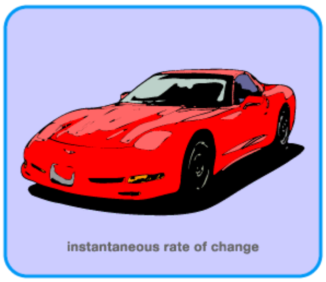
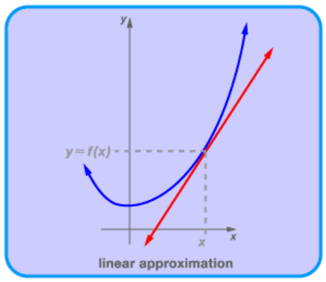
Unit: Curve Sketching **Module:** Introduction

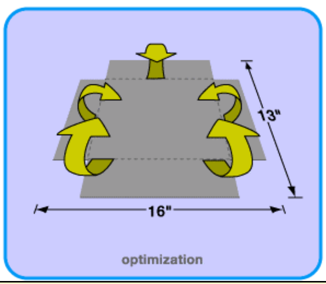
Introduction to Curve Sketching

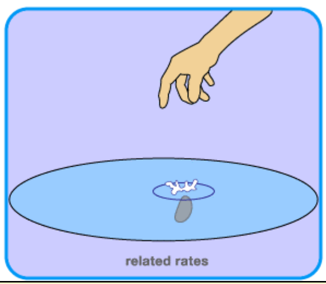
key concepts:

- Applications of the derivative include motion problems, linear approximations, optimization, related rates, and curve sketching.
- The techniques used in algebra for graphing functions do not demonstrate subtle behaviors of curves. You can use the derivative to describe the curvature of a graph more accurately.







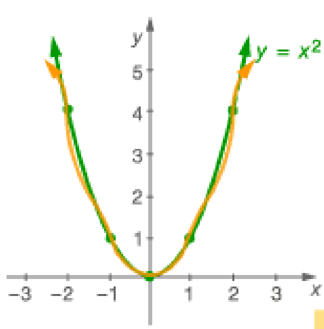


The derivative has many real-world applications, ranging from motion problems to related rates.

But the derivative is more powerful still. You can use the derivative to more accurately sketch the graph of a function.

The parabola

Graph the parabola $y = x^2$.



x-value	y-value
0	0
1	1
2	4
−1	1
−2	4

What keeps the curve from wiggling?

It is not possible to know what the graph of a function looks like with certainty using only algebra.

Calculus will prove that the curve does not wiggle.

A function $f(x)$ is decreasing on a closed interval if the first derivative is negative for all x in the corresponding open interval.
A function $f(x)$ is increasing on a closed interval if the first derivative is positive for all x in the corresponding open interval.

In algebra, you probably learned to sketch functions by plotting points. Then you connected the points together, assuming that they connected smoothly and without any wiggles.

However, you cannot be sure that graphs do not wiggle with the naïve approach used in algebra.

Calculus will prove that the graph does not wiggle.

Unit: Curve Sketching **Module:** Introduction

Three Big Theorems

key concepts:

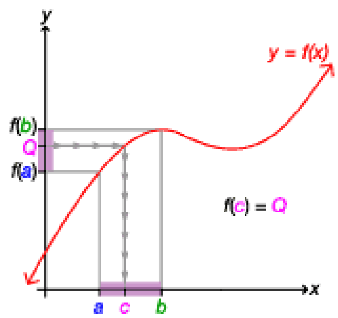
- The **intermediate value theorem**: If f is continuous on a closed interval $[a,b]$ and Q is any number between $f(a)$ and $f(b)$ then there is at least one number c in $[a,b]$ such that $f(c) = Q$.
- **Rolle's theorem**: If f is continuous on $[a,b]$, differentiable on (a,b), and $f(a) = f(b)$, then there is at least one number c in (a,b) such that $f'(c) = 0$.
- The **mean value theorem**: If f is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one number c in (a,b) such that $\dfrac{f(b)-f(a)}{b-a} = f'(c)$.

The intermediate value theorem

The intermediate value theorem deals with the values a function takes on between two points.

If a function is continuous then all intermediate values are taken on.

There must be a point c between a and b such that the function equals Q.



The **intermediate value theorem** proves that continuous functions cannot jump around. If a continuous function goes through two different points, it must also go through each and every value between the two points.

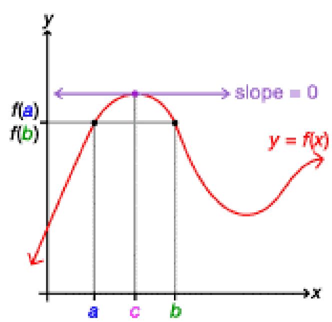
Rolle's theorem

For this theorem, the function must be both continuous (no breaks) and differentiable (smooth).

Suppose $f(a) = f(b)$.

What goes up must come down, so there must be a turning point.

There must be a point between a and b where the derivative is equal to zero.

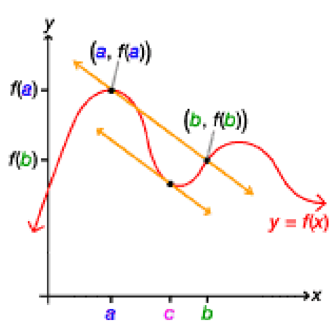


Rolle's theorem is about tangent lines. Specifically, Rolle's theorem says that if a continuous, differentiable function produces the same range value for two different domain values, then there must be a point between the two domain values where the function has a horizontal tangent.

The mean value theorem

The function must be both continuous and differentiable for the mean value theorem to apply.

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$



The **mean value theorem** is a generalization of Rolle's theorem. Instead of looking for horizontal tangents, the mean value theorem says that there will be at least one tangent line with slope equal to the slope of the secant line connecting the two specified points.