Unit: Limits **Module:** The Concept of the Limit

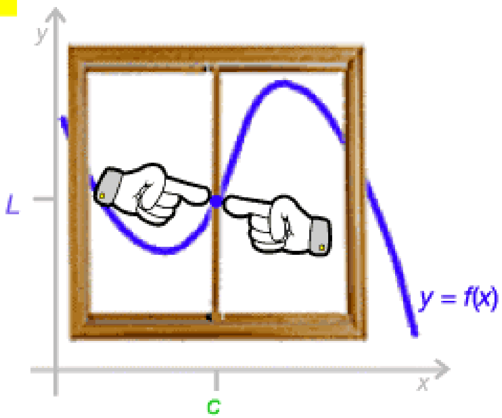
The Formal Definition of a Limit

key concepts:

- The concept of a **limit** can be expressed exactly by describing it in terms of tiny neighborhoods that are mapped around a point.
- The **formal definition of a limit**: Let f be a function defined on an open interval containing c (except possibly at c itself) and let L be a real number. If for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < | x - c | < \delta$ and $| f(x) - L | < \epsilon$ then the limit as x approaches c exists and equals L.

Intuitive idea of a limit

$\lim_{x \to c} f(x) = L$



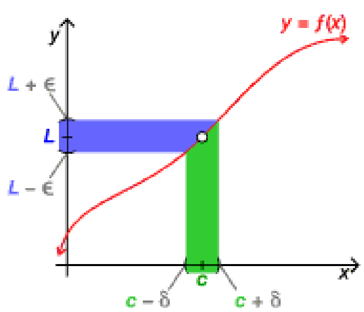
The idea of a **limit** as the value that you close in on from both directions can be intuitively described very easily.

Remember, the limit is the value that your fingers get arbitrarily close to when closing in on a particular x-value.

One of the challenges of mathematics is to take these intuitive ideas and express them formally.

The formal definition of a limit

$\lim_{x \to c} f(x) = L$



Points close to L can be found by points close to c.

Let f be a function defined on an open interval containing c (except possibly at c itself).

Let L be a real number.

If for all $\epsilon > 0$
there exists a $\delta > 0$
such that $0 < |x-c| < \delta$
and $|f(x)-L| < \epsilon$

then the limit as x approaches c exists and equals L.

So, you can get close to L by getting close to c.

Let f be a function defined on an open interval containing c (except possibly at c itself), and let L be a real number. If for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < |x - c| < \delta$ and $|f(x) - L| < \epsilon$ then the limit as x approaches c exists and equals L.

The formal definition of a limit starts with a function defined on an open interval of radius δ around the x-value where you are taking the limit.

If the limit exists, then every x-value in that interval is mapped to a y-value in another interval of radius ϵ that contains the limit.

The trick is to show that shrinking one of the intervals shrinks the other interval. To do so, you must find a relationship between ϵ and δ. If the limit exists, then there will be some sort of correlation between $| x - c |$ and $| f(x) - L |$.

Once you establish that relationship, then you have found the δ (in terms of ϵ) for which the limit holds. For example, if $| f(x) - L | = 3 | x - c |$, then you can choose $\delta = \epsilon/3$. Thus, given any offset, you can select δ such that the y-value is within that offset.

Here is the formal definition of a limit.

6947 –rev 06/18/2001