Calculus Lecture Notes

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Unit: Limits **Module:** Evaluating Limits

Two Techniques for Evaluating Limits

key concepts:

- When evaluating the limit of a compound fraction, try to simplify the fraction by finding the lowest common denominator.
- An expression involving a binomial can often be simplified by multiplying by the **conjugate** of the binomial. Given a binomial expression (*a* + *b*), the conjugate is the expression (*a* – *b*).

The lowest common denominator

Evaluate $\lim_{x\to 0} \dfrac{\frac{1}{x+1}-1}{x}$.

✓Look for ways to simplify fractions.

Attempting direct substitution with this limit results in an indeterminate form.

This expression is a compound fraction; it has a fraction in the numerator. You will need to simplify the numerator by finding a common denominator.

$$\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{x} = \lim_{x\to 0} \frac{\frac{1}{x+1}-\frac{x+1}{x+1}}{x}$$

Find the lowest common denominator of the numerator.

$$= \lim_{x\to 0} \frac{\frac{\cancel{1}-x\cancel{-1}}{x+1}}{x}$$

$$= \lim_{x\to 0} \frac{\frac{-x}{x+1}}{\frac{x}{1}}$$

Cancel.

$$= \lim_{x\to 0} \left(\frac{-\cancel{x}}{x+1}\cdot\frac{1}{\cancel{x}}\right)$$

Flip the denominator and multiply.

Dividing by a fraction is accomplished by multiplying by its reciprocal. Cancellation removes the 0/0 culprit.

$$= \lim_{x\to 0} \frac{-1}{x+1}$$

Cancel.

$$= -\frac{1}{0+1}$$

Substitute directly.

$$= -1$$

Now direct substitution produces the value of the limit.

Using the conjugate

Evaluate $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$.

Here is another limit where direct substitution results in an indeterminate form. However, there is nothing to factor and nothing to combine.

$$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} \left(\frac{\sqrt{x+4}-2}{x}\right)\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right)$$

You can remove the radical from the numerator if you multiply by its **conjugate**. The numerator is of the form *a* – *b*, so multiply the numerator and denominator by *a* + *b*. In this way you are essentially multiplying by 1.

$$= \lim_{x\to 0} \frac{x+4-\cancel{2\sqrt{x+4}}+\cancel{2\sqrt{x+4}}-4}{x\left(\sqrt{x+4}+2\right)}$$

$$= \lim_{x\to 0} \frac{x+\cancel{4}-\cancel{4}}{x\left(\sqrt{x+4}+2\right)}$$

Cancel.

$$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+4}+2\right)}$$

Cancel.

Notice that the radical is now in the denominator. You may not think that you have made any progress, but now you can cancel the factors of *x* in the numerator and denominator.

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2}$$

Cancel.

$$= \frac{1}{\sqrt{4}+2}$$

Substitute directly.

Direct substitution then produces the value of the limit.

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$