Calculus Lecture Notes

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Unit: Computational Techniques **Module:** The Power Rule

A Shortcut for Finding Derivatives

key concepts:

- Using the definition to find the derivative of a function is very time-consuming. However, there is a shortcut when dealing with variables raised to rational powers that makes finding derivatives easier. This shortcut is called the **power rule**.
- The power rule states that if N is a rational number, then the function $f(x) = x^N$ is differentiable and $f'(x) = Nx^{N-1}$.

Looking for patterns

Q: Is there some hidden structure or pattern to the derivative?

A: Yes, there is a pattern.

function	derivative
$f(x) = 2x^2$	$f'(x) = 4x^1$
$P(t) = t^2 + t^1$	$P'(t) = 2t^1 + 1t^0$
$f(x) = 5x^2 - x^1 + 3x^0$	$f'(x) = 10x^1 - 1x^0 + 0$
$P(t) = t^2 - 7t^1$	$P'(t) = 2t^1 - 7t^0$
$f(x) = \frac{1}{x}$	$f'(x) = -\frac{1}{x^2}$
$f(x) = \sqrt{x} = x^{1/2}$	$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

For each term:

- ✓ Multiply the coefficient by the original exponent.
- ✓ Subtract 1 from the exponent.

This is a table of some functions and their derivatives.

If you look carefully, you can see a pattern between the powers of the terms of the function and the powers of the terms of the derivative.

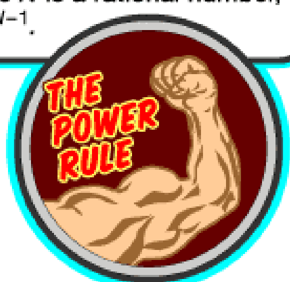
You can also find a pattern between the powers of the terms of the functions and the constants of the terms of the derivatives.

In each case, the power of the term of the derivative is one less than the power of the corresponding term of the function.

Also, the constant multiple of each term of the derivative is equal to the constant multiple of the corresponding term of the function times the power of that term.

The shortcut

If $f(x) = x^N$ where N is a rational number, then $f'(x) = Nx^{N-1}$.



Inspirational Example

$f(x) = x^{100}$

$f'(x) = 100x^{100-1}$

$= \boxed{100x^{99}}$

This pattern gives rise to a shortcut called the **power rule**.

The power rule works on any term made up of a variable raised to a rational power.

To use the power rule, take the exponent of the original term and multiply it by the term. Then reduce the exponent by one.