

Calculus Lecture Notes

Unit: Special Functions

Module: Trigonometric Functions

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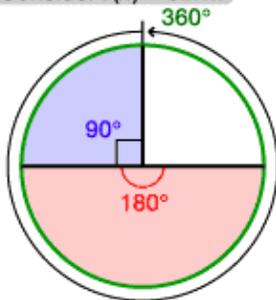
Graphing Trigonometric Functions

key concepts:

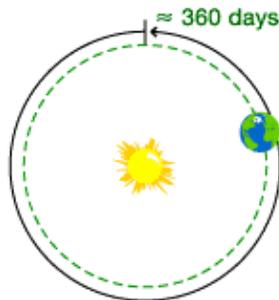
- Angles can be measured in degrees or radians. When using calculus, it is best to work with **radian measure**.
- To convert from degrees to radians, multiply by $\pi/180$. To convert from radians to degrees, multiply by $180/\pi$.
- The graphs of the trigonometric functions are periodic. Use the sine and cosine curves to extrapolate the graphs of tangent, secant, cosecant, and cotangent.

Introducing radian measure

Consider $f(x) = \sin x$.



Q Why do you use 360 as the number of degrees in a circle?



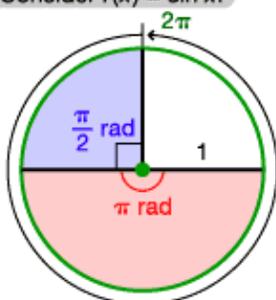
A Because there are about 360 days in a year.

In Geometry, most students used **degree measure** to describe angles.

However, degree measure is just one arbitrary way of describing an angle.

Converting from degrees to radians

Consider $f(x) = \sin x$.



example

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$5^\circ = 5 \left(\frac{\pi}{180} \right) \text{ rad}$$

$$= \frac{5\pi}{180} \text{ rad}$$

$$= \frac{\pi}{36} \text{ rad}$$

In Calculus, it is easier to use **radian measure**.

To convert between radians and degrees, it is important to remember that 360° is equal to 2π radians. From this relationship you can find any conversion you need.

There is a formula for converting between radians and degrees. To convert from degrees to radians, multiply by $\pi/180$. To convert from degrees to radians, multiply by $180/\pi$.

Degrees	Radians
360°	$2\pi \text{ rad}$
180°	$\pi \text{ rad}$
90°	$\frac{\pi}{2} \text{ rad}$
45°	$\frac{\pi}{4} \text{ rad}$

example

$$\left(\frac{180^\circ}{\pi} \right) = 1 \text{ rad}$$

$$\frac{\pi}{6} \text{ rad} = \frac{\pi}{6} \left(\frac{180^\circ}{\pi} \right)$$

$$= \frac{180^\circ}{6}$$

$$= 30^\circ$$

remember

Once around a circle is 2π radians.

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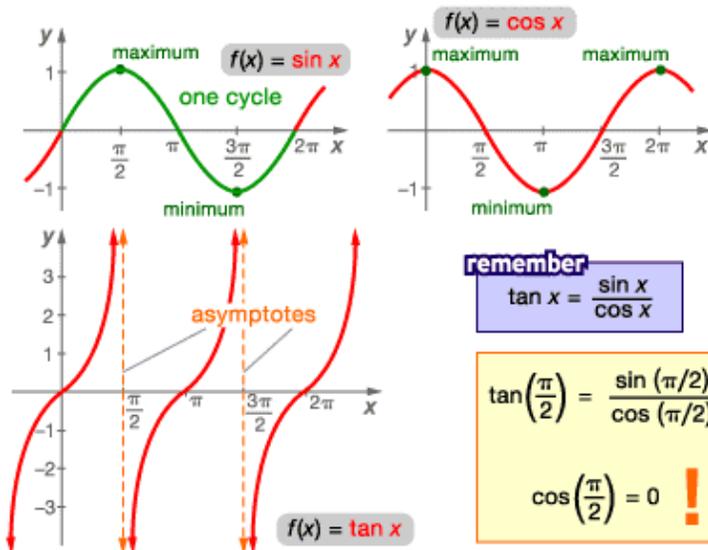
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Graphing Trigonometric Functions

Graphing the tangent function

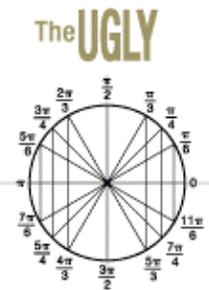


Notice that the graphs of the sine function and the cosine function are continuous. The graphs are also periodic—they repeat the same pattern over and over.

You can use the graphs of the sine function and the cosine function to graph other trig functions. Express the trig function you want to graph in terms of sine and cosine. Then evaluate that expression for different values of x .

Important values

Degrees	Radians	The GOOD The BAD	
		$\sin \theta$	$\cos \theta$
0°	0 rad	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$
30°	$\frac{\pi}{6}$ rad	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$
45°	$\frac{\pi}{4}$ rad	$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$ rad	$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
90°	$\frac{\pi}{2}$ rad	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$



Here is a table of important angles and the sine and cosine of those angles. You should either commit these values to memory or be able to derive them quickly.

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Do not forget these essential identities.