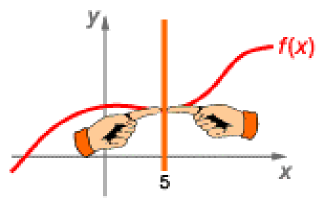
Unit: Limits **Module:** Evaluating Limits

Evaluating Limits

key concepts:

- The **limit** of a function is the range value that the function approaches as you get closer to a particular domain value.
- To evaluate a limit at a value where a function is well behaved, substitute the value into the function expression.
- Limits that produce **indeterminate forms** may or may not exist. An indeterminate form is a signal that more work is needed to evaluate the limit.

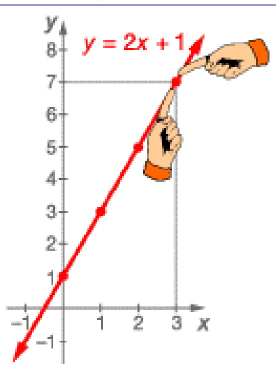


✓ If your fingers come together, then the limit exists.

notation: $\lim_{x \to 5} f(x)$

Limits allow you to study the behavior of a function near a certain x-value. If the function approaches the same value on either side of that x-value, then the limit exists.

This limit is read as "the limit as x approaches 5 of f of x."



Evaluate $\lim_{x \to 3} (2x + 1)$.

$$\lim_{x \to 3} (2x + 1) = 7$$

✓ If a function is well behaved, then the limit will equal the function at that point.

You can evaluate limits of well behaved functions by substituting the x-value into the limit expression.

Notice that the value of the function given by $y = 2x + 1$ at $x = 3$ is the same as the limit as x approaches 3 of $2x + 1$.

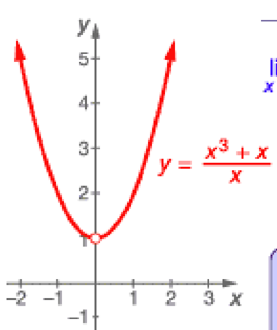
Evaluate $\lim_{x \to 0} \frac{x^3 + x}{x}$.

note: Keep writing the limit until you successfully direct substitute.

Direct substitution results in $\frac{0^3 + 0}{0} \to \frac{0}{0}$. indeterminate form

remember: Division by 0 is undefined.

The limit needs more work!



$$\lim_{x \to 0} \frac{x^3 + x}{x} = \lim_{x \to 0} \frac{x(x^2 + 1)}{x}$$

$$= \lim_{x \to 0} (x^2 + 1)$$

$$= (0)^2 + 1 \quad \text{Substitute directly.}$$

$$= 1$$

$\frac{x(x^2 + 1)}{x} = x^2 + 1$ for $x \neq 0$.

Therefore, $\lim_{x \to 0} \frac{x(x^2 + 1)}{x} = \lim_{x \to 0} (x^2 + 1)$.

For some limits, direct substitution will result in an **indeterminate form** such as 0/0. This expression cannot be evaluated since division by 0 is not defined.

An indeterminate form is a sign that you need to do more work.

In this case, you can factor the expression and cancel the x in the numerator with the x in the denominator. You can then substitute 0 in for each occurrence of x and determine the value of the limit. This limit is 1, which agrees with the graph of the function.

When you cancel you have to promise that the denominator will never be 0. However, the limit is studying the function near $x = 0$ and not at that value. Therefore direct substitution is allowed.