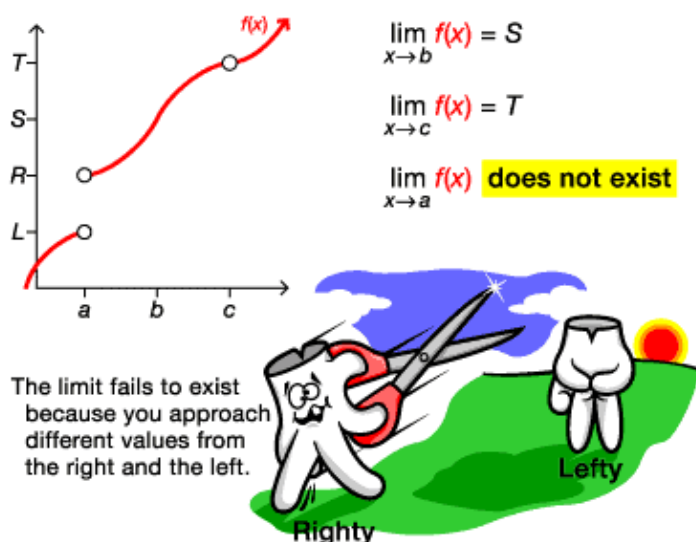**Unit:** Limits   **Module:** The Concept of the Limit

One-Sided Limits

key concepts:

● It is sometimes useful to examine limits from strictly the left or right side. Such limits are **one-sided limits**. A **left-handed limit** is the value the function approaches only from the left (increasing). A **right-handed limit** is the value the function approaches only from the right (decreasing).

● A limit exists only if the left-handed and right-handed limits both exist and are equal.

Existence of a limit

$$\lim_{x \to b} f(x) = S$$

$$\lim_{x \to c} f(x) = T$$

$$\lim_{x \to a} f(x) \quad \textbf{does not exist}$$

The limit fails to exist because you approach different values from the right and the left.
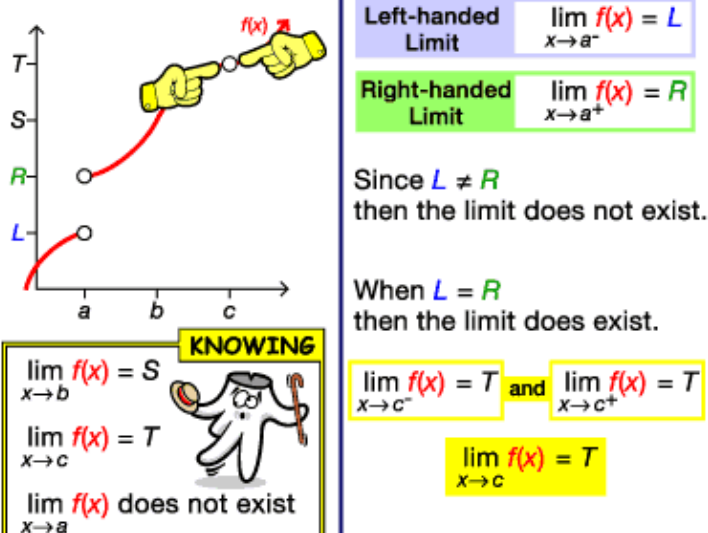
Righty   Lefty

A limit exists when you can show that the function gets arbitrarily close to a certain point. It is important to note that the definition of a limit requires the function to approach the same point from both sides.

Sometimes it is useful to consider the behavior of a function on only one side of a point. The value a function approaches on such an interval is called a **one-sided limit**.

A one-sided limit can be considered to the left or to the right of the point in question. A limit to the left of the point is called a **left-handed limit**. A limit to the right of the point is called a **right-handed limit**.

Notice that if the left-handed limit and the right-handed limit disagree then you have shown that the limit does not exist because the function approaches different values from the two sides.

One-sided limits

Left-handed Limit	$\lim\limits_{x \to a^-} f(x) = L$
Right-handed Limit	$\lim\limits_{x \to a^+} f(x) = R$

Since $L \neq R$ then the limit does not exist.

When $L = R$ then the limit does exist.

KNOWING

$$\lim_{x \to b} f(x) = S$$

$$\lim_{x \to c} f(x) = T$$

$$\lim_{x \to a} f(x) \text{ does not exist}$$

$$\lim_{x \to c^-} f(x) = T \quad \textbf{and} \quad \lim_{x \to c^+} f(x) = T$$

$$\lim_{x \to c} f(x) = T$$

When working with one-sided limits, there is some notation that you need to know.

A small superscripted "+" or "−" above the domain value being approached indicates a one-sided limit. A "−" indicates a left-handed limit, since all the values used in the domain must be less than the given number. A "+" indicates a right-handed limit because all the values used must be greater than the given number.

If the left-handed limit and the right-handed limit both exist and agree, then the limit of the function is equal to the two one-sided limits.