Calculus Lecture Notes

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Unit: Computational Techniques **Module:** The Chain Rule

Introduction to the Chain Rule

key concepts:

- A **composite function** is made up of layers of functions inside of functions. Some techniques of differentiation become very cumbersome when applied to composite functions.
- The **chain rule** states that if $f(x) = g(h(x))$, where g and h are differentiable functions, then f is differentiable and $f'(x) = g'(h(x)) \cdot h'(x)$.

A cooking metaphor

Some functions, called composite functions, have layers that must be peeled away in order to find their derivatives.

A **composite function** is a function that results from applying a function to the results of another function.

Each different function that is applied can be thought of as a layer of the composite function.

To find the derivative of a composite function, you must look at each layer.

Looking for patterns

Let $f(x) = (3x^2 + 1)^2$. Find $f'(x)$.

$$f(x) = (3x^2 + 1)^2$$
$$= (3x^2 + 1)(3x^2 + 1)$$

$$\boxed{f'(x) = 2(3x^2 + 1)(6x)}$$

If $f(x) = (\text{BLOP})^2$,

then $f'(x) = 2(\text{BLOP}) \cdot (\text{BLOP})'$

outside′ inside′

But what if the problem had been this?

Let $f(x) = (3x^2 + 1)^{200}$. Find $f'(x)$.

$$f(x) = (\text{BLOP})^{200}$$
$$f'(x) = 200(\text{BLOP})^{199} \cdot (\text{BLOP})'$$

outside′ inside′

$$\boxed{f'(x) = 200(3x^2 + 1)^{199} \cdot (6x)}$$

The **chain rule** is a shortcut for finding the derivative of a composite function. The chain rule must be used for each layer of the composite function.

The chain rule states that the derivative of a composition of two functions is equal to the derivative of the outer function evaluated at the inner function times the derivative of the inner function.

Consider the inside of the composite function as a "blop." Take the derivative of that piece as though the "blop" was just x. Then multiply that result by the derivative of the "blop."

Notice that the chain rule can simplify the process of finding some derivatives.