Unit: Implicit Differentiation **Module:** Implicit Differentiation Basics

Finding the Derivative Implicitly

key concepts:

- **Leibniz notation** is another way of writing derivatives. This notation will be helpful when finding the derivatives of relations that are not functions.
- **Implicit differentiation** uses the chain rule in a creative way to find the derivative of functions in implicit form.

A recap

RECALL

Leibniz notation :

$\frac{dy}{dx}$ means "the derivative of y with respect to x." (a noun)

$\frac{d}{dx}$ means "take the derivative with respect to x." (a verb)

$y = x^2$ explicit form

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = 2x$$

An equation is in **explicit form** when one variable is directly equal to an expression made up of the other variable.
An equation is in **implicit form** when neither variable is isolated on one side of the equal sign.

Leibniz notation is another way of writing derivatives.

Notice that Leibniz notation can work like an operation, instructing you to find the derivative of something.

An **explicit equation** is an equation where one of the variables is equal to an expression made up of the other variable. The equation is explicitly defined in terms of the second variable.

Explicit equations usually describe functions.

Implicit differentiation

Given $x^2 + y^2 = 1$ find $\frac{dy}{dx}$.

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x^2 + y^2) = 0$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + \frac{d}{dx}(y^2) = 0$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}\left[(y)^2\right]$$

$$= 2y \cdot \frac{d}{dx}(y)$$

$$= 2y \cdot \frac{dy}{dx}$$

Notice:
The derivative is given in terms of both x and y. There might be more than one point associated with a particular x–value.

An **implicit equation** is not organized nicely like an explicit equation. To find the derivative of an implicit equation you can either solve the equation for one of the variables (putting it in explicit form) or you can use **implicit differentiation**.

Start by taking the derivative of both sides of the implicit equation with respect to one of the variables. Work each term piece by piece.

When you encounter a term with a variable different from the one you are differentiating with respect to, treat that variable like a blop and use the chain rule.

After you have differentiated each piece, isolate the dy/dx-term.

Notice that the derivative of y with respect to x is an expression containing both x and y. When using the derivative, it is important to substitute both the x-value and the y-value into the expression.