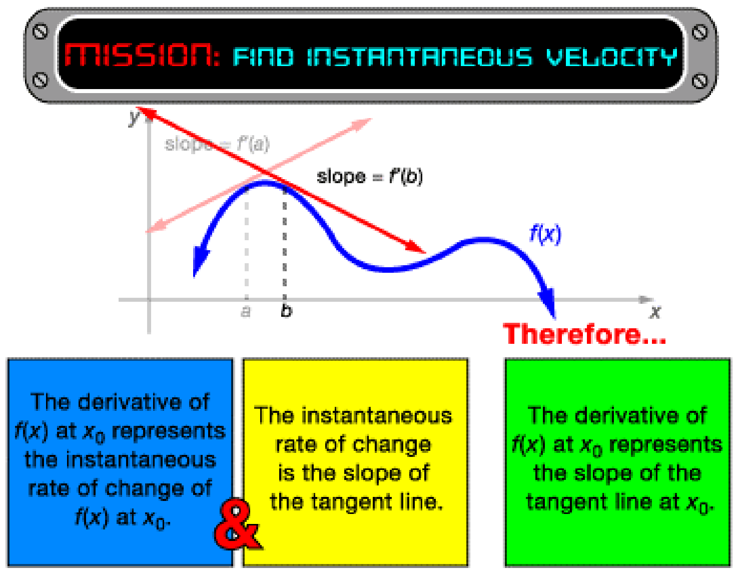
Unit: Practical Application of the Derivative **Module:** Position and Velocity

Acceleration and the Derivative

key concepts:

- **Velocity** is the rate of change of position. **Acceleration** is the rate of change of velocity.
- Tangent lines can be used to approximate functions that are difficult to evaluate. The slopes of tangent lines can be used to **optimize** outputs such as profits and areas.

Focusing on the big picture



Velocity is the rate of change of position. **Acceleration** is the rate of change of velocity. So the velocity function is the derivative of the position function and the acceleration function is the derivative of the velocity function.

The connection between instantaneous rate, the derivative, and the slope of the tangent line give rise to many different applications of differential calculus.

Using slopes of tangent lines

The equations of tangent lines help you approximate function values.

The slopes of tangent lines help you maximize profit and area.

Tangent lines can be used to approximate the values of hard-to-evaluate functions as well as the roots of functions.

The derivative can also be used to **optimize** function outputs. The behavior of tangent lines can tell you where functions attain maximum and minimum values.

Solving Word Problems Involving Distance and Velocity

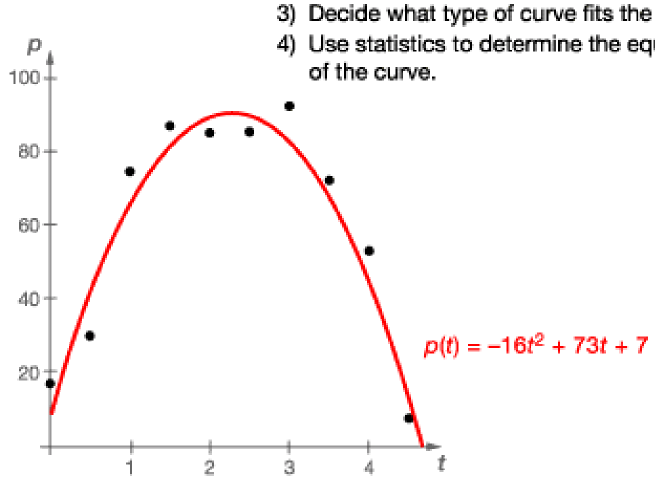
key concepts:

- By collecting and analyzing physical data, functions can be found that model physical events.
- For an object to attain a maximum position, its instantaneous velocity must equal 0.
- **Acceleration** measures the rate of change of velocity with respect to time. Acceleration is the second derivative of position and the first derivative of velocity.

Functions that model physical data

How do you find a function that describes a physical event?

Steps for modeling physical data:
1) Perform an experiment.
2) Collect data.
3) Decide what type of curve fits the data.
4) Use statistics to determine the equation of the curve.



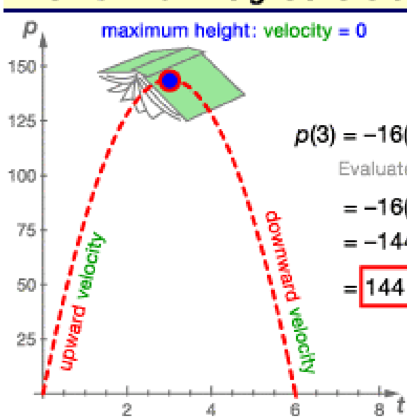
Functions are often used to mathematically describe certain events. Using an equation to describe a real world phenomenon is called mathematical modeling.

Before you can use calculus to study a situation, the situation must be modeled.

Modeling requires you to construct an experiment and gather data. Once enough data is accumulated you try to fit an equation to the data points to describe them mathematically. Entire courses are devoted to the techniques of modeling.

Scientific theories and formulas are examples of mathematical modeling.

The maximum height of the book



The position of the calculus book:

$p(t) = -16t^2 + 96t$

t is in seconds, and $p(t)$ is in feet.

$p(3) = -16(3)^2 + 96(3)$

Evaluate $p(t)$ at $t = 3$.

$= -16(9) + 96(3)$

$= -144 + 288$

$= \boxed{144 \text{ ft}}$

At $t = 3$ sec the book is at its maximum height.

Challenge:
1. What is the maximum height attained by the book? **144 ft.**
2. At what time does the book hit the ground?
3. How fast is the book moving when it hits the ground?

The altitude of a book thrown into the air can be modeled by a quadratic equation.

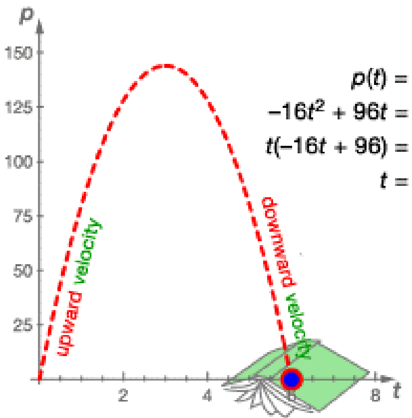
You can apply calculus to a mathematical model to discover other facts about the event.

To determine when the object reached its maximum height, you can find the derivative of the position function. When the velocity of the book changes from positive to negative the book changes from rising to falling. It is at this moment that the book is at its maximum height, namely when the slope of the tangent line is equal to 0.

Set the derivative equal to 0 to find the time that the book reaches its maximum height. Plug that time into the position function to determine the maximum height.

Solving Word Problems Involving Distance and Velocity

The book hits the ground



The position of the calculus book:

$p(t) = -16t^2 + 96t$

t is in seconds, and $p(t)$ is in feet.

$$p(t) = 0$$
$$-16t^2 + 96t = 0$$
$$t(-16t + 96) = 0 \quad \text{Factor.}$$
$$t = 0 \quad \text{or} \quad -16t + 96 = 0$$
$$-16t = -96$$
$$t = \frac{-96}{-16} = \frac{48}{8}$$
$$= \frac{24}{4} = \frac{12}{2}$$
$$\boxed{= 6 \text{ sec}}$$

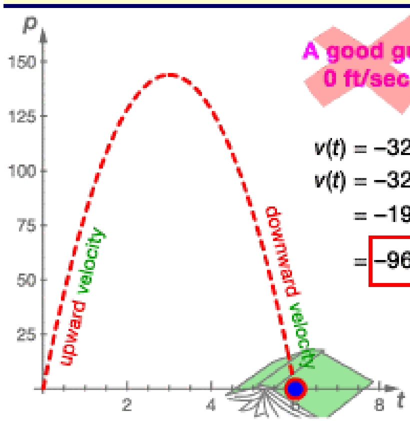
Challenge:

1. What is the maximum height attained by the book? **144 ft.**
2. At what time does the book hit the ground? **6 seconds.**
3. How fast is the book moving when it hits the ground?

The book hits the ground when its position (altitude) is equal to 0. To determine the time when this happens, set the position equation equal to 0.

Notice that it is important to consider what different actions mean when working with models. For example, when you set the variable equal to a specific time what you are actually doing is finding the position of the object at that time. Setting the position function equal to a value tells you the time at which the object was at that height.

Acceleration



~~A good guess: 0 ft/sec~~

The position of the calculus book:

$p(t) = -16t^2 + 96t$

t is in seconds, and $p(t)$ is in feet.

$$v(t) = -32t + 96$$
$$v(t) = -32(6) + 96$$
$$= -192 + 96$$
$$= \boxed{-96 \text{ ft/sec}}$$

$$a(t) = v'(t)$$
$$= \boxed{-32 \text{ ft/sec}^2}$$

Summit:

1. What is the maximum height attained by the book? **144 ft.**
2. At what time does the book hit the ground? **6 seconds.**
3. How fast is the book moving when it hits the ground? **–96 ft/sec.**

Jerk is the rate of change of acceleration with respect to time.

Finding the velocity of the book when it hits the ground requires you to know the time the book hit the ground and the velocity equation. Just plug that time into the velocity equation to find the speed.

Although the book will be traveling 0 ft/sec immediately after the book hits the ground, notice that the question asks for the instantaneous velocity the moment the book hits the ground, not immediately after.

The rate of change of velocity is called **acceleration**. Acceleration describes how the velocity of an object is changing. Find the acceleration function by taking the derivative of the velocity function.

The way that acceleration changes can also be examined. The rate of change of acceleration is called the **jerk**. The jerk function is the derivative of the acceleration function. The jerk function does not have many practical applications in motion.