Calculus Lecture Notes

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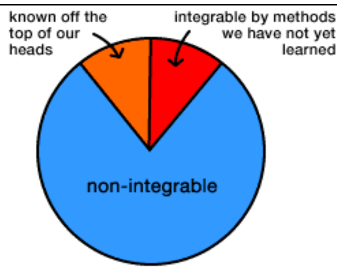
Unit: The Basics of Integration **Module:** Integration Using Tables

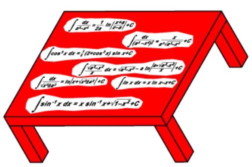
Introduction to the Integral Table

key concepts:

- The majority of functions cannot be integrated.
- We can consult integral tables when evaluating difficult integrals.
- Software applications can find the numerical value of definite integrals or the symbolic representation of indefinite integrals.



Only a small fraction of mathematical functions can be integrated. Most are non-integrable. Of the functions that can be integrated, some are easily recognized and can be integrated using obvious methods. Other functions are integrable, but are more difficult to figure out.



When faced with an integral for which there is no obvious method of integration, integral tables, which mathematicians have developed over the years, can be used.

Use an integral table to evaluate

$$\int \frac{\sqrt{4-x^2}}{x}\,dx$$

$$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C,\ a>0$$

The result is $$\int \frac{\sqrt{4-x^2}}{x}\,dx = \sqrt{4-x^2} - 2\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$$

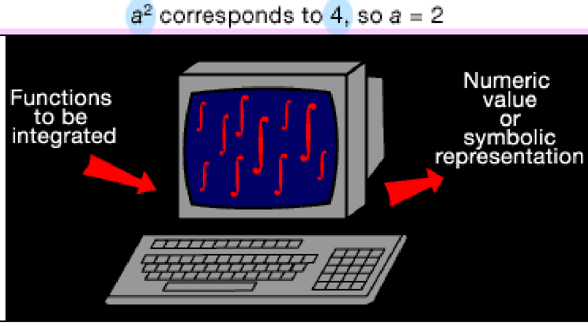
a^2 corresponds to 4, so $a = 2$

The integral in this example is complicated and not integrable by any obvious technique.

The integrand can be represented as $\frac{\sqrt{a^2+x^2}}{x}$, where a is 2.

Looking at an integral table, you can find a solution to this integral.

Substituting 2 for *a* provides the result for this difficult integral.



In the last several years, software applications have been developed which can find the numerical value of definite integrals or the symbolic representation of indefinite integrals.

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Unit: Techniques of Integration **Module:** Integration Using Tables

Making *U*-Substitutions

key concepts:

- The ***u*-substitution** method is an integration technique that undoes the chain rule.
- To use *u*-substitution, choose a *u* such that the integrand contains *u* and *du*. Then substitute and solve the simplified integral.

Choosing the u-substitution expression

Example

Evaluate $\int (3x^2-1)^{100}\, x\,dx = \int u^{100}\,\frac{1}{6}\,du$ Make the *u*-substitution.

Look for a pattern.

$=\frac{1}{6}\int u^{100}\,du$ Bring out the constant.

$=\frac{1}{6}\frac{u^{101}}{101}+C$ Don't forget the constant.

$=\frac{u^{101}}{606}+C$

Insert $3x^2-1$ in place of *u*.

$$\boxed{=\frac{(3x^2-1)^{101}}{606}+C}$$

Let $u = 3x^2-1$

$\frac{du}{dx}=6x$

$du = 6x\,dx$

$\frac{1}{6}du = x\,dx$

This gives you $x\,dx$ like you have in the integral.

An integral is a good candidate for ***u*-substitution** when you can express one part of the integrand as the derivative of the other part. Here $3x^2 - 1$ makes a good choice for *u* since its derivative is outside the parentheses.

To find out what the *du*-term should be you must take the derivative of *u*, like in the box.

Then you can move constants around so that the expression without the *du*-term matches exactly the leftover term in the integrand.

Once the terms match up, you can substitute.

Now the integral can be solved directly with the power rule. Make sure that you find the final answer in terms of *x* and not *u*.

U-substitution under a radical

Example

Evaluate $\boxed{\int \frac{x}{\sqrt{1-2x^2}}\,dx} = \int \frac{-1/4}{\sqrt{u}}\,du$ Make the *u*-substitution.

$=-\frac{1}{4}\int u^{-1/2}\,du$ Bring out the constant.

$=-\frac{1}{4}\left(2u^{1/2}\right)+C$

$=-\frac{1}{2}u^{1/2}+C$

$=-\frac{1}{2}\left(1-2x^2\right)^{1/2}+C$ Insert $1-2x^2$ in place of *u*.

$$=\boxed{-\frac{1}{2}\sqrt{1-2x^2}+C}$$

Let $u = 1-2x^2$

$\frac{du}{dx}=-4x$

$du = -4x\,dx$

$-\frac{1}{4}du = x\,dx$

This gives you $x\,dx$ like you have in the integral.

This integral seems very complicated unless you notice that the expression inside the radical makes a good choice for *u*.

Constants can be moved freely inside and outside the integral. Putting the constant outside yields an integral that you can solve directly.

Don't forget to add the constant of integration to indefinite integrals.

NOTICE: It is very important to state your answers in terms of *x* and not *u*.

Tip

Check your work by differentiating the result you get from integration.

Integration and differentiation are reverse processes. So if you want to make sure that your answer is correct just differentiate your result.