

Calculus Lecture Notes

Unit: The Basics

Module: Precalculus Review

Graphing Lines

key concepts:

- A **graph** is a way of illustrating a set of ordered pairs. One of the easiest objects to graph is the line. Lines have direction, but no thickness.
- The **slope-intercept form**, $y = mx + b$, and the **point-slope form**, $(y - y_1) = m(x - x_1)$, are two means of describing lines.
- When writing the equation of a line, the point-slope form is easier to use than the slope-intercept form, because you can use any point.

Two ways to describe lines algebraically

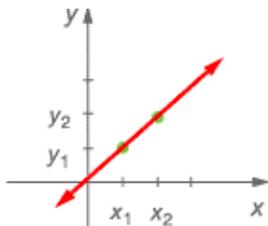
Slope-intercept form

$$y = mx + b$$

↑ slope
↑ y-intercept

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

↑ slope



You can describe a line by an equation that relates the x-values and y-values of the points on the line. One form of the equation of a line is the **slope-intercept form**. This form makes it easy to graph a line, because the y-intercept b and the slope m show up distinctly.

The slope is the pitch of the line. To calculate it, you will need two points on the line. Label their y-coordinates y_1 and y_2 respectively, and their x-coordinates x_1 and x_2 , respectively. Divide $y_2 - y_1$ by $x_2 - x_1$. If you change the order of the coordinates, you must change the order of both the x-coordinates and the y-coordinates.

EXAMPLE: A line passes through the points (3,2) and (-1,4). What is the equation of this line in slope-intercept form?

Finding the slope

$$m = \frac{4 - 2}{-1 - 3} = -\frac{1}{2}$$

Finding the y-intercept

$$y = mx + b$$

$$y = -\frac{1}{2}x + b$$

$$(4) = -\frac{1}{2}(-1) + b$$

$$4 = \frac{1}{2} + b$$

$$b = 4 - \frac{1}{2}$$

$$b = \frac{7}{2}$$

$$y = mx + b$$

↑ slope
↑ y-intercept

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Here is an example that gives two points and asks you to find the equation of the line passing through them.

First calculate the slope. You can think of it as the difference of the y-values divided by the difference of the x-values.

Neither of the points you are given is the y-intercept, so you will have to calculate it. Since the line must pass through $(-1, 4)$, you can substitute those coordinates in place of x and y . This leaves b as the only unknown.

After you solve for b , substitute its value and the slope into the slope-intercept form.

The point-slope form

$$(y - y_1) = m(x - x_1)$$

↑ slope
↑ x-coordinate of a point on the line

↑ y-coordinate of a point on the line

If your goal is to write the equation of a line, you may find it easier to use the **point-slope form**. First, use any two points to calculate the slope. Then use the coordinates of any point on the graph to arrive at the equation of the line.