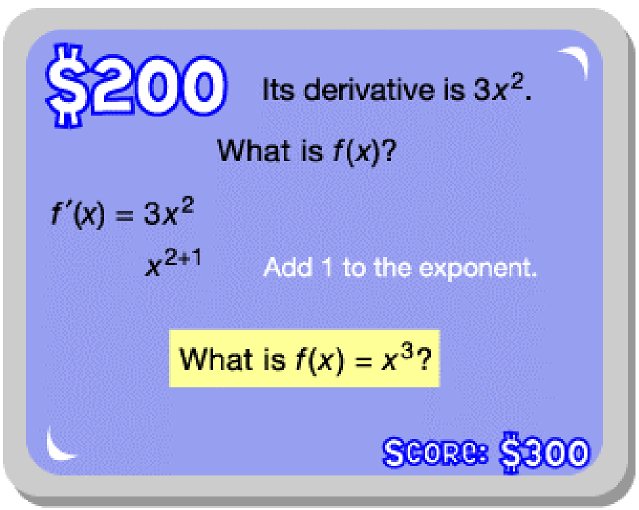
Unit: The Basics of Integration **Module:** Antiderivatives

Antidifferentiation

key concepts:

- If you want to undo the derivative, try using the derivative formulas in reverse.
- **Antidifferentiation** is a process or operation that reverses differentiation.
- Given two functions, f and F, F is an **antiderivative** of f if $F'(x) = f(x)$.

The derivative-undoing game



Suppose you are given the derivative of a function. How could you determine what the original function was?

A good start would be to think about the different differentiation formulas in reverse. For example, since the power rule for derivatives requires you to subtract one from the exponent, perhaps you should add one when trying to undo the derivative.

The beginning of integral calculus

What is an **antiderivative** of $F(x) = x^7$?

$$G(x) = \frac{1}{8}x^{7+1}$$

$$= \frac{1}{8}x^8$$

Notice:

$G'(x) = F(x)$

So G is an **antiderivative** of F.

Given two functions, f and F, F is an **antiderivative** of f if $F'(x) = f(x)$. The process or operation of finding antiderivatives is called **antidifferentiation**.

The equation that you would have had before you took the derivative is called an **antiderivative**.

There is another way to think of it. An antiderivative of a function, f, is another function, F, whose derivative, F', is equal to the first function, f.

You might think of an antiderivative as the function you had before you took the derivative. However, you do not always have to take a derivative to find an antiderivative.

The process of finding an antiderivative is called **antidifferentiation**.

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Antiderivatives of Powers of *x*

key concepts:

- Given two functions, *f* and *F*, *F* is an **antiderivative** of *f* if $F'(x) = f(x)$. **Antidifferentiation** is a process or operation that reverses differentiation.
- If a function has an antiderivative, then it has an infinite number of antiderivatives. The most general antiderivative includes an arbitrary **constant of integration**.
- The properties of antidifferentiation mirror the properties of differentiation.

Untangling the derivative

$\int f(x)dx$ instructs you to find the most general **antiderivative** of *f*.

Why do we use this symbol?

Stay tuned!

$$\int f(x)dx = F(x)$$

The derivative of *F* is *f*.

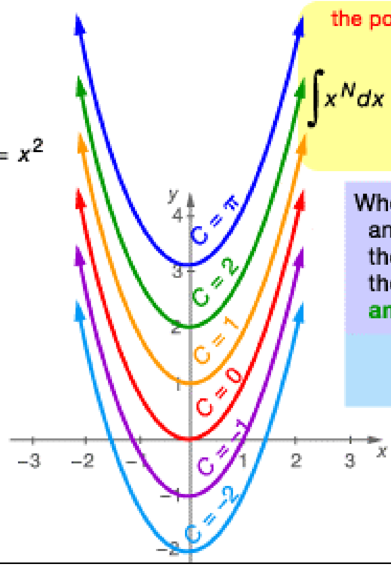
$$F'(x) = f(x)$$

F is an **antiderivative** of *f*.

This expression is called an **indefinite integral**. The integral symbol instructs you to find the most general **antiderivative**.

Using the power rule

$$\int 2x\,dx = x^2$$



the power rule for integration

$$\int x^N dx = \begin{cases} \dfrac{x^{N+1}}{N+1} \boxed{+\,C} & N \neq -1 \\ \ln|x| \boxed{+\,C} & N = -1 \end{cases}$$

When asked to evaluate an indefinite integral, the answer should be the most general **antiderivative**.

$$\int 2x\,dx = \boxed{x^2 \boxed{+\,C}}$$

most general **antiderivative**

constant of integration

Notice that a function can have many different antiderivatives.

In fact, you can add any constant to one of the antiderivatives and the result is still an antiderivative of the function.

Notice that none of these graphs describes the most general antiderivative. Each one is very specific. To describe the most general antiderivative you must add an arbitrary constant to the antiderivative.

C is called the **constant of integration**.

the **constant multiple rule** for integration: $\int cf(x)dx = c\int f(x)dx$

the **sum rule** for integration: $\int [f(x)+g(x)]dx = \int f(x)\,dx + \int g(x)\,dx$

Some of the rules for evaluating integrals are a lot like the rules for finding derivatives.

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Antiderivatives of Trigonometric and Exponential Functions

key concepts:

- Given two functions, f and F, F is an **antiderivative** of f if $F'(x) = f(x)$. **Antidifferentiation** is a process or operation that reverses differentiation.
- Discover integration formulas by looking at differentiation formulas backwards.

Antiderivatives of more exotic functions

Here are some antiderivative formulas.

Formula	Comment
$\int \cos x\,dx = \sin x + C$	The derivative of $\sin x$ is $\cos x$.
$\int \sin x\,dx = -\cos x + C$	The derivative of $\cos x$ is $-\sin x$.
$\int \sec^2 x\,dx = \tan x + C$	The derivative of $\tan x$ is $\sec^2 x$.
$\int \csc^2 x\,dx = -\cot x + C$	The derivative of $\cot x$ is $-\csc^2 x$.
$\int \tan x\,dx = ?$	Stay tuned!
$\int e^x dx = e^x + C$	The derivative of e^x is itself.
$\int a^x dx = \dfrac{a^x}{\ln a} + C$	The derivative of a^x is $a^x \ln a$.
$\int \ln x\,dx = ?$	Answer this in Calculus II.

Notice that some functions that are easy to differentiate are not as easy to integrate. It is generally the case that it is easier to differentiate than integrate.

Antiderivatives in action

$$\boxed{\text{Evaluate } \int \left[\sin x + e^x - \left(\sqrt[3]{x}\right)^2\right] dx.}$$

To evaluate this indefinite integral, start by applying the sum rule.

$$\int \left[\sin x + e^x - \left(\sqrt[3]{x}\right)^2\right] dx = \int \sin x\,dx + \int e^x dx + \int -\left(\sqrt[3]{x}\right)^2 dx$$

Now you can evaluate the integral of each term individually.

$$= -\cos x + e^x + \int -\left(\sqrt[3]{x}\right)^2 dx$$

$$= -\cos x + e^x - \frac{3}{5}x^{5/3} + C$$

Remember, when using the power rule for integration, you must multiply by the reciprocal of the new exponent.

check your work:

$$\frac{d}{dx}\left[-\cos x + e^x - \frac{3}{5}x^{5/3} + C\right] = \sin x + e^x - \frac{5}{3}\left(\frac{3}{5}x^{2/3}\right)$$

$$= \sin x + e^x - x^{2/3}$$

You can always check that your answer is correct by taking the derivative.