Parabolas

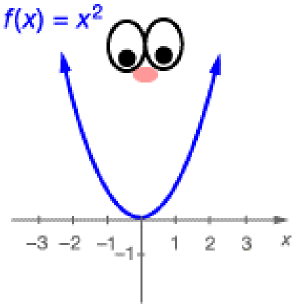
key concepts:

- The graph of a second-degree polynomial expression is a **parabola**. A parabola consists of the set of all points in a plane that are equidistant from a fixed line (the directrix) and a fixed point not on the line (the focus).
- When graphing functions, start by looking for ways to simplify their expressions. Always promise that the denominator will not equal zero when you cancel.
- The **distance formula** is an application of the **Pythagorean theorem**.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

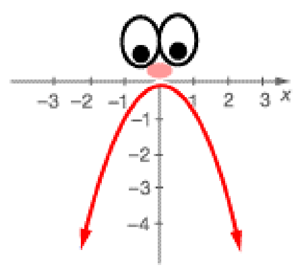
Looking at parabolas

$f(x) = x^2$



"happy-faced"
The positive coefficient makes the **parabola** open upwards.

$f(x) = -x^2$

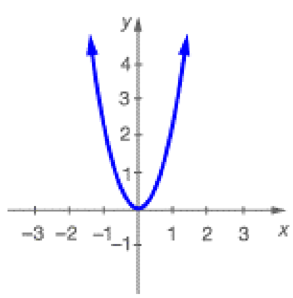


"sad-faced"
The negative coefficient makes the **parabola** open downwards.

In general, a **parabola** can be expressed by $f(x) = ax^2 + bx + c$. Each of the constants a, b, and c has a different effect on the appearance of the parabola.

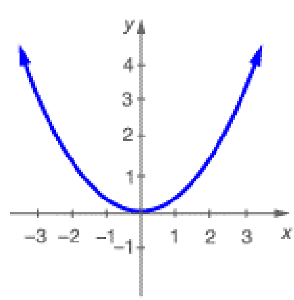
If the coefficient of the x-squared term is positive, the parabola will open upwards. You can think of it as a happy-faced parabola. If the coefficient of x-squared is negative, the parabola will open downwards as a sad-faced parabola.

$f(x) = 3x^2$



"happy-faced"
A coefficient greater than 1 makes the **parabola** tighter.

$f(x) = \frac{1}{2}x^2$



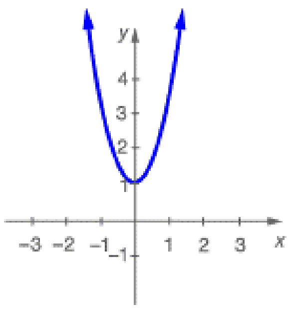
"happy-faced"
A coefficient between 0 and 1 makes the **parabola** wider.

If the coefficient of the x-squared term is greater than 1 (or less than –1), the parabola will be stretched vertically, making it look thinner and tighter.

Conversely, if the coefficient of the x-squared term is between 0 and 1 (or between 0 and –1), the parabola will be compressed vertically, making it look wider.

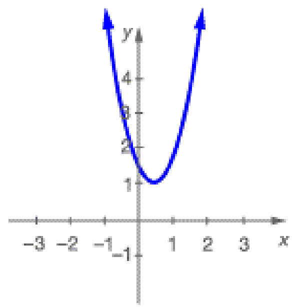
Parabolas

$f(x) = 3x^2 + 1$



"happy-faced"
Adding a constant moves the **parabola** vertically.

$f(x) = 3x^2 - 2x + 1$



"happy-faced"
Adding a multiple of x moves the **parabola** around.

The constant c has the effect of shifting the parabola vertically. For the function on the far left, the constant 1 moves the parabola upwards one unit. A negative constant would move the parabola downwards.

Finally, a coefficient of x has the effect of moving the parabola around in a manner that is a bit more difficult to predict.

Graphing functions

Sketch the graph of $f(x) = \frac{x^3 + x}{x}$.

Start by looking for ways to simplify the expression.

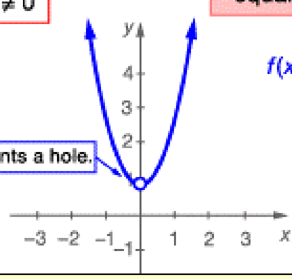
$$\frac{x^3 + x}{x} = \frac{\cancel{x}(x^2 + 1)}{\cancel{x}}, \quad x \neq 0$$

$$\boxed{f(x) = x^2 + 1, x \neq 0}$$

When you cancel, make sure to promise that the denominator will never equal zero.

$f(x) = \frac{x^3 + x}{x}$

The open dot represents a hole.

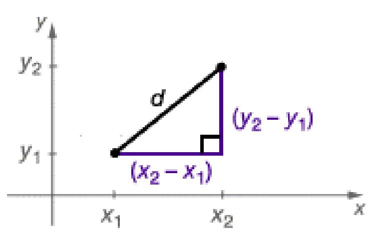


Here is a function expression that does not resemble the general form for a parabola.

Notice that you can factor x from the numerator. When you cancel it with the x in the denominator, you must agree not to evaluate the function at $x = 0$.

The expression that results from the cancellation is that of a happy-faced parabola shifted up one unit. Make sure to leave a hole in the graph at $x = 0$. The function is not defined at that point.

Calculating distances



Disregard the negative answer since d is a length and lengths must be positive.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

the **distance formula**

To determine the distance between two points (x_1, y_1) and (x_2, y_2), use the **Pythagorean theorem**. By connecting the points with a line segment you can construct a right triangle whose legs are parallel to the x- and y-axes, respectively. The lengths of the legs are given by $x_2 - x_1$ and $y_2 - y_1$. You can then solve for the length of the hypotenuse d, which is the distance between the two points.

The formula you produce is called the **distance formula**. You can either memorize the formula or use the Pythagorean theorem to derive it when you need it.