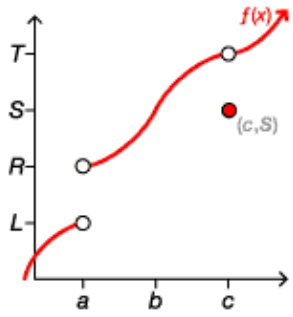
Continuity and Discontinuity

key concepts:

- A function is **continuous** at a point if it has no breaks or holes at that location.
- Three conditions must be met for a function to be continuous at a point.

Continuity



A discontinuity is a break or a rip in a curve.

A function is continuous when it doesn't have any rips or breaks.

A function *f* is **continuous** at a point *c* if:

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

? What does it mean for a function to be "nice"?

A That the function doesn't have any breaks or rips.

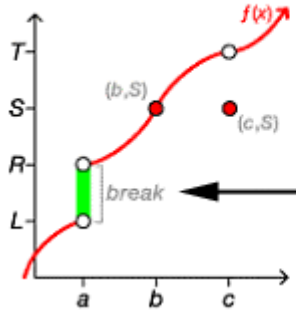
Some functions behave exactly how you expect them to. Others jump around, have points in odd places, and generally behave strangely. If the curve of a function is well behaved at a given point, then the function is said to be **continuous** at that point. Otherwise the function is **discontinous** at that point.

Three conditions must be met for a function to be continuous at a point.

1. The function must be defined at that point.
2. The limit of the function at that point must exist.
3. The function and the limit must be equal.

Although continuity is defined point by point, if a curve is continuous for all values then it is okay to say that the function itself is continuous.

Discontinuity



A break or jump discontinuity occurs when the function suddenly stops and starts again at a different value. In jump discontinuities, the limit does not exist.

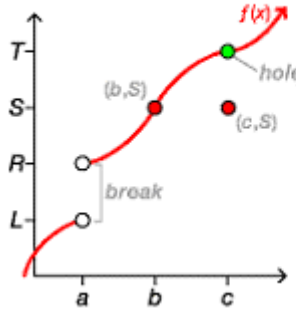
When a function has a break, the limit does not exist. This means that it has a non-removable discontinuity

There are two ways a function can be discontinuous.

The first way is called a **jump discontinuity** or a break. Jump discontinuities occur when the left-handed and right-handed limits do not agree with each other. When a function has a jump discontinuity that means its limit does not exist; therefore the discontinuity is **non-removable**.

The greatest integer function is an example of a function with jump discontinuities. Look for jump discontinuities any time you work with piecewise-defined functions.

Discontinuity



When a function has a hole, the limit exists. This means that the function has a removable discontinuity

A hole or point discontinuity occurs when there is a single point missing or strangely defined. In point discontinuities, the limit exists but the limit and function do not agree.

The second type of discontinuity is a **point discontinuity** or a hole. Point discontinuities occur when the limit exists but disagrees with the function. When a function has point discontinuity that means that the discontinuity is **removable**.

Point discontinuities are often seen when dealing with rational functions. Look for point discontinuities when dealing with piecewise-defined functions as well.