Calculus Lecture Notes

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Unit: An Introduction to Derivatives **Module:** Understanding the Derivative

The Derivative

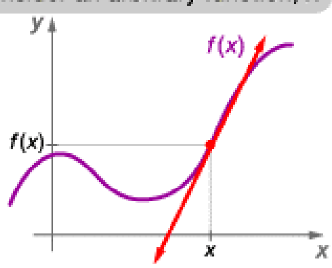
key concepts:

- **Tangent lines** are graphic representations of **instantaneous rates of change**.
- To find the slope of a tangent line, take the limit as the change in the independent variable approaches zero.
- The **derivative** is a function that gives you the instantaneous rate and slope of the tangent line at a point. The derivative got its name from the fact that it is derived from another function.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Generalizing instantaneous rate

Consider an arbitrary function, *f*.



The average rate of change between two points is equal to the slope of the secant line.

The instantaneous rate of change at a point is equal to the slope of the line tangent to the curve at that point.

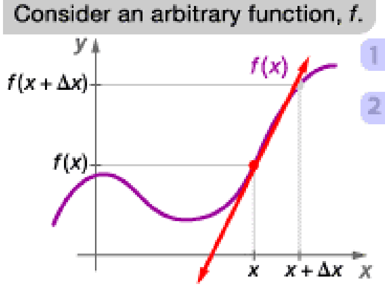
Q What is the instantaneous rate at *x*?

A The slope of the tangent line.

One way to approximate the instantaneous rate of change at a point is to calculate the average rate of between that point and another point nearby. The closer the second point is, the better the approximation will be.

As the distance between the two points diminishes to zero, the line becomes tangent to the curve. The slope of that line is the instantaneous rate of change of the function.

Consider an arbitrary function, *f*.



1 Consider the interval $[\,x, x+\Delta x\,]$.

2 Find the **average rate of change**.

$$R_{\text{average}} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

3 Take the limit as Δx approaches zero.

$$R_{\text{instant}} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

"prime"

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

"the derivative of *f*"

Here is a systematic approach for finding the instantaneous rate of change of a function *f* at a point $(x, f(x))$.

First, consider a nearby point whose *x*-coordinate is off by small amount Δx. Its coordinates will be $(x + \Delta x, f(x + \Delta x))$. Second, express the average rate of change between the two points.

Finally, use a limit to reduce the offset to zero.

The limit is called the **derivative** of *f* at *x* and is denoted by a prime symbol " ′ ". For a specific value of *x*, the derivative produces the slope of the line tangent to the function at that point. More generally, the expression defines the derivative function, which takes an *x*-value as its input and produces the slope of the corresponding tangent line.