Calculus Lecture Notes

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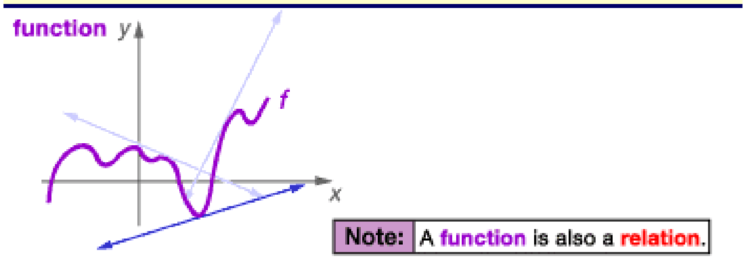
Unit: Implicit Differentiation **Module:** Implicit Differentiation Basics

Introduction to Implicit Differentiation

key concepts:

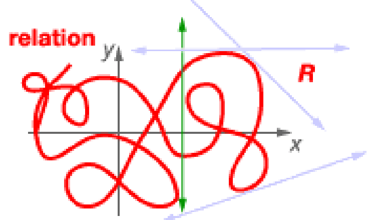
- The definition of the derivative empowers you to take derivatives of functions, not relations.
- **Leibniz notation** is another way of writing derivatives. This notation will be helpful when finding derivatives of relations that are not functions.

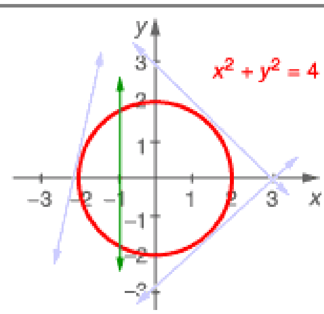
Differentiation and relations



Note: A function is also a relation.

not a function





A **function** is a set of ordered pairs in which each domain value is mapped to at most one range value.

A **relation** is a set of ordered pairs. Relations can map a single domain value to many range values.

Notice that a function counts as a relation, but a relation is not necessarily a function. A function is a special type of relation.

You can check to see if a graph represents a relation or a function by using the **vertical line test**.

You have not learned how to take the derivative of a relation. But notice that relations should have derivatives, since a relation can have a tangent line.

The circle is a common example of a relation. Notice that the circle fails the vertical line test. The circle still has tangents however.

A slow-motion derivative

Suppose $y = 3x^2 - 4x + 1$.

Find $\frac{dy}{dx}$.

Vocabulary lesson:

$\frac{d}{dx}$ means "take the derivative with respect to x."

$\frac{d}{dx}$ is a verb.

$$y = 3x^2 - 4x + 1$$

$$\frac{d}{dx}(y) = \frac{d}{dx}(3x^2 - 4x + 1)$$

Take the derivative of both sides of the equation.

$\frac{d}{dx}(y)$ means "take the derivative of y with respect to x."

$$\frac{d}{dx}(y) = \frac{dy}{dx}$$

$$\frac{d}{dx}(3x^2 - 4x + 1) = \frac{d}{dx}(3x^2) - \frac{d}{dx}(4x) + \frac{d}{dx}(1) = 6x - 4$$

$$\frac{dy}{dx} = 6x - 4$$

This method will be extendable to relations.

You can use **Leibniz notation** to make finding the derivative of a relation easier.

To use Leibniz notation, simply take the derivative of each side of the equation separately.

Notice that the derivative of y with respect to x has a special name in Leibniz notation, namely dy/dx.

Take the derivative of the right side of the equation piece by piece.

This piece-by-piece approach will also work when finding the derivative of a relation.