

Unit: Limits

Module: The Concept of the Limit

## The Limit Laws

### key concepts:

- Since limits are just numbers, a lot of the properties of real numbers also apply to limits.
- The limit of a sum:  $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- The limit of a difference:  $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- The limit of a product:  $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- The limit of a quotient: For  $\lim_{x \rightarrow a} g(x) \neq 0$ ,  $\lim_{x \rightarrow a} \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$
- The scalar multiple rule for limits:  $\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} f(x)$
- The power rule for limits:  $\lim_{x \rightarrow a} [f(x)]^n = \left[ \lim_{x \rightarrow a} f(x) \right]^n$

### Limits of combinations of functions

You can combine known limits to make more complicated limits easier.

**Suppose You Know**  
 $\lim_{x \rightarrow a} f(x) = L$      $\lim_{x \rightarrow a} g(x) = M$

⊕ The limit of a sum of two functions is equal to the sum of the two limits.

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) = L + M$$

⊖ The limit of a difference of two functions is equal to the difference of the limits.

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x) = L - M$$

⊗ The limit of a product of two functions is equal to the product of the limits.

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = L \cdot M$$

⊘ The limit of a quotient of two functions is equal to the quotient of the limits.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}, M \neq 0$$

### Just For Fun!

The limit of a constant times a function is equal to the constant times the limit.

$$\lim_{x \rightarrow a} [c f(x)] = c \left[ \lim_{x \rightarrow a} f(x) \right] = c \cdot L$$

Taking the limit of a function is an operation, but the resulting limit is just a number. So it makes sense that limits follow a lot of the same properties that numbers do.

The limit of a sum of two functions is equal to the sum of the limits.

The limit of a difference of two functions is equal to the difference of the limits.

The limit of a product of two functions is equal to the product of the limits.

The limit of a quotient of two functions is equal to the quotient of the limits, provided that the denominator does not equal zero.

The limit of a function times a constant is equal to the constant times the limit.

In addition, the limit of a function raised to a power is equal to the limit raised to that power.