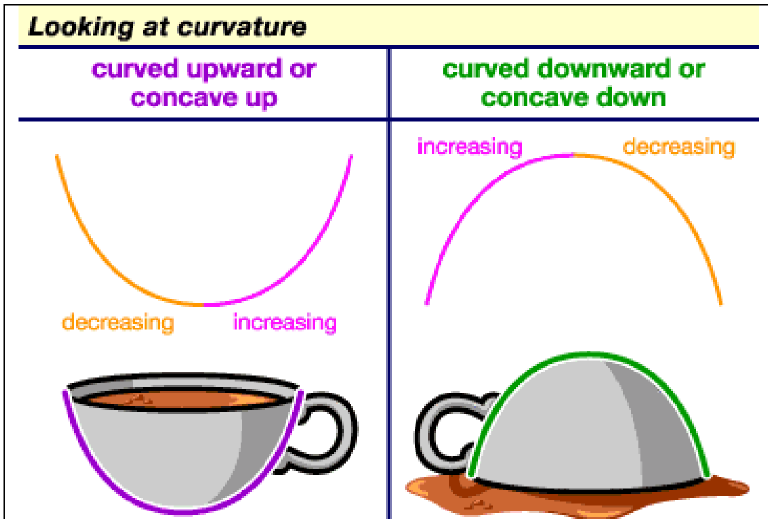
Unit: Curve Sketching **Module:** Concavity

Concavity and Inflection Points

key concepts:

- The **concavity** of a graph can be determined by using the second derivative
- If the second derivative of a function is positive on a given interval, then the graph of the function is **concave up** on that interval. If the second derivative of a function is negative on a given interval, then the graph of the function is **concave down** on that interval.
- Points where the graph changes concavity are called **inflection points**.

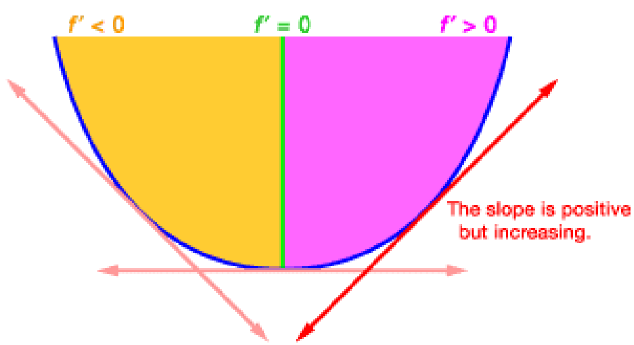


Given a function, you can determine where it is increasing and where it is decreasing. The next property to examine is curvature or **concavity**.

Notice that the graph on the left is decreasing and then increasing, but it is curved upward. The graph is said to be **concave up**. It resembles the outline of a coffee cup that is upright.

On the right, the graph is curved downward. It is said to be **concave down**. This time it resembles the outline of an overturned coffee cup.

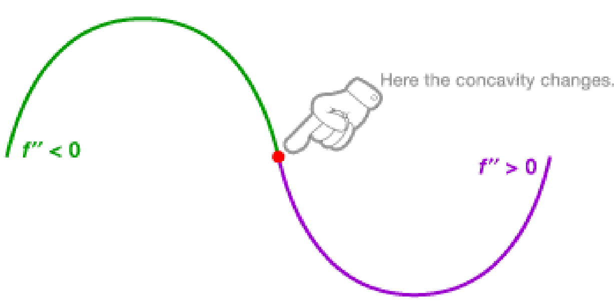
To determine the concavity of a function you will need to study the behavior of its derivative.



Notice that the slopes of the tangent lines start out negative, then become zero, and finally become positive. They are increasing. Therefore the derivative is increasing.

Another way of saying that the derivative is increasing is to say that the second derivative is positive.

You can conclude that if the second derivative is positive, then the function is concave up. Similarly, If the second derivative is negative, then the function is concave down.



This graph is concave down on the left and concave up on the right. The point where the concavity changes is called an **inflection point**.

An inflection point can only occur where the second derivative is zero or undefined.

