

# Calculus Lecture Notes

Unit: The Basics

Module: Overview

## Average Rates of Change

**key concepts:**

- A change in one variable is often indicated by the Greek letter **delta** ( $\Delta$ ).
- Since division by 0 is undefined, you cannot find **instantaneous rates of change** with algebra alone.

**Average rates of change**

**The first question of calculus** How do you find instantaneous rates of change (velocities)?

Did Prof. Burger break the law at the instant he passed the road sign?

$$R = \frac{\Delta D}{\Delta T}$$

$$R = \frac{30}{3/2}$$

Divide the change in distance by the change in time.

$$= 30 \cdot \frac{2}{3}$$

$$= 20 \text{ mph}$$

This is the average velocity (or rate of change) over the entire bike ride.

Suppose Professor Burger completes a 30-mile bike ride in 1.5 hours. The formula for **average rate of change** tells you that his average velocity for the entire ride is equal to the total distance covered divided by the time it took to cover that distance.

Notice that the rate equation is written differently here. That triangular symbol is the Greek letter **delta** ( $\Delta$ ). Mathematicians and scientists use delta to indicate a change in a particular variable.

The change in distance ( $\Delta D$ ) is 30 miles. The change in time ( $\Delta T$ ) is  $3/2$  hours. So the average rate of change in Professor Burger's position over the entire bike ride is equal to 20 miles per hour.

**Instantaneous rates of change**

**The first question of calculus** How do you find instantaneous rates of change (velocities)?

Did Prof. Burger break the law at the instant he passed the road sign?

**BIG PROBLEM**

$$R = \frac{\Delta D}{\Delta T}$$

$\frac{0}{0}$  because we are looking at a single point.  
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$\frac{0}{0} = ?$

~~$0 = 1$  is undefined.~~

Anything divided by 0 is undefined.

Anything divided by itself is 1.

**NOT A NUMBER!**

Now suppose that 20 miles into the ride there is a speed limit sign. If the speed limit is 20 miles per hour, did Professor Burger break the law at the instant he passed the sign?

One way to try to find **instantaneous rate of change** would be to consider the exact point where Professor Burger passed the sign.

Since you are looking at a single moment in time, the change in position is equal to zero. The change in time is also equal to zero. This is because you are looking at a single point, not an interval.

The formula for average rate will give you zero divided by zero. But what does that equal?

There are several different ways to look at  $0/0$ . But none of these ways is right. Just remember that  $0/0$  is not a number.

You will analyze  $0/0$  in depth when we discuss limits.