Calculus Lecture Notes

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Unit: Special Functions **Module:** Logarithmic Functions

The Derivative of the Natural Log Function

key concepts:

- The derivative of the **natural exponential function** is itself. Use the chain rule to find the derivative of the composition of the natural exponential function and another function.
- The natural exponential function can be used to define the derivative of the **natural log function**. Use the chain rule to find the derivative of the composition of the natural log function and another function.

Differentiating the natural log function

Q: What is the derivative of the natural log function?

If $f(x) = \ln x$, then $f'(x) = ?$

Use the exponential function.

Remember: $e^{\ln x} = x$

Let $M(x) = e^{\ln x}$.	$M(x) = x$
$M'(x) = e^{\ln x} f'(x)$	$M'(x) = 1$

Use the chain rule.

$$e^{\ln x} f'(x) = 1$$

$$x f'(x) = 1$$

A: $f'(x) = \dfrac{1}{x}$ Divide by x.

You can find the derivative of the **natural log function** if you know the derivative of the **natural exponential function**.

Consider the function given by the number e raised to the power $\ln x$. The log identities prove that this expression is equal to x.

Take the derivative of both equations. Notice that the derivatives must be equal. Moreover, the derivative of this expression includes the derivative of the natural log function as one of its factors.

Isolate that factor. The result is the derivative of the natural logarithmic function.

You can use a similar process to find the derivative of any **log function**.

Using the derivative of the natural log

Example:

$f(x) = \ln x^3$

note: $\ln x^3$ does not mean $(\ln x)^3$.

$f'(x) = \dfrac{1}{\cancel{x^3}x} \cdot 3\cancel{x^2}$ Use the chain rule.

$= \boxed{\dfrac{3}{x}}$ Cancel.

A Sneaky Way:

$f(x) = \ln x^3$

$= 3\ln x$ Bring the exponent out in front.

$f'(x) = 3 \cdot \dfrac{1}{x}$

$= \boxed{\dfrac{3}{x}}$

You will often need to use the chain rule when finding the derivative of a log function.

Think of the argument of the log as a blop. The chain rule states that you must multiply the result by the derivative of the blop.

Notice that you can use the properties of logs to make finding some derivatives easier.

309 –rev 06/18/2001