Calculus Lecture Notes

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Unit: An Introduction to Derivatives **Module:** Some Special Derivatives

The Derivative of the Square Root Function

key concepts:

- The derivative of $f(x) = \sqrt{x}$ is $f'(x) = \dfrac{1}{2\sqrt{x}}$.
- To find the instantaneous rate of change of an object at a specific time, substitute the time into the derivative of the position function.
- To find the time when an object is moving a particular speed, set the derivative equal to that speed and solve for the independent variable.

Instantaneous rate of change

Q Suppose the bear's position (in feet) is given by $f(x) = \sqrt{x}$ where x is in seconds. What is the instantaneous rate of change of the bear when $x = 4$?

To find the instantaneous rate of change given the position function, start by finding the derivative.

Finding the derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(\frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}\right)\left(\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}\right)$$

Multiply numerator and denominator by the conjugate of the numerator.

To find the derivative of the **square root function** you will have to multiply the numerator and the denominator by the conjugate of the numerator.

$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{(\Delta x)\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{(\cancel{\Delta x})\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Evaluating the derivative

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}}$$

Plug into the derivative to find the instantaneous rate.

A $f'(4) = \dfrac{1}{4}$ ft/sec

The derivative of $f(x) = \sqrt{x}$ is $f'(x) = \dfrac{1}{2\sqrt{x}}$.

Evaluate the derivative at a specific time to find the instantaneous rate of change at that time.

Follow-up questions

Q1 When was the bear traveling at a rate of 2 ft/sec?

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{4} \text{ ft/sec}$$

To find when an object is moving at a particular speed, set the derivative equal to that rate.

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$2 = \frac{1}{2\sqrt{x}}$$

Set the derivative equal to the speed that you are interested in.

$$\frac{1}{\sqrt{x}} = 4$$

$$1 = 4\sqrt{x}$$

$$\frac{1}{4} = \sqrt{x}$$

A1 $x = \dfrac{1}{16}$ sec

Solve the derivative for the independent variable.