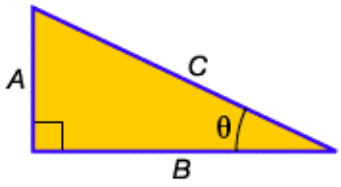
Unit: Special Functions **Module:** Trigonometric Functions

Review of Trigonometry

key concepts:

- The **Pythagorean theorem** relates the sides of a right triangle. Each **trigonometric function** matches the ratio of two sides of a right triangle to one of the angles.
- The **Pythagorean identity**: $\sin^2 x + \cos^2 x = 1$.

Trig facts



Trig functions

$\sin\theta = \frac{A}{C}$ opposite / hypotenuse

$\cos\theta = \frac{B}{C}$ adjacent / hypotenuse

$\tan\theta = \frac{A}{B}$ opposite / adjacent

Pythagorean theorem

$A^2 + B^2 = C^2$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Trig functions	Reciprocal trig functions
$\sin\theta = \frac{A}{C}$	$\csc\theta = \frac{C}{A} = \frac{1}{\sin\theta}$
$\cos\theta = \frac{B}{C}$	$\sec\theta = \frac{C}{B} = \frac{1}{\cos\theta}$
$\tan\theta = \frac{A}{B}$	$\cot\theta = \frac{B}{A} = \frac{1}{\tan\theta}$

BE CAREFUL!

The reciprocal of $\sin\theta$ is $\csc\theta$, not $\sec\theta$.

The reciprocal of $\cos\theta$ is $\sec\theta$, not $\csc\theta$.

$(\sin x)^2 = \sin^2 x$ ≠ $\sin x^2 = \sin(x^2)$

sine of x times sine of x — sine of the quantity x^2

$\sin^2 x$ accepted notation

Trigonometry examines the relationships between the lengths of the sides of a right triangle with regards to one of the acute angles of that triangle.

The most important identity to remember with regards to trigonometry is the **Pythagorean theorem**. This theorem gives rise to trigonometry.

The three most basic trig functions are the **sine function**, the **cosine function**, and the **tangent function**. Each is defined for a given angle by the ratio of the sides of a right triangle.

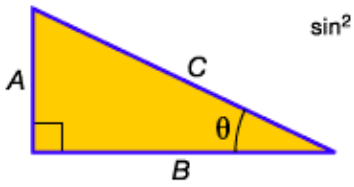
Notice that the tangent function can be expressed in terms of the sine and cosine functions.

The reciprocal trig functions are defined by reciprocal relationships with the three basic trig functions. The reciprocal functions are the **cosecant function**, the **secant function**, and the **cotangent function**.

Notice that the reciprocal of $\sin\theta$ is $\csc\theta$, not $\sec\theta$.

There is some notation that you should be aware of regarding trig functions. Notice that if a trig function is raised to a power, then the exponent can either appear after the name of the trig function or outside of the function surrounded by parentheses. If you do not include the parentheses, it is easy to confuse the meaning of the exponent.

Relationships between the trig fuctions



Pythagorean theorem

$A^2 + B^2 = C^2$

$$\sin^2\theta + \cos^2\theta = \left(\frac{A}{C}\right)^2 + \left(\frac{B}{C}\right)^2$$

$$= \frac{A^2}{C^2} + \frac{B^2}{C^2}$$

$$= \frac{A^2 + B^2}{C^2}$$ Combine the fractions.

$$= \frac{C^2}{C^2}$$ Use the Pythagorean theorem.

$$= 1$$ Simplify.

There are many relationships between the different trigonometric functions. The most fundamental is the **Pythagorean identity**.

The Pythagorean identity states that the square of the sine of any angle plus the square of the cosine of that angle is equal to 1.

You can prove the Pythagorean identity using the Pythagorean theorem.