Calculus Lecture Notes

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Unit: An Introduction to Derivatives **Module:** Some Special Derivatives

The Derivative of the Reciprocal Function

key concepts:

- The derivative of $f(x) = x^{-1}$ is $f'(x) = -x^{-2}$.
- To find the equation of a line tangent to a curve: take the derivative, evaluate the derivative at the point of tangency to find the slope, and substitute the slope and the point of tangency into the point-slope form of a line.

Finding the slope

Challenge: Find the equation of the tangent line at $x = 3$.

Let $f(x) = \frac{1}{x}$.

Finding the derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

Find a common denominator.

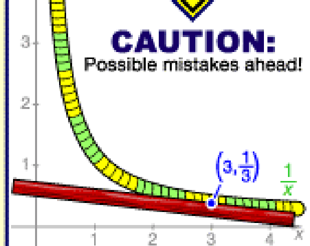
$$= \lim_{\Delta x \to 0} \frac{\left(\frac{x}{x}\right)\frac{1}{x+\Delta x} - \left(\frac{x+\Delta x}{x+\Delta x}\right)\frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x}{x(x+\Delta x)} - \frac{x+\Delta x}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{-\Delta x}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = -\frac{1}{x^2}$$



The **derivative** of $f(x) = \frac{1}{x}$ is $f'(x) = -\frac{1}{x^2}$.

The function $f(x) = x^{-1}$ is called the **reciprocal function** because for any value of x the function produces the reciprocal of x as its output.

Remember, $\frac{1}{x^n}$ can be expressed as x^{-n}.

To find the equation of a line tangent to the reciprocal function at a point, start by finding the derivative. Notice that you must find a common denominator for the numerator in order to evaluate the limit.

By combining the terms in the numerator it is possible to simplify the expression and cancel some Δx-terms.

The derivative of $f(x) = x^{-1}$ is $f'(x) = -x^{-2}$.

Putting it all together

Challenge: Find the equation of the tangent line at $x = 3$.

Let $f(x) = \frac{1}{x}$.

Evaluating the derivative

$f'(x) = -\frac{1}{x^2}$ slope machine

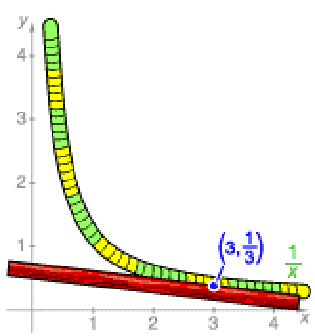
$f'(3) = -\frac{1}{3^2}$

$= -\frac{1}{9}$ slope

Using the point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$$



Once you have found the derivative of the reciprocal function, evaluate the derivative at the point of tangency to find the slope of the tangent line.

Use the slope and the point of tangency to express the equation of the line in point-slope form.

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