

# Calculus II Lecture Notes

Unit: Special Functions

Module: Logarithmic Functions

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## Evaluating Logarithmic Functions

### key concepts:

- **Remember: Change of base theorem:**

$$\log_b x = \frac{\log_a x}{\log_a b}$$

allows revising a logarithm problem to be in a base that is easier to use in solving the problem.

- A **logarithm** indicates the exponent to which you raise a certain base to produce a given value. The inverse of a **logarithmic function** is an **exponential function**.
- Logs to the base 10 are written without a base. Logs to the base  $e$  are indicated by the symbol "ln."
- $\log_B (AC) = \log_B A + \log_B C$
- $\log_B \left(\frac{A}{C}\right) = \log_B A - \log_B C$
- $\log_B (A^C) = C \cdot \log_B A$

### The logarithm frontier

**Remember: A LOG IS AN EXPONENT.**

$$\log_B A = C$$

**Q: What does this statement mean?**

"The log to the base  $B$  of  $A$  equals  $C$ ."

$$\mathbf{A:} \log_B A = C \Leftrightarrow B^C = A$$

└ exponent  
└ base

A **logarithm** is another way of writing an equation that involves an exponential term.

Always remember that a logarithm is an exponent. Whatever the log equals is actually the exponent of the equivalent equation.

$C$  is the exponent to which you must raise  $B$  in order to get  $A$ .

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## Evaluating Logarithmic Functions

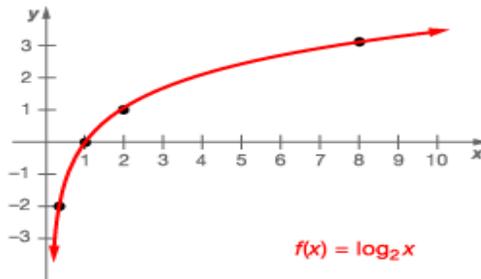
### Graphing a logarithmic function

$$\log_2 8 = 3$$

$$\log_2 \frac{1}{4} = -2$$

$$\log_2 2 = 1$$

$$\log_2 1 = 0$$



The base of a **logarithmic function** remains constant.

Graph a logarithmic function by plotting some points. Notice that for domain values between 0 and 1, this logarithmic curve produces negative range values.

Logarithmic functions are only defined for positive domain values. The logarithmic function is strictly increasing.

### Linking logarithms and exponents

$$\log_B B^A = A$$

the exponent to which you raise  $B$  to get  $B^A$

$$B^{\log_B A} = A$$

$B$  raised to the exponent to which you raise  $B$  to get  $A$

#### Remember

A log is an exponent.

$$\log_B B^A = A$$

$$B^{\log_B A} = A$$

Remember, a log is an exponent.

$\log_B B^A = A$  is a fancy way of saying “the exponent to which you must raise  $B$  to get  $B$  is  $A$ .”

$B^{\log_B A} = A$  is a fancy way of saying “ $B$  raised to the exponent to which you must raise  $B$  to get  $A$  is  $A$ .”

A log written without a base is assumed to have base 10, which is also called the **common log**.

A log with base  $e$  is called the **natural log** and is abbreviated “ln.”

It is a good idea to commit these identities to memory. You can derive them from the definition of a logarithm if you forget them.

## Evaluating Logarithmic Functions

### Properties of logarithms

$\log_B(AC) = \log_B A + \log_B C$  The log of a product is the sum of the logs.

Remember:  $N^A N^B = N^{A+B}$

$\log_B\left(\frac{A}{C}\right) = \log_B A - \log_B C$  The log of the quotient is the difference of the logs.

Remember:  $\frac{N^A}{N^B} = N^{A-B}$

$\log_B(A^c) = c \log_B A$

Remember:  $(N^A)^B = N^{AB}$

Here are some additional important logarithmic identities.

The log of a product of two numbers is equal to the sum of the log of the two numbers.

The log of a quotient of two numbers is equal to the log of the numerator minus the log of the denominator.

The log of a variable raised to a power is equal to the product of the power and the log of the variable.

Notice that there are no identities for the log of a sum or for the product of two logs.