Unit: Limits **Module:** Evaluating Limits

Limits and Indeterminate Forms

key concepts:

- To evaluate a **limit** at a value where a function is well behaved, substitute the value into the function expression.
- Limits that produce **indeterminate forms** may or may not exist. An indeterminate form is a signal that more work is needed to evaluate the limit
- If direct substitution produces zero divided by a non-zero number, then the limit equals zero. If it produces a non-zero number divided by zero the limit is undefined.

Evaluate $\lim_{x\to -1}\left(x^3+\sqrt{x+5}\right)$.

$$\begin{aligned}\lim_{x\to -1}\left(x^3+\sqrt{x+5}\right) &= (-1)^3+\sqrt{(-1)+5}\\ &= (-1)^3+\sqrt{4}\\ &= (-1)^3+2\\ &= -1+2\\ &= \boxed{1}\end{aligned}$$

Always try direct substitution as your first step when evaluating a **limit**.

In this example direct substitution leads to the value of limit. 2

Evaluate $\lim_{x\to 5}\dfrac{x^2-4x-5}{x^2-6x+5}$.

$$\begin{aligned}\lim_{x\to 5}\frac{x^2-4x-5}{x^2-6x+5} &= \lim_{x\to 5}\frac{(x+1)\cancel{(x-5)}}{\cancel{(x-5)}(x-1)}\\ &= \lim_{x\to 5}\frac{(x+1)}{(x-1)}\\ &= \frac{(5)+1}{(5)-1}\\ &= \frac{6}{4}\\ &= \boxed{\frac{3}{2}}\end{aligned}$$

When you first use direct substitution with this limit, it will produce the **indeterminate form** 0/0. Try factoring the numerator and denominator. When you do, you will find that they have a common factor of $x - 5$. Cancel the common factors, keeping in mind that this is a limit.

The resulting expression can be evaluated using direct substitution.

Evaluate $\lim_{x\to 3}\dfrac{x^2-x-6}{x^2+3x+2}$.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-x-6}{x^2+3x+2} &= \frac{(3)^2-(3)-6}{(3)^2-3(3)+2}\\ &= \frac{0}{20}\\ &= \boxed{0}\end{aligned}$$

✔ This expression is not an indeterminate form.

Evaluate $\lim_{x\to -1}\dfrac{x+5}{x^2-1}$.

$$\frac{(-1)+5}{(-1)^2-1} \quad \frac{4}{0}$$

✘ $\frac{4}{0}$ is undefined (not indeterminate).

✘ **Therefore the limit does not exist.**

✔ This expression is not an indeterminate form.

Before you begin factoring, make sure you really have an indeterminate form. Zero divided by any number other than zero is not indeterminate. It equals zero.

In addition, any non-zero number divided by zero is not indeterminate. It is undefined. If a limit produces this type of expression, then the limit does not exist.

A quick recap of zero

$\frac{0}{0}$	**indeterminate form** The limit needs more work.
$\frac{0}{\text{not } 0}$	The limit equals 0.
$\frac{\text{not } 0}{0}$	The limit is undefined.

Here is a summary of quotients involving zero. Only the first one is an indeterminate form.