Calculus Lecture Notes

Unit: An Introduction to Derivatives **Module:** Understanding the Derivative

Finding Instantaneous Velocity

key concepts:

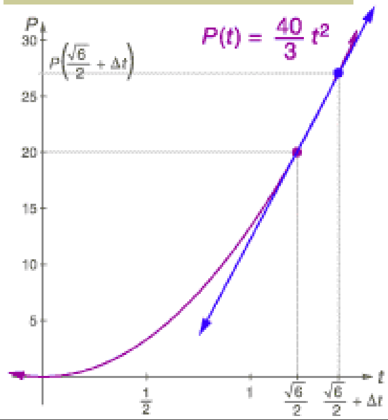
- **Average rate of change** is equal to the slope of the **secant line** between the two points being considered. **Instantaneous rate of change** is equal to the slope of the **tangent line** at the point being considered.
- By examining the average rate of change along an interval of length Δt, you can set the length to be as large or small as you like.
- To find the instantaneous rate, take the limit of the average rate on the interval $[\,t, t + \Delta t\,]$ as Δt approaches zero.

Setting up the limit

Prof. Burger went on a 30 mile bike ride. The ride took 1.5 hours. 20 miles into the ride, he passed a 20 mph speed limit sign. Did Prof. Burger break the law?

✓ Find the average velocity using a **secant line**.

✓ Find the instantaneous velocity using a **tangent line**.



$$R_{average} = \frac{P\left(\frac{\sqrt{6}}{2} + \Delta t\right) - 20}{\cancel{\frac{\sqrt{6}}{2}} + \Delta t - \cancel{\frac{\sqrt{6}}{2}}} = \frac{P\left(\frac{\sqrt{6}}{2} + \Delta t\right) - 20}{\Delta t}$$

Cancel and simplify the denominator.

You have now seen that there is a connection between rates of change and slopes of lines.

Consider the **average rate of change** between the point $\left(\frac{\sqrt{6}}{2}, 20\right)$ and a point whose x-coordinate is shifted over by a small offset, Δt. The coordinates of that point are $\left(\frac{\sqrt{6}}{2} + \Delta t, P\left(\frac{\sqrt{6}}{2} + \Delta t\right)\right)$. Substitute these values into the formula for average rate of change.

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Q Did Prof. Burger break the law?

$P(t) = \frac{40}{3}t^2$

$$R_{instant} = \lim_{\Delta t \to 0} \frac{P\left(\frac{\sqrt{6}}{2} + \Delta t\right) - 20}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\frac{40}{3}\left(\frac{\sqrt{6}}{2} + \Delta t\right)^2 - 20}{\Delta t}$$

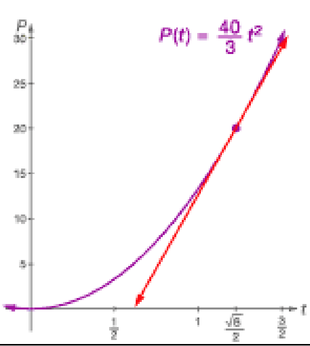
$$= \lim_{\Delta t \to 0} \frac{\left[\cancel{20} + \frac{40}{3}\sqrt{6}\,\Delta t + \frac{40}{3}(\Delta t)^2\right] - \cancel{20}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{\Delta t}\left(\frac{40}{3}\sqrt{6} + \frac{40}{3}\Delta t\right)}{\cancel{\Delta t}}$$

$$= \lim_{\Delta t \to 0} \left(\frac{40}{3}\sqrt{6} + \frac{40}{3}\Delta t\right)$$

$$= \frac{40}{3}\sqrt{6}\ \boxed{\approx 32.6598}$$

A Yes!



To find the **instantaneous rate of change**, you want to reduce the offset to 0. Set up the limit as Δt approaches 0.

Expand the limit expression and simplify wherever possible.

Cancel the Δt factors from the numerator and the denominator to remove the indeterminate form.

Evaluate the expression as Δt approaches 0.

Professor Burger was traveling at about 33 mph when he passed the 20 mph speed limit sign.