Calculus Lecture Notes

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Unit: An Introduction to Derivatives **Module:** Understanding the Derivative

Instantaneous Rate

key concepts:

- Substitute the specified time into the position function to find the location of an object at that time. Substitute the specified time into the derivative of the position function to find the velocity of an object at that time.
- The derivative of f at x is given by $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$ provided the limit exists.

The crab problem

A crab is crawling along the edge of your desk. Its location (in feet) at time t (in seconds) is given by $P(t) = t^2 + t$.

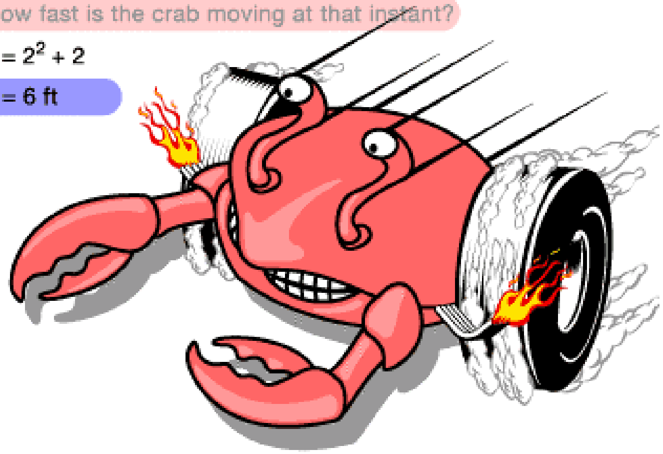
position function

Q1 Where is the crab located after 2 seconds?

Q2 How fast is the crab moving at that instant?

$P(2) = 2^2 + 2$

A1 $= 6$ ft



Suppose you are given the position function for a particular object.

To find the location of the object at a specific time, just substitute that time into the position function.

The second question

A crab is crawling along the edge of your desk. Its location (in feet) at time t (in seconds) is given by $P(t) = t^2 + t$.

Q1 Where is the crab located after 2 seconds? A1 $P(2) = 6$ ft

Q2 How fast is the crab moving at that instant?

$$P'(t) = \lim_{\Delta t \to 0} \frac{P(t+\Delta t) - P(t)}{\Delta t}$$

The **derivative** of the position function represents the velocity.

$$= \lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 + (t+\Delta t) - (t^2+t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{t^2} + 2t\Delta t + (\Delta t)^2 + \cancel{t} + \Delta t - \cancel{t^2} - \cancel{t}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{2t\Delta t + (\Delta t)^2 + \Delta t}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{\Delta t}(2t + \Delta t + 1)}{\cancel{\Delta t}}$$

$$= \lim_{\Delta t \to 0} (2t + \Delta t + 1)$$

$P'(t) = 2t + 1$

$P'(t) = 2t + 1$

$P'(2) = 2(2) + 1$

To find the velocity of an object at a specific time given its position function you need to take the derivative.

Start with the definition of the derivative.

Substitute the position function into the definition.

Expand the expression and cancel any terms that do not contain Δt.

Factor a Δt out of the numerator and cancel it with the denominator.

Direct substitution results in the derivative. Now evaluate the derivative at the specific time to find the velocity of the object.