Calculus Lecture Notes

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Unit: Computational Techniques **Module:** The Power Rule

A Quick Proof of the Power Rule

key concepts:

- In math, it is not enough to find patterns. Once you find one, it is necessary to prove that it holds in general. To prove the **power rule** for integer exponents, use the **binomial theorem** to express the general case.
- The power rule states that if N is a rational number, then the function $f(x)=x^N$ is differentiable and $f'(x)=Nx^{N-1}$.

The **power rule** states that if N is a rational number, then the function $f(x)=x^N$ is differentiable and $f'(x)=Nx^{N-1}$. If $f(x)=x^N$ then $f'(x)=Nx^{N-1}$. Proving specific cases is not difficult. The challenge is proving the general case.	If the **power rule** is true, finding derivatives will be much easier and quicker. But how do you know that the rule is true for all cases? It is not enough to see that it holds sometimes. You must prove that the rule works.
the binomial theorem: $(a+b)^n=\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}x^{n-k}y^k$ ***Proving the general case*** $f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^N-x^N}{\Delta x}$ $=\lim_{\Delta x\to 0}\frac{(x+\Delta x)(x+\Delta x)(x+\Delta x)(x+\Delta x)\cdots(x+\Delta x)-x^N}{\Delta x}$ $=\lim_{\Delta x\to 0}\frac{\cancel{x^N}+Nx^{N-1}\Delta x+(\Delta x)^2(\text{stuff})-\cancel{x^N}}{\Delta x}$ $=\lim_{\Delta x\to 0}\frac{Nx^{N-1}\Delta x+(\Delta x)^2(\text{stuff})}{\Delta x}$ $=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}\left[Nx^{N-1}+\Delta x(\text{stuff})\right]}{\cancel{\Delta x}}$ $=\lim_{\Delta x\to 0}Nx^{N-1}+\cancel{\Delta x(\text{stuff})}$ $=\boxed{Nx^{N-1}}$ There are N ways to generate a term with Δx raised to the first power. All the remaining terms will have $(\Delta x)^2$ as a factor. **Note:** The **binomial theorem** applies only to integer values of k. Therefore, the following proof demonstrates the power rule for $N\in\mathbb{N}$.	Use the **binomial theorem** to prove the power rule for integer powers. The binomial theorem illustrates how a binomial expands when raised to any given power. Consider the function $f(x)=x^N$. Notice that the binomial term raised to a power creates many additional terms that must be considered. However, the binomial theorem shows that only the first two terms are important, since all other terms will have a factor of $(\Delta x)^2$. Taking the limit after canceling the Δx-terms eliminates all of the extra terms generated by expanding the binomial. Notice that the binomial theorem only holds for integers, so this proof only works for integer powers.