

Unit: Limits

Module: Evaluating Limits

An Overview of Limits

key concepts:

- The **limit** is the range value that a function approaches as you get closer to a particular domain value.
- An **indeterminate form** is a mathematically meaningless expression.

Limits that stay crunchy in milk

example
Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$. $\lim_{x \rightarrow 2} \frac{0}{0}$ **indeterminate form**
Direct substitution would result in $\frac{0}{0}$.

✓ Look for a way to cancel terms.

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} &= \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} \\ &= \lim_{x \rightarrow 2} (x + 2) \quad \text{Cancel.} \\ &= 4 \end{aligned}$$



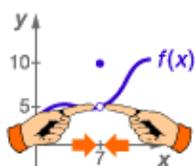
This **limit** involves an unusual variable.

Remember to use direct substitution as a first step in evaluating limits. In this case, direct substitution produces the familiar **indeterminate form** of 0/0.

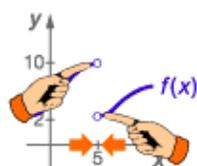
Proceed by factoring the numerator, which is a difference of two squares.

Use cancellation to simplify the limit expression and then apply direct substitution to arrive at the result.

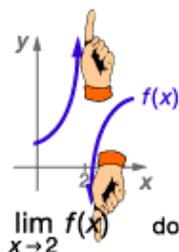
Existence of the limit



$$\lim_{x \rightarrow 7} f(x) = 5$$



$$\lim_{x \rightarrow 5} f(x) \text{ does not exist}$$



$$\lim_{x \rightarrow 2} f(x) \text{ does not exist}$$

The existence of limits can be demonstrated graphically. On the far left, the graph shows that near $x = 7$ the function is approaching the same value from both the left and the right. The limit exists and equals that value, even though the function takes on a different value at $x = 7$.

On the near left, the graph approaches different values on either side of $x = 5$. Since the two one-sided limits have different values, the limit of the function does not exist.

Here is an example of a function that is approaching very large values from the one side and very small values from the other. Therefore the limit does not exist.