Calculus Lecture Notes

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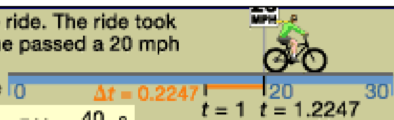
Unit: An Introduction to Derivatives **Module:** Understanding the Derivative

Rates of Change, Secants, and Tangents

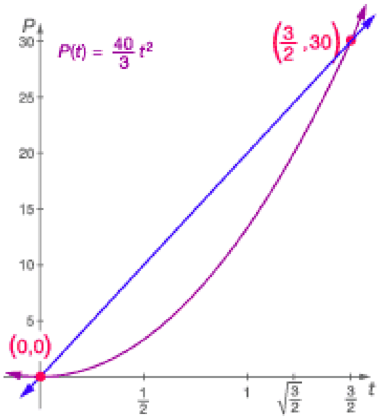
key concepts:

- Approximate **instantaneous rate of change** by finding the **average rate of change** on a small interval around the point in question.
- Represent average rate of change graphically by a **secant line**. Average rate of change is equal to the slope of the secant line between the two points being considered.
- Represent instantaneous rate of change graphically by a **tangent line**.
- To find the slope of a tangent line, take the limit of the function as the change in the independent variable approaches zero.

Prof. Burger went on a 30 mile bike ride. The ride took 1.5 hours. 20 miles into the ride, he passed a 20 mph speed limit sign. Did Prof. Burger break the law when he passed the speed limit sign?



Prof. Burger's position function: $P(t) = \frac{40}{3}t^2$



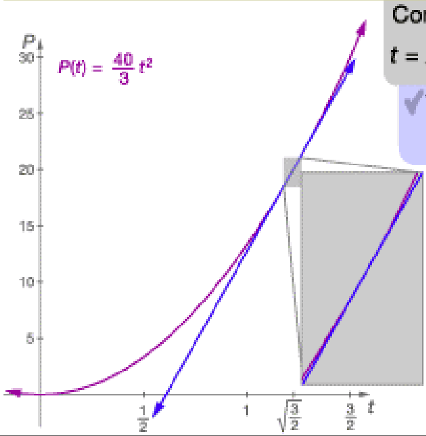
$\frac{\Delta D}{\Delta T}$ is a lot like $\frac{\Delta y}{\Delta x}$.

$$m = \frac{\Delta y}{\Delta x} = \frac{(30)-(0)}{(3/2)-(0)} = \frac{30}{3/2} = \boxed{20}$$

$$R = \frac{\Delta P}{\Delta T} = \frac{(30)-(0)}{(3/2)-(0)} = \frac{30}{3/2} = \boxed{20}$$

Another way of studying Professor Burger's bike ride is by graphing the position function. The result is a parabola.

The line connecting the starting and ending points is called a **secant line**. Notice that the slope of the secant line is the same as the **average rate of change** of position over the entire trip.



Consider the interval from $t = \sqrt{\frac{3}{2}} \approx 1.2247449$ to $t = 1.23$.

The average rate of change between two points is equal to the slope of the secant line.

$$P\left(\sqrt{3/2}\right) = \frac{40}{3}\left(\sqrt{3/2}\right)^2 = 20$$

$$P(1.23) = \frac{40}{3}(1.23)^2 = 20.172$$

$$R = \frac{P(1.23) - P\left(\sqrt{3/2}\right)}{1.23 - \sqrt{3/2}} = \frac{20.172 - 20}{1.23 - 1.2247} = \boxed{32.729932\ldots}$$

One way to turn the average rate of change into a better approximation of **instantaneous rate of change** is to reduce the length of the interval.

As you can see, this interval is so short that the secant line is almost tangent to the graph of the position function.

Calculating the slope of this secant line produces an average rate of change of 32.73 mph.

A smaller interval will produce an even better approximation.

$$\boxed{\text{instantaneous rate} = \lim_{\Delta T \to 0} \frac{\Delta D}{\Delta T}}$$

As the length of the interval becomes 0, the instantaneous rate becomes the limit of the change in the position function.