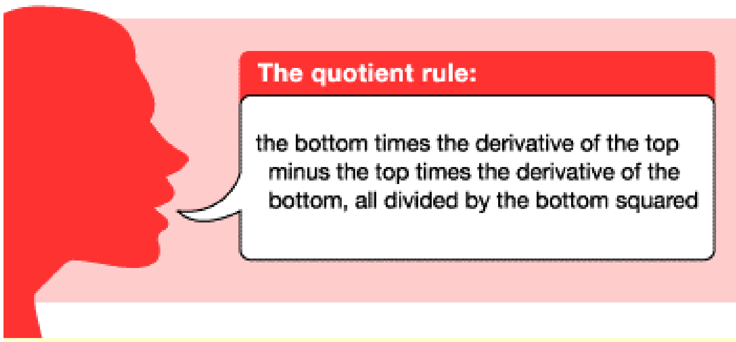
Unit: Computational Techniques **Module:** The Product and Quotient Rules

The Quotient Rule

key concepts:

- Similar to the **product rule**, the derivative of a quotient of two functions is not necessarily equal to the quotient of the derivatives.
- The **quotient rule** states that if $q(x) = f(x)/g(x)$, where f and g are differentiable functions, then q is differentiable except where $g(x) = 0$ and
$$q'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}.$$



The derivative of a quotient of two functions is not necessarily equal to the quotient of the derivatives.

Much like the **product rule**, there is a shortcut you can use to find the derivative of a quotient. The shortcut is called the **quotient rule**.

Try to remember this chant when using the quotient rule.

The quotient rule

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Here is the formula for the quotient rule.

$$\left(\frac{x^2 + x}{x}\right)' = \frac{x(2x+1) - (x^2+x)(1)}{x^2}$$
$$= \frac{2x^2 + x - (x^2 + x)}{x^2}$$
$$= \frac{\cancel{2}x^2 + x - \cancel{x^2} - x}{x^2}$$
$$= \frac{x^2 + \cancel{x} - \cancel{x}}{x^2}$$
$$= \frac{x^2}{x^2} = \boxed{1}$$

To use the quotient rule you will need to find the derivative of the numerator and the denominator. Then just combine the different pieces according to the chant.

Combining terms and canceling can often simplify the result.

Another example

Given $q(x) = \dfrac{6x^3 + 2x^2 - 14x}{3x^2 - 6}$, find $q'(x)$.

$$\left(\frac{6x^3 + 2x^2 - 14x}{3x^2 - 6}\right)' = \frac{(3x^2 - 6)(18x^2 + 4x - 14) - (6x^3 + 2x^2 - 14x)(6x)}{(3x^2 - 6)^2}$$

Here is another example of the quotient rule.

Multiply the denominator by the derivative of the numerator, subtract the numerator times the derivative of the denominator, and divide everything by the denominator squared.