The Derivatives of Trigonometric Functions

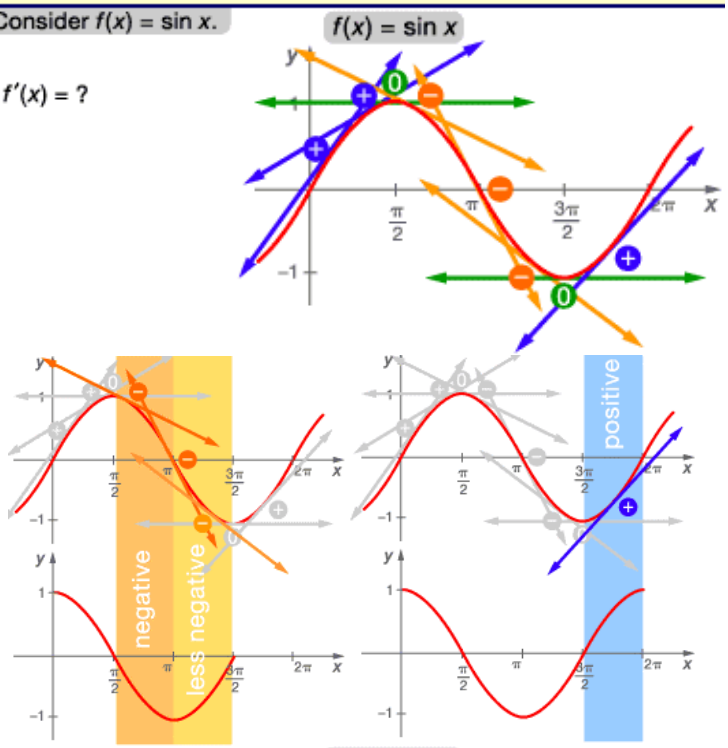
key concepts:

- If $f(x) = \sin x$, $f'(x) = \cos x$. If $f(x) = \cos x$, $f'(x) = -\sin x$.
- Use the derivatives of sine and cosine along with different differentiation techniques to find the derivatives of the other trigonometric functions.

The derivative of sine

Consider $f(x) = \sin x$.

$f'(x) = ?$

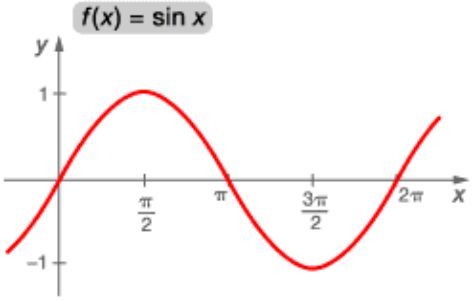


It is not clear what the derivative of the sine function is when you apply the formula for the derivative.

However, you can get a good idea what the graph of the derivative looks like by considering the way that the slopes of its tangent lines change.

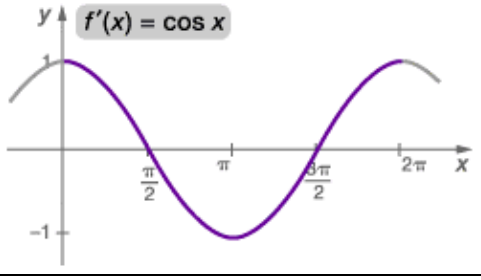
Notice that the tangent lines start with positive slopes. Then the slopes become negative. Then the slopes become positive again.

If you plot the values of the slopes on a graph, you will trace out the cosine curve.



For $f(x) = \sin x$, $f'(x) = \cos x$.

The derivative of the sine function is the cosine function.



remember
The derivative at a point represents the slope of the tangent line to the function at that point.

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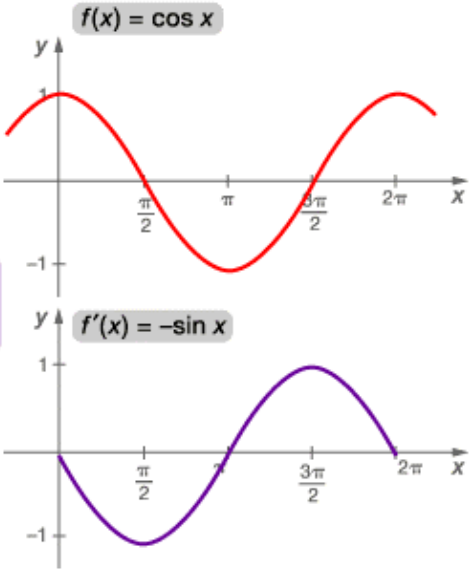
The derivative of cosine

Consider $f(x) = \cos x$.



For $f(x) = \cos x$, $f'(x) = -\sin x$.

Prof. Burger's guess: The derivative of cosine is sine.

The same process can be used on the cosine function. However, the results are a little unexpected.

The derivative of the cosine function is the negative sine function.

The derivative of tangent

Consider $f(x) = \tan x$.

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$

remember **quotient rule:**

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$f'(x) = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$ Use the quotient rule.

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

the Pythagorean identity: $\sin^2 x + \cos^2 x = 1$

$$= \frac{1}{\cos^2 x}$$ Apply the Pythagorean identity.

$$= \sec^2 x$$

For $f(x) = \tan x$, $f'(x) = \sec^2 x$.

Find the derivatives of other trigonometric functions by expressing them in terms of sine and cosine and then applying different computational techniques.

For example, the tangent function can be expressed as a quotient of the sine and cosine functions. So finding the derivative of the tangent function requires the quotient rule.

The derivative of the tangent function is the square of the secant function.