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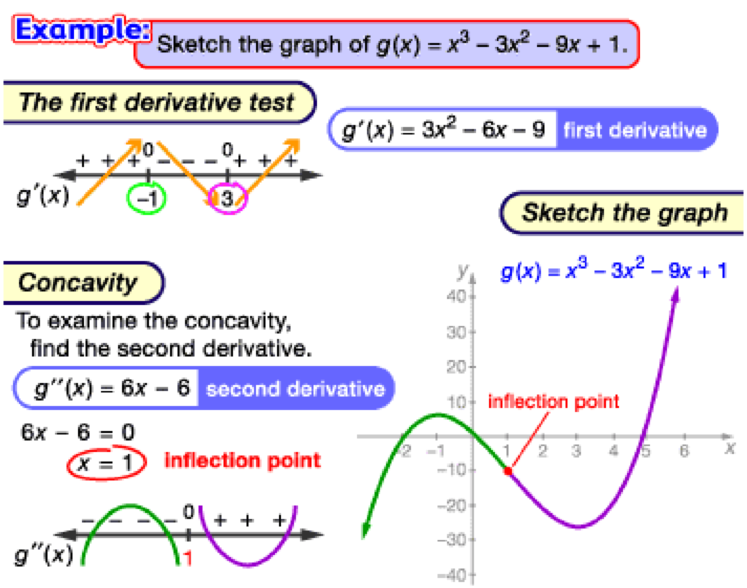
Using the Second Derivative to Examine Concavity

key concepts:

- The second derivative can be used to determine where the graph of a function is **concave up** or **concave down** and to find **inflection points**.
- Knowing the critical points, local extreme values, increasing and decreasing regions, the **concavity**, and the inflection points of a function enables you to sketch an accurate graph of that function.



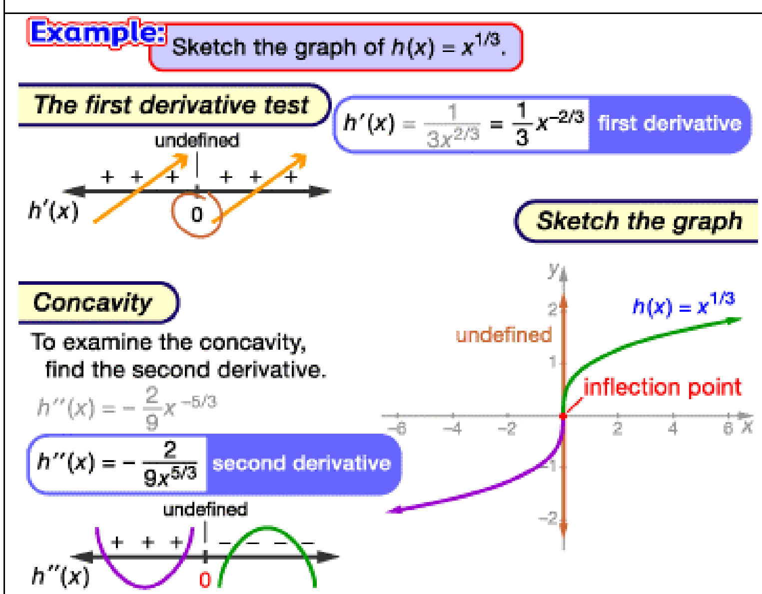
Knowing the **concavity** of a function can help you make a better sketch of its curve. Recall that the graph of a function is **concave up** if the derivative is increasing and **concave down** if the derivative is decreasing.

To determine the behavior of the derivative, you will need its derivative, i.e. the second derivative of the function.

Set the second derivative equal to zero to determine possible **inflection points**, which are characterized by a change in concavity

Then make a sign chart for the second derivative. If the second derivative is negative, then the derivative is decreasing and the function is concave down. Similarly, if the second derivative is positive, then the derivative is increasing and the function is concave up.

Since the concavity changes at the point where the second derivative equals zero, it is an inflection point, after all.



Once again, take the second derivative of the function in order to determine its curvature.

The second derivative is never equal to zero, but it is undefined at $x = 0$. An inflection point might exist there.

Make a sign chart for the second derivative. Since the function changes from concave up to concave down at $x = 0$, it is an inflection point. Notice that the tangent line at $x = 0$ has an undefined slope.