Unit: Computational Techniques **Module:** The Power Rule

Uses of the Power Rule

key concepts:

- The **power rule** states that if *N* is a rational number, then the function $f(x) = x^N$ is differentiable and $f'(x) = Nx^{N-1}$.
- Given a differentiable function *f* and a constant *c*, the **constant multiple rule** states that $[c \cdot f(x)]' = c[f'(x)]$.
- Given two differentiable functions *f* and *g*, the **sum rule** states that $[f(x) + g(x)]' = f'(x) + g'(x)$.

The power rule in action

If $f(x) = x^N$, then $f'(x) = Nx^{N-1}$.

Find the derivative.

$f(x) = x^2$

$f'(x) = 2x^1$

$= 2x$

$f(x) = x^{2000}$

$f'(x) = 2000x^{1999}$

$f(x) = x^1$

$f'(x) = 1x^0$

$= 1 \cdot 1 = 1$

$f(x) = 15x^0$

$f'(x) = 0$

> The derivative of a constant function is zero.
> The derivative of a linear function is a constant.
> For example: $f(x) = 3x$
> $f'(x) = 3$

The **power rule** allows you to find the derivative of certain functions without having to use the definition of the derivative.

To use the power rule, copy the exponent in front of the function and reduce the power by one.

Notice that the power rule works for strange powers as well, such as 1 and 0.

Remember, the derivative of a constant function is zero. The derivative of a linear function is a constant.

A function times a constant

Find the derivative.

$f(x) = 16x^3$

$f'(x) = 16(3x^2)$

$= 48x^2$

$g(x) = 10x^9$

$g'(x) = 10(9x^8)$

$= 90x^8$

Combining the power rule with other derivative rules makes it even more powerful. One such rule is the **constant multiple rule**.

The constant multiple rule states that the derivative of a constant times a function is equal to the constant times the derivative of the function.

The sum or difference of two functions

Find the derivative.

$f(x) = 2x^3 + 6x$

$f'(x) = 6x^2 + 6$

$f(x) = 2x^3 - 6x$

$f'(x) = 6x^2 - 6$

The **sum rule** lets you take the derivative of a function term by term.

Notice that you can use the constant multiple rule, the sum rule, and the power rule all together to find a single derivative.