Unit: An Introduction to Derivatives **Module:** Using the Derivative

More on Instantaneous Rate

key concepts:

- Set the position function equal to a specific location to find the time that an object reaches that point. Substitute the specified time into the derivative of the position function to find the velocity of an object at that time.
- Set the derivative of the position function equal to zero to find when the object stops.

The first question

The red sports car is traveling, and its position P (in miles) at time t (in hours) is given by $P(t) = t^2 - 7t$. position function

Q1 When is the car 30 miles from where it started?

You need an equation that relates time and position.

Use the position function.

$$P(t) = t^2 - 7t$$

$t^2 - 7t = 30$ Set $P(t)$ equal to 30.

$t^2 - 7t - 30 = 0$ Pull all terms to one side.

$(t + 3)(t - 10) = 0$ Factor.

$\cancel{t + 3 = 0}$ $\quad t - 10 = 0$

$\cancel{t = -3}$ $\quad$ **A1** $t = 10$ hr

Suppose you are given the position function for a particular object.

To find the time that an object reaches a particular location, set the position function equal to that location and solve for t.

Notice that some answers may not make sense. For example, time cannot be negative.

Q2 What is the velocity at the very moment the car is 30 miles away?

$$P'(t) = \lim_{\Delta t \to 0} \frac{P(t + \Delta t) - P(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^2 - 7(t + \Delta t) - (t^2 - 7t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{t^2 + 2t\Delta t + (\Delta t)^2 - 7t - 7\Delta t - t^2 + 7t}{\Delta t} \quad \text{Expand.}$$

$$= \lim_{\Delta t \to 0} \frac{2t\Delta t + (\Delta t)^2 - 7\Delta t}{\Delta t} \quad \text{Simplify.}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta t(2t + \Delta t - 7)}{\Delta t} \quad \text{Factor out } \Delta t.$$

$$= \lim_{\Delta t \to 0} (2t + \Delta t - 7) \quad \text{Cancel.}$$

$$P'(t) = 2t - 7$$

$P'(10) = 2(10) - 7$ = 13 mph **A2**

QUESTION COUNTER 1 2 3

You need the derivative to find the velocity of an object at a specific time given the position function.

Start with the definition of the derivative.

Substitute the position function into the definition.

Expand the expression and cancel any terms that do not contain Δt.

Factor a Δt out of the numerator and cancel it with the denominator.

Direct substitution results in the derivative.

Now evaluate the derivative at the specific time to find the velocity of the object.

Q3 When does the car stop? **A3** $t = 3.5$ hr

The car stops when its velocity equals zero.

$$P'(t) = 2t - 7$$

$2t - 7 = 0$ To find the time when the velocity is equal to zero, set the derivative of the position function equal to zero and solve for t.

$2t = 7$

$$\boxed{t = \frac{7}{2} \text{ or } 3.5 \text{ hr}}$$

An object stops when its velocity is equal to zero.

To find the time when an object stops, set the derivative of the position function equal to zero and solve for t.