

The Product Rule

- The derivative of a product of two functions is not necessarily equal to the product of the two derivatives.
- The product rule states that if $p(x) = f(x)g(x)$, where f and g are differentiable functions, then p is differentiable and $p'(x) = f(x)g'(x) + g(x)f'(x)$.

Looking for patterns

$$f(x) = (4x^3 + 1)(x^2 - 1)$$

$$f'(x) = 20x^4 - 12x^2 + 2x$$

$$f'(x) = 8x^4 + 12x^4 - 12x^2 + 2x$$

$$= 8x^4 + 2x + 12x^4 - 12x^2$$

$$= (4x^3 + 1)(2x) + (x^2 - 1)(12x^2)$$

the **first** times the
derivative of the **second**
plus the **second** times
the derivative of the **first**

Given $p(x) = (5x^3 + 6x^2 - 1)\left(3x^9 - \frac{1}{2}x + 7\right)$, find $p'(x)$.

$$p'(x) = (5x^3 + 6x^2 - 1)\left(27x^8 - \frac{1}{2}\right) + \left(3x^9 - \frac{1}{2}x + 7\right)(15x^2 + 12x)$$

To find the derivative of a product of two functions you can first simplify the functions and then find the derivative of the result.

At first glance, there does not seem to be any pattern to the derivative.

But if you decompose the product just right, you can see that the derivative is actually made up of four pieces: the original two functions and their derivatives.

The product rule is a shortcut for taking the derivative of a product of two functions. Remember the product rule by using the chant to the left.

The derivative of a product of two functions is equal to the first times the derivative of the second plus the second times the derivative of the first.

Using the product rule

Given $f(x) = (2x^4 - 7x - 3)\left(\sqrt{x} - \frac{1}{x} + 1\right)$, find $f'(x)$.

$$f'(x) = (2x^4 - 7x - 3)\left(\frac{1}{2\sqrt{x}} + \frac{1}{x^2}\right) + \left(\sqrt{x} - \frac{1}{x} + 1\right)(8x^3 - 7)$$

the **first** times the
derivative of the **second**
plus the **second** times
the derivative of the **first**

$$\sqrt{x} = x^{1/2}$$

$$\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{x} = x^{-1}$$

$$-1x^{-2} = -\frac{1}{x^2}$$

Expanding this function would take a lot of work. But the product rule makes finding the derivative easy.

You might need to do some side calculations to use the product rule.

Remember the chant! It is a simple way to remember the product rule.