Unit: Computational Techniques **Module:** The Chain Rule

Combining Computational Techniques

key concepts:

- The **chain rule** states that if $f(x) = g(h(x))$, where g and h are differentiable functions, then f is differentiable and $f'(x) = g'(h(x)) \cdot h'(x)$.
- Some functions are actually combinations of other functions, such as products or quotients. To differentiate these functions, it may be necessary to use several computational techniques and to use some more than once.

A lesson on notation

function notation	$f(x) = 3x^2$	$y = 3x^2$	y-notation
prime notation	$f'(x) = 6x$	$\frac{dy}{dx} = 6x$	Leibniz notation

slope $= \frac{\Delta y}{\Delta x}$	derivative $= \frac{dy}{dx}$

! $\frac{dy}{dx}$ means "the derivative of y with respect to x."

The **chain rule** can be expressed in Leibniz notation as $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

There are several ways to describe functions. Often the ***y*-notation** is used instead of **function notation**.

When looking for derivatives in function notation, the prime symbol (′) is often used to indicate a derivative. The prime symbol can be used when working in y-notation as well, however **Leibniz notation** has certain advantages over **prime notation**.

Notice that the connection between the derivative and slope is more apparent with Leibniz notation.

Leibniz notation is also easier to use when remembering some formulas, such as the chain rule.

Putting it all together

Let $y = \underbrace{\frac{(2x+1)^3(3x^2)}{(x^2+1)^2}}$ (quotient). Find $\frac{dy}{dx}$.

The quotient rule:
$$q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$
the bottom times the derivative of the top minus the top times the derivative of the bottom, all divided by the bottom squared

Laundry list:

Use the **quotient rule**.
- Use the **product rule** while using the **quotient rule**.
  - Use the **chain rule** while using the **product rule**.
- Use the **chain rule** while using the **quotient rule**.

The product rule:
$$p'(x) = f(x)g'(x) + g(x)f'(x)$$
the first times the derivative of the second plus the second times the derivative of the first

The chain rule:
$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$\frac{dy}{dx} = \frac{(x^2+1)^2\left[(2x+1)^3(6x) + (3x^2)(3)(2x+1)^2(2)\right] - (2x+1)^3(3x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

Notice that this function is actually a combination of several other functions. Specifically, the function involves a quotient, a product, and two composite functions.

Start with the outermost combination. You can tell which combination to start with by thinking about what you would do last by order of operations.

Once you start the **quotient rule**, you will need to take the derivative of the numerator. To do so you will need the **product rule**.

The numerator involves a **composite function**. To find its derivative you will need the **chain rule**.

Finding the derivative of the denominator will also require the chain rule.