Calculus Lecture Notes

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Unit: Special Functions **Module:** Logarithmic Functions

Using the Derivative Rules with Transcendental Functions

key concepts:

- Some functions are combinations of other functions, such as products or quotients. To differentiate these functions, it may be necessary to use several computational techniques, possibly more than once.
- **Transcendental functions** have unusual derivatives.

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}\left[e^x\right] = e^x \qquad \frac{d}{dx}[\ln x] = \frac{1}{x}$$

Using the chain rule

Differentiate $f(x) = \sin^2(2x^3+1)$.

$$= [\sin(2x^3+1)]^2$$

LAUNDRY LIST:
- chain rule ✓
- chain rule ✓

$$f'(x) = 2[\sin(2x^3+1)]\cos(2x^3+1)(6x^2)$$

$$= \boxed{12x^2\sin(2x^3+1)\cos(2x^3+1)}$$

A **transcendental function** is a function that cannot be expressed in terms of a variable raised to a power. You can use all of the different derivative rules when working with transcendental functions.

To find the derivative of a composition of a composite function you will need to use the chain rule twice. Each additional composite function within a function will require an additional chain rule.

Notice that the exponent is the outside of this expression. Next is the sine function. The argument of the sine function is the final function.

An example with trig and exponential functions

Differentiate $g(x) = \cos(x) \bullet e^{\sin x}$.

LAUNDRY LIST:
- product rule ✓
- chain rule ✓

$$g'(x) = \cos(x) \bullet e^{\sin x} \bullet \cos x + e^{\sin x} \bullet (-\sin x)$$

$$= (\cos^2 x)e^{\sin x} - e^{\sin x}\sin x$$

$$= \boxed{e^{\sin x}(\cos^2 x - \sin x)}$$ Factor out $e^{\sin x}$.

Here two functions are combined by multiplication. In addition, the second function is a composite function. To find the derivative you will need the product rule and the chain rule.

Start with the product rule. Remember that you will need to use the chain rule when asked to find the derivative of the second piece of the product.

Using the quotient rule

Consider $y = \dfrac{\tan x}{1+\ln x}$.

Find $\dfrac{dy}{dx}$.

LAUNDRY LIST:
- quotient rule

$$\boxed{\frac{dy}{dx} = \frac{(1+\ln x)\sec^2 x - \tan x(1/x)}{(1+\ln x)^2}}$$

This function is made up of the quotient of two other functions. Notice that none of these functions are composite functions.

Be careful when finding the derivative of the tangent function. If you do not remember the formula you can derive it by converting tangent to sines and cosines.