Unit: Introduction to Derivatives **Module:** Using the Derivative

Slope of a Tangent Line

key concepts:

- To find the slope of a tangent line, evaluate the derivative at the point of tangency.
- The derivative of f at x is given by $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ provided the limit exists.

Finding the slope of a tangent line

Example: Find the slope of the line tangent to $f(x) = 2x^2$ at $x = 3$.

Taking the derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2[x^2 + 2x\Delta x + (\Delta x)^2] - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 2x^2}{\Delta x}$$

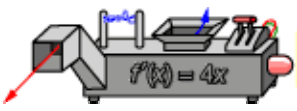
$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x} \quad \frac{0}{0} \text{ culprit}$$

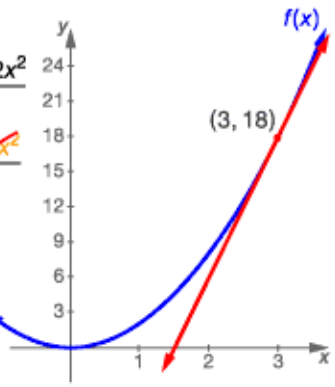
$$= \lim_{\Delta x \to 0} (4x + 2\Delta x)$$

$$= 4x + 0$$

$$f'(x) = 4x$$



slope-producing machine



To find the slope of a line tangent to a curve at a given point, it is necessary to take the derivative.

Start with the definition of the derivative.

Substitute the function into the definition.

Expand the expression so you can find pieces that cancel.

Every term that does not have a Δx should cancel away.

Factor a Δx out of the remaining expression.

Cancel the Δx with the one in the denominator.

Now evaluate the resulting limit by direct substitution.

The resulting equation is the derivative of the function f. Notice that the derivative is not the answer to the question. There is more work to do.

Finding the slope of a tangent line

Example: Find the slope of the line tangent to $f(x) = 2x^2$ at $x = 3$.

Taking the derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2[x^2 + 2x\Delta x + (\Delta x)^2] - 2x^2}{\Delta x}$$

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$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$$

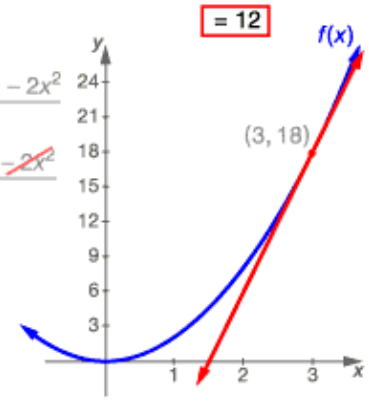
$$= \lim_{\Delta x \to 0} (4x + 2\Delta x)$$

Evaluating the slope

$$f'(x) = 4x$$

$$f'(3) = 4(3)$$

$$= 12$$



Now that you know the derivative of f, find the slope of the tangent line by plugging the point of tangency into the derivative

The resulting number is the slope of the tangent line.

The derivative gives you the slope